

Engineering decoherence for two-qubit systems interacting with a classical environment

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We address the dynamics of a two-qubit system interacting with a classical dephasing environment driven by a Gaussian stochastic process. Upon introducing the concept of *entanglement-preserving time*, we compare the degrading effects of different environments, e.g. those described by Ornstein–Uhlenbeck (OU) or fractional noise. In particular, we consider pure Bell states and mixtures of Bell states and study the typical values of the entanglement-preserving time for both independent and common environments. We found that engineering environments towards fractional Gaussian noise is useful to preserve entanglement as well as to improve its robustness against noise. We also address entanglement sudden death by studying the *sudden-death time* as a function of the initial negativity. We found that: (i) the sudden-death time is bounded from below by an increasing function of the initial negativity, (ii) the sudden-death time depends only slightly on the process used to describe the environment and exhibits typicality. Overall, our results show that engineering the environment has only a slight influence over the entanglement sudden-death time, while it represents a valuable resource to increase the entanglement-preserving time, i.e. to maintain entanglement closer to the initial level for a longer interaction time.

Keywords: Entanglement; classical noise; Gaussian stochastic process; decoherence.

1. Introduction

The unavoidable interaction of a quantum system with its environment generally causes decoherence and a loss of quantumness. On the other hand, the possibility to

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perform quantum operations within the coherence time of a quantum system lies at the heart of quantum information processing. A deep understanding of the decoherence mechanisms in quantum systems, together with the capability to engineer the environment in order to reduce its detrimental effects, are thus essential steps toward the development of quantum technologies.

The interaction of a quantum system with its environment may be described using either a classical or a quantum mechanical picture of the latter. Understanding whether and in which conditions the two descriptions are equivalent is still a debated topic.^{1–4} When the environment has many degrees of freedom and/or a structured noise spectrum, a quantum description may be challenging, and the approximations may be crude enough to prevent a reliable description of the dynamics. In these situations, a classical description may be convenient and also more accurate. Several systems of interest belong to these categories and many efforts have been devoted to study situations where quantum systems are affected by classical noise. Examples include the dynamics of quantum correlations,^{5–13} the simulation of motional averaging,¹⁴ or decoherence in solid state qubits^{15–25} and the characterization of the environment using quantum probes.^{26,27} When the environment affecting the quantum system may be described as collection of fluctuators, a Gaussian statistics for the noise can be assumed.^{28,29} Moreover, the Gaussian approximation is valid even in the presence of non-Gaussian noise, as far as the coupling with the environment is weak.^{30,31}

In this paper, we address the dynamics of entanglement for a two-qubit system subject to a classical noise induced by a Gaussian stochastic process. Specifically, we consider the case where the typical frequencies of the system are larger compared to those of the environment, so that the system dynamics can be described as a pure dephasing.^{32–36,24} Dephasing induced by classical noise has been studied previously,^{5,37} and it is known to induce a monotonic decay of entanglement, including the phenomenon of sudden death³⁸ i.e. the transition from an entangled to a separable state after a finite interaction time. Here we quantitatively compare the degrading effects of different kinds of environments by considering the *entanglement-preserving time* and the *entanglement sudden-death time* and by studying their dependence on the nature and on the parameters of the stochastic process that models the environment. We focus on two paradigmatic examples of Gaussian processes describing normal and anomalous diffusion processes: the Ornstein–Uhlenbeck (OU) process^{25,39,40} and the fractional Gaussian noise.⁴¹

This paper is organized as follows: In Sec. 2, we describe the physical model that accounts for the system-environment interaction and introduce the Gaussian processes that drive the noise. In Sec. 3, we look at the dynamics of the system and analyze in some detail the dependence of the entanglement-preserving time and the entanglement sudden-death (ESD) time on the nature of the Gaussian process and the initial state of the system. Section 4 closes the paper with some concluding remarks.

2. The Physical Model

We consider a system of two non-interacting, identical qubits, characterized by the same energy splitting ω_0 and coupled to two external classical fluctuating fields. The effective Hamiltonian is thus of the form

$$\mathcal{H}(t) = \mathcal{H}_1(t) \otimes \mathbb{I}_2 + \mathbb{I}_1 \otimes \mathcal{H}_2(t), \quad (1)$$

where the local Hamiltonians are

$$\mathcal{H}_i(t) = [\omega_0 + \lambda B_i(t)] \sigma_z. \quad (2)$$

Here, λ is a coupling constant and $B_i(t)$ is an external classical field acting on each qubit, which we describe by means of a zero-mean Gaussian stochastic process. We consider both the case in which the two qubits are interacting with two independent environments, i.e. $B_1(t)$ and $B_2(t)$ are totally uncorrelated, and the case in which the two qubits are subject to a common environment, $B_1(t) = B_2(t)$.

The Hamiltonian in Eq. (1) models an effective interaction between a quantum system and a noisy environment having characteristic frequencies much smaller than the typical frequencies of the system ω_0 . The Hamiltonian in Eqs. (1) and (2) can also describe a two-level quantum degree of freedom coupled to a classical degree of freedom, for example the spin of a spin- $\frac{1}{2}$ particle undergoing a diffusion process in an external field.

A Gaussian process can be described completely by its second order statistics, i.e. by its mean μ and its autocorrelation function K , in formula:

$$\mu(t) = \mathbb{E}[B(t)] = 0, \quad (3)$$

$$K(t, t') = \mathbb{E}[B(t)B(t')], \quad (4)$$

where $\mathbb{E}[\cdot]$ denotes the average over all possible realizations of the process $B(t)$. The characteristic function of a Gaussian process is defined as⁴²:

$$\mathbb{E} \left[\exp \left(i \int_0^t ds f(s) B(s) \right) \right] = \exp \left[-\frac{1}{2} \int_0^t \int_0^t ds ds' f(s) K(s, s') f(s') \right], \quad (5)$$

where $f(t)$ is an arbitrary function of time. If $f = \kappa$ is constant with respect to time, Eq. (5) rewrites as:

$$\mathbb{E} \left[\exp \left(\pm i \kappa \int_0^t ds B(s) \right) \right] = \exp \left[-\frac{1}{2} \kappa^2 \beta(t) \right], \quad (6)$$

where

$$\beta(t) = \int_0^t \int_0^t ds ds' K(s, s'). \quad (7)$$

In this work, we focus on two paradigmatic Gaussian processes: the OU process and the fractional Gaussian noise (fGn). The OU process describes a diffusion process

with friction and it is characterized by the autocorrelation function

$$K_{\text{OU}}(t - t') = \frac{\gamma}{2} \exp(-\gamma|t - t'|), \quad (8)$$

where $\gamma = \tau^{-1}$ plays the role of a memory parameter and τ is the correlation time of the process. For increasing γ the noise spectrum becomes broader and in the limit $\gamma \gg 1$ one achieves white noise. The fGn describes anomalous diffusion processes, with a diffusion coefficient proportional to t^{2H} , where $H \in (0, 1)$ is known as the Hurst parameter. The autocorrelation function may be written as:

$$K_{\text{fGn}}(t - t') = \frac{1}{2} (|t|^{2H} + |t'|^{2H} - |t - t'|^{2H}). \quad (9)$$

When $H = 1/2$ we have $K_{\text{fGn}}(t - t') = \min(t, t')$ and the fGn reduces to the Wiener process (i.e. Brownian motion). When $H > \frac{1}{2}$, the increments of the process have positive correlation and the regime is called super-diffusive; when $H < \frac{1}{2}$, we are in the sub-diffusive regime and the increments are negatively correlated.

The β functions (7) for the OU and fGn processes are given by:

$$\beta_{\text{OU}}(t) = \frac{1}{\gamma} (e^{-\gamma t} + \gamma t - 1), \quad (10)$$

$$\beta_{\text{fGn}}(t) = \frac{t^{2H+2}}{2H+2}. \quad (11)$$

The evolution operator $U(t)$ for a given realization of the process $B_i(t)$, is expressed as:

$$\begin{aligned} U(t) &= \exp \left[-i \int_0^t \mathcal{H}(s) ds \right] \\ &= \exp \{ -i[\omega_0 t + \lambda \varphi_1(t)] \sigma_z \} \otimes \exp \{ -i[\omega_0 t + \lambda \varphi_2(t)] \sigma_z \}, \end{aligned} \quad (12)$$

where we defined the phase noise $\varphi_i(t) = \int_0^t ds B_i(s)$. If the system is initially prepared in the state ρ_0 , the density matrix at a time t is given by the expected value of the evolved density matrix over all possible realizations of the stochastic processes, i.e.

$$\rho(t) = \mathbb{E}[U(t)\rho_0 U^\dagger(t)]. \quad (13)$$

As initial state, we consider a system prepared in a Bell-state mixture:

$$\begin{aligned} \rho_0 &= c_1 |\Phi^+\rangle \langle \Phi^+| + c_2 |\Phi^-\rangle \langle \Phi^-| + c_3 |\Psi^+\rangle \langle \Psi^+| + c_4 |\Psi^-\rangle \langle \Psi^-| \\ &= \frac{1}{4} \left(\mathbb{I} + \sum_{i=1}^3 a_i \sigma_i \otimes \sigma_i \right), \end{aligned} \quad (14)$$

where $|\Phi^\pm\rangle = \frac{1}{\sqrt{2}} (|00\rangle \pm |11\rangle)$, $|\Psi^\pm\rangle = \frac{1}{\sqrt{2}} (|01\rangle \pm |10\rangle)$, and the σ_i are the three Pauli matrices. The coefficients c_i satisfy the condition $\sum c_i = 1$, and are related to the a_i

through the equalities:

$$\begin{aligned} a_1 &= c_1 - c_2 + c_3 - c_4, \\ a_2 &= -c_1 + c_2 + c_3 - c_4, \\ a_3 &= c_1 + c_2 - c_3 - c_4. \end{aligned} \quad (15)$$

We evaluate the entanglement by means of the negativity

$$N(\rho) = 2 \left| \sum_i \lambda_i^- \right|, \quad (16)$$

where λ_i^- are the negative eigenvalues of the partial transpose of the system density matrix. Negativity is zero for separable states and one for maximally entangled states, such as pure Bell states.

3. Results

3.1. Independent environments

Here, we consider the case of independent environments, i.e. each qubit is coupled to its own environment, described by the stochastic field $B_i(t)$. In order to obtain the evolved density matrix of the system, we calculate the expectation value in Eq. (13) over all possible realizations of the two uncorrelated processes $B_1(t)$ and $B_2(t)$. The evolved density matrix for the two qubits can be written explicitly by using Eq. (6). We find

$$\rho(t) = \frac{1}{2} \begin{pmatrix} (c_1 + c_2) & 0 & 0 & e^{-4\lambda^2\beta - 4i\omega_0 t}(c_1 - c_2) \\ 0 & (c_3 + c_4) & e^{-4\lambda^2\beta}(c_3 - c_4) & 0 \\ 0 & e^{-4\lambda^2\beta}(c_3 - c_4) & (c_3 + c_4) & 0 \\ e^{-4\lambda^2\beta + 4i\omega_0 t}(c_1 - c_2) & 0 & 0 & (c_1 + c_2) \end{pmatrix}, \quad (17)$$

that is, a pure dephasing map. By applying the local unitary transformation $e^{i\omega_0 t\sigma_z} \otimes e^{i\omega_0 t\sigma_z}$, we can write $\rho(t)$ in the diagonal Bloch form

$$\rho(t) = \frac{1}{4} (\mathbb{I} + e^{-4\lambda^2\beta(t)} a_1 \sigma_x \otimes \sigma_x + e^{-4\lambda^2\beta(t)} a_2 \sigma_y \otimes \sigma_y + a_3 \sigma_z \otimes \sigma_z), \quad (18)$$

where a_1, a_2 and a_3 are the components of the initial state ρ_0 . Since the density matrix (18) depends on time only through the function $\beta(t)$, the system will reach the separable steady state $\rho(t) = \frac{1}{4} (\mathbb{I} + a_3 \sigma_z \otimes \sigma_z)$ for $t \rightarrow \infty$. The trajectories of the evolved states in the a_i -parameter space are shown in Fig. 1(a). We notice that, with the exception of initial Bell states, the trajectories of the system actually enter the set of separable states at a finite time, thus showing a sudden death of entanglement.

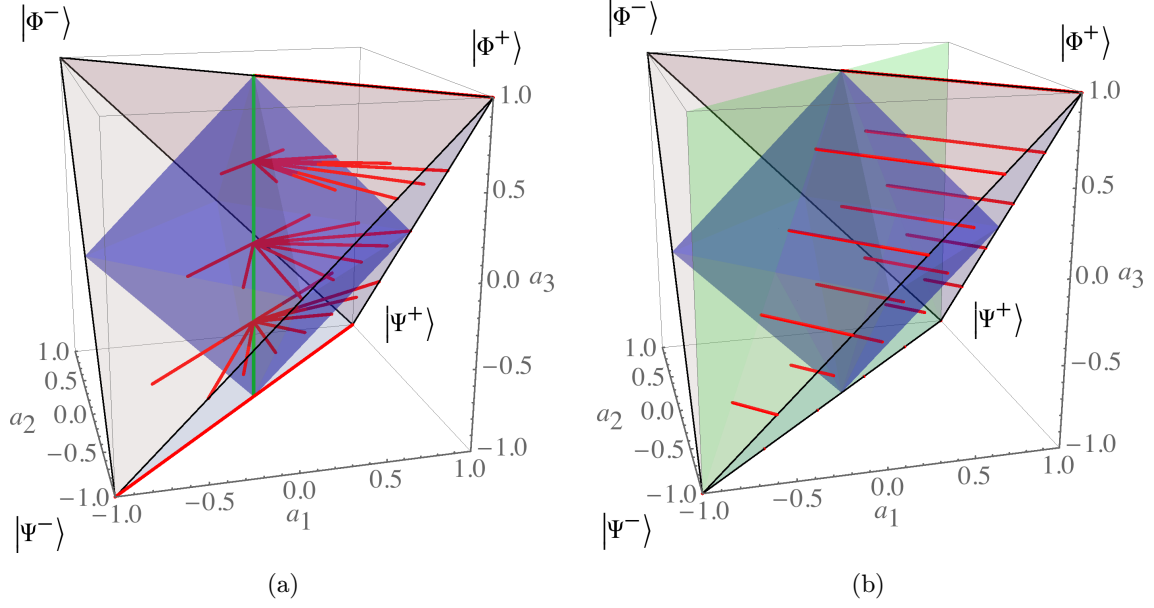


Fig. 1. (Color online) Trajectories of the system in the space of parameters $\{a_1, a_2, a_3\}$, for two independent environments (a) and for a common environment (b). The Bell-state mixtures, Eq. (14), form a tetrahedron. The set of separable states is the dark-blue octahedron. The initial states are Bell-state mixtures that lie on the surface of the tetrahedron. For independent environments, the trajectories converge to the green line $a_1 = a_2 = 0$. For a common environment, the trajectories are directed orthogonally to the plane $a_1 = a_2$, shown in green. In both cases, a_3 remains constant.

The negativity as a function of time, for an initial arbitrary Bell-state mixture, is given by:

$$N(t) = \frac{1}{2} \left(|c_1 + c_2 + e^{-4\lambda^2\beta(t)}(c_3 - c_4)| + |c_1 + c_2 - e^{-4\lambda^2\beta(t)}(c_3 - c_4)| \right. \\ \left. + |e^{-4\lambda^2\beta(t)}(c_1 - c_2) + c_3 + c_4| + |-e^{-4\lambda^2\beta(t)}(c_1 - c_2) + c_3 + c_4| \right) - 1. \quad (19)$$

As we can see from Eq. (19), the evolution of negativity does not depend on the energy splitting ω_0 of the two qubits.

3.2. Common environment

If the two qubits interact with the same environment, we can assume that $B_1(t) = B_2(t) = B(t)$ and thus

$$U(t) = \exp\{-i[\omega_0 t + \lambda\varphi(t)]\sigma_z\} \otimes \exp\{-i[\omega_0 t + \lambda\varphi(t)]\sigma_z\}. \quad (20)$$

The evolved density matrix at time t is given by

$$\rho(t) = \frac{1}{2} \begin{pmatrix} (c_1 + c_2) & 0 & 0 & e^{-8\lambda^2\beta - 4i\omega_0 t}(c_1 - c_2) \\ 0 & (c_3 + c_4) & (c_3 - c_4) & 0 \\ 0 & (c_3 - c_4) & (c_3 + c_4) & 0 \\ e^{-8\lambda^2\beta + 4i\omega_0 t}(c_1 - c_2) & 0 & 0 & (c_1 + c_2) \end{pmatrix} \quad (21)$$

and the Bloch-diagonal form of the state (after a local unitary transformation $e^{i\omega_0 t \sigma_z} \otimes e^{i\omega_0 t \sigma_z}$) is

$$\rho(t) = \frac{1}{4} \left\{ \mathbb{I} + \frac{1}{2} [e^{-8\lambda^2 \beta(t)} (a_1 - a_2) + a_1 + a_2] \sigma_x \otimes \sigma_x + \frac{1}{2} [e^{-8\lambda^2 \beta(t)} (a_2 - a_1) + a_1 + a_2] \sigma_y \otimes \sigma_y + a_3 \sigma_z \otimes \sigma_z \right\}. \quad (22)$$

In this case, the negativity as a function of time for an initial arbitrary mixture of Bell states, is

$$N(t) = \frac{1}{2} [|e^{-8\lambda^2 \beta(t)} (c_1 - c_2) + c_3 + c_4| + |e^{-8\lambda^2 \beta(t)} (c_2 - c_1) + c_3 + c_4| + |1 - 2c_3| + |1 - 2c_4| - 2]. \quad (23)$$

The trajectories in the Bell-state tetrahedron are shown in Fig. 1(b). They run orthogonally to the plane $a_1 = a_2$. By looking at the figure, we notice that the system experiences ESD when the initial state has $a_3 > 0$, except for mixtures of $|\Phi^+\rangle$ and $|\Phi^-\rangle$, for which $N(t) \rightarrow 0$ only for $t \rightarrow \infty$. For those Bell-state mixtures that are entangled and for which $a_3 < 0$, the trajectory runs parallel to the surface of the octahedron and hence negativity is constant over time. This set also includes the two Bell states $|\Psi^\pm\rangle$ which are stable states for the dephasing dynamics.

3.3. Entanglement-preserving time

The effect of the longitudinal field is to induce decoherence in the form of a dephasing. The entanglement, computed by the negativity, decays monotonically in time, as shown in Fig. 2. In particular, depending on the initial state different behaviors of quantum correlations appear: for initial Bell states, the negativity goes

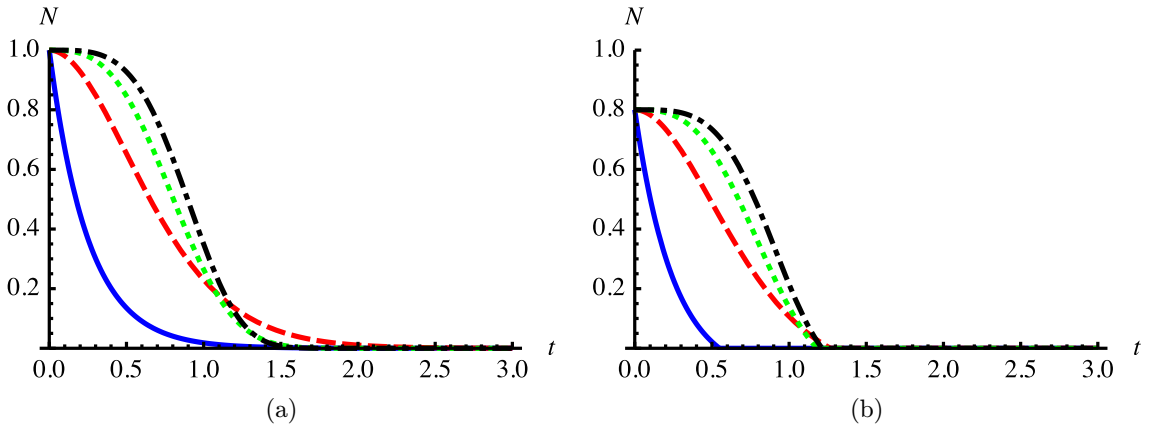


Fig. 2. (Color online) Negativity as a function of the interaction time for an initially pure Bell state (a) and for the mixture $\rho = \frac{1}{10}|\Phi^+\rangle\langle\Phi^+| + \frac{9}{10}|\Psi^+\rangle\langle\Psi^+|$ (b) interacting with independent environments driven by different stochastic processes: white noise (solid blue), OU with $\gamma = 1$ (red dashed), Wiener (green dotted), fGn with $H = 0.9$ (dot-dashed black). For pure Bell states, the negativity decreases smoothly to zero, while for mixtures of Bell states ESD appears.

asymptotically to zero, as a smooth function of time; on the contrary, if the initial state is a mixture of Bell states, entanglement displays sudden death, reaching zero abruptly. For a fixed initial state, the robustness of quantum correlations depends on the nature of the considered stochastic process: different expressions of the β function give different decaying velocities for entanglement.

We now investigate the role of the different considered processes in enhancing the system's ability to retain its coherence. To be quantitative, we define the *entanglement-preserving time* t^* as the time at which the negativity of the system falls below a certain threshold, that we fix at the ratio $r = 99\%$ of the initial negativity. We first consider the case in which the initial state is a Bell state. In this case, the negativity as a function of time is easily found to be

$$N_{\text{se}}(t) = \exp[-4\lambda^2\beta(t)], \quad (24)$$

$$N_{\text{ce}}(t) = \exp[-8\lambda^2\beta(t)], \quad (25)$$

for the independent-environment and common-environment case, respectively. Hereafter, we fix the value of the coupling constant to $\lambda = 1$. Upon introducing the quantity $\beta^* = -1/4 \log(r) \simeq 0.0025$, we may write the entanglement-preserving time as in Table 1, where we show the dependencies of t^* on the parameters of the processes, i.e. the inverse of the correlation time γ for the OU process and the Hurst parameter H for the fractional noise. We also report the results for white noise (i.e. OU for $\gamma \rightarrow \infty$) and the Wiener process (i.e. fGn with $H = \frac{1}{2}$).

The entanglement-preserving time for OU and fGn is shown in Fig. 3 as a function of the characteristic parameters γ and H . For the OU process, in the limit of a quasi-static field, i.e. $\gamma \rightarrow 0$, the entanglement-preserving time diverges, $t^* \rightarrow \infty$, such that the system retains its coherence indefinitely, while in the Markovian limit, $\gamma \rightarrow \infty$, $t^* \rightarrow \beta^*$, recovering the behavior typical of the white noise. In the case of fGn, the dependence of t^* on H is well approximated by a linear relation and the higher the diffusion coefficient, the longer the entanglement-preserving time. We also notice that, for vanishing H , t^* is comparable to the OU process with $\gamma = 1$. Indeed, we have that $\beta_{\text{OU}}(t) \simeq \frac{1}{2}\gamma t^2$ for small t and $\beta_{\text{fGn}}(t) \simeq \frac{1}{2}t^2$ for vanishing H . For general mixtures of Bell states, t^* is always smaller than the case of pure Bell states.

Table 1. The entanglement-preserving time t^* for different environments and for an initial pure Bell state. The quantity β^* is given by $\beta^* = -1/4 \log(r) \simeq 0.0025$ and $W(z)$ is the Lambert function, i.e. the principal solution of $z = W \exp W$.

Process	t^*
OU	$\frac{1}{\gamma}[\gamma\beta^* + W(-e^{-\gamma\beta^*-1}) + 1]$
White noise	β^*
fGn	$[(2H + 2)\beta^*]^{\frac{1}{2H+2}}$
Wiener	$[3\beta^*]^{1/3}$

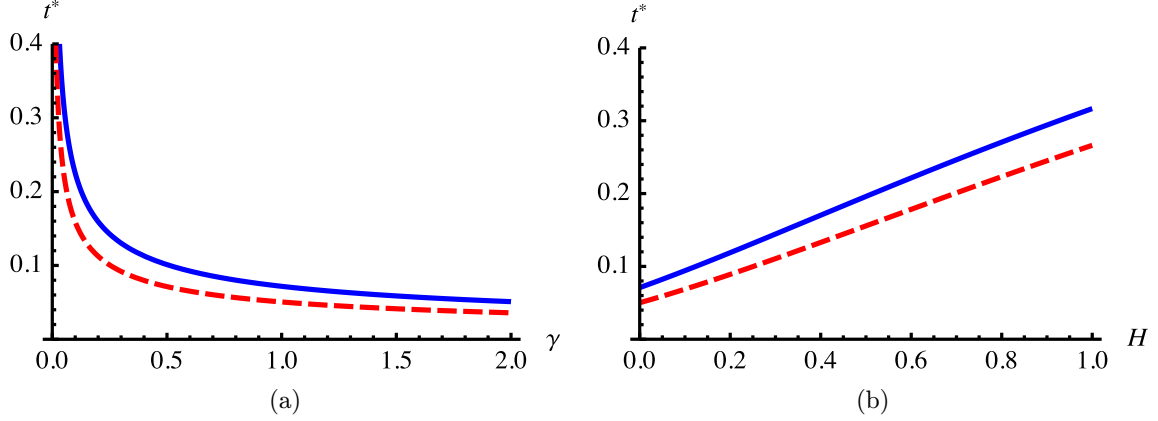


Fig. 3. (Color online) The entanglement-preserving time t^* as a function of the characteristic parameter of the external field. We show results for OU (a) and fractional Gaussian noise (b) and for the case of independent (solid blue) and common (red dashed) environments.

In Fig. 4, we show t^* as a function of the initial negativity N_0 for a set of randomly generated initial Bell-mixed states interacting with OU and fGn external fields (blue and red points respectively) either independently Fig. 4(a) or as a common environment Fig. 4(b). As it is apparent from the plots, the larger is the initial entanglement, the longer is the preserving time. This is true both in the case of independent and common environments. In the former case, the entanglement-preserving time is longer than in case of a common bath, for a fixed value of the initial negativity. In both scenarios, the entanglement is more robust in the case of fGn, rather than the OU process, with longer values of the preserving time t^* .

By looking at Fig. 4 we see that the values of t^* are not much dispersed. Rather, they concentrate around typical values which strongly depend on the kind of environment and only slightly on the initial negativity itself. Besides, the value of t^* is

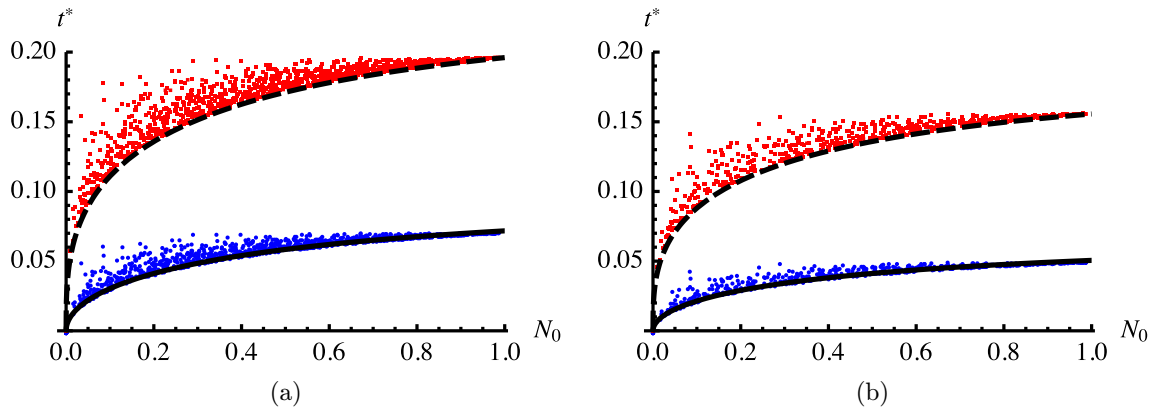


Fig. 4. (Color online) The entanglement-preserving time t^* (for a ratio $r = 0.99$ to the initial negativity) as a function of the initial negativity N_0 for randomly chosen initial Bell-state mixtures. We show results for the OU process with $\gamma = 1$ (blue points) and the Wiener process, i.e. fractional Gaussian noise with $H = 1/2$ (red points). The solid and dashed black lines are the lower bounds for t^* for the OU and Wiener process respectively, obtained from Eq. (26). (a) Independent environments, (b) common environment.

bounded from below by an increasing function of the initial negativity, the analytical expression of which can be obtained by determining the entanglement-preserving time for mixtures of a Φ and a Ψ Bell state. In this case, for a given ratio r to the initial negativity, t^* satisfies the equation

$$\beta(t^*) = \frac{1}{4A} \log \left[\frac{N_0 + 1}{N_0(2r - 1) + 1} \right], \quad (26)$$

where $A = 1$ for independent environments and $A = 2$ for a common environment. From Eq. (26) we obtain lower bounds to t^* as a function of N_0 , which are shown (solid and dashed black lines) in Fig. 4.

3.4. Entanglement sudden-death time

As previously discussed, the interaction of the two-qubit system with the external classical field induces a sudden death of entanglement for most of the Bell-state mixtures. In this section we study how the nature of the stochastic Gaussian process affects the *entanglement sudden-death time*, t_{ESD} , i.e. the time at which the state becomes separable and its negativity goes to zero.

In Fig. 5, we show t_{ESD} versus the initial negativity N_0 for randomly generated Bell-state mixtures for the OU process and the fGn with $H = \frac{1}{2}$. We can see that t_{ESD} is bounded from below by a monotonically increasing function of negativity, which itself diverges for $N_0 \rightarrow 1$, i.e. as the initial state gets closer to a pure Bell state. The analytical expression of this function is obtained by considering initial states belonging to a face of the Bell-state tetrahedron, and thus easily follows from Eq. (26) by substituting $r = 0$. We have

$$\beta(t_{\text{ESD}}) = \frac{1}{4A} \log \left(\frac{1 + N_0}{1 - N_0} \right), \quad (27)$$

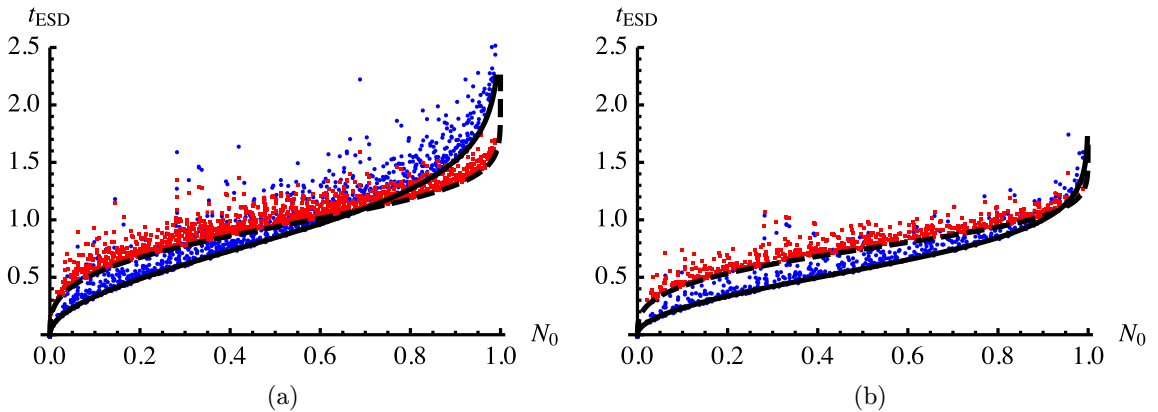


Fig. 5. (Color online) The entanglement sudden-death time t_{ESD} as a function of the initial negativity N_0 for randomly chosen (initial) Bell-state mixtures, for the OU process with $\gamma = 1$ (blue) and the Wiener process, i.e. fractional Gaussian noise with $H = 1/2$ (red). (a) Independent environments. The solid and dashed lines are the lower bounds for t_{ESD} for the OU and Wiener process respectively, obtained from Eq. (27). (b) Common environment.

where $A = 1$ for the independent-environments case and $A = 2$ for the common-environment case. Survival time is thus longer for larger values of the initial entanglement. In the case of independent environments the lower bound is larger than in the case of a common environment, confirming the tendency of entanglement to be more robust in the case of independent noises affecting the two qubits. As opposed to the entanglement-preserving time, the behavior of t_{ESD} is comparable for the two considered processes.

4. Conclusion

The decoherence caused by the interaction of a quantum system with the external environment is one of the main obstacles to the large scale deployment of quantum communication protocols and quantum information processing. A deep understanding of the decoherence mechanisms and the ability to engineer the environment are thus in order to obtain more robust quantum correlations and to design robust implementations of quantum technologies.

In this paper, we have addressed the dynamics of a two-qubit system interacting with classical noise generated by a stochastic Gaussian process and leading to a dephasing time evolution. In particular, we considered two diffusion processes: the OU process, characterized by a decoherence time γ^{-1} and the fGn, characterized by the Hurst parameter H . We computed the time evolved density matrix of the two-qubit system by performing the average over the stochastic processes, both in the case of independent and common environments. We have characterized the trajectories of the system inside the set of mixtures of Bell-states and shown the occurrence of sudden death of entanglement for certain sets of initial quantum states.

We introduced the entanglement-preserving time t^* and the ESD time t_{ESD} in order to analyze the effects of the nature of noise on the decoherence mechanism. We found that t^* is larger for fGn than OU process and that a larger initial entanglement corresponds to a longer preserving time. We also found that t^* is bounded from below by an increasing function of the initial negativity and that independent environments degrade quantum correlations more weakly than a common one. Also the entanglement sudden-death time t_{ESD} is bounded from below by a (different) increasing function of the initial negativity but, contrarily to the preserving time, has comparable values for the two considered processes.

Overall, our results indicate that engineering the environment has only a slight influence over the ESD time, while it represents a valuable resource to increase the entanglement-preserving time, i.e. to maintain entanglement closer to the initial level for a longer interaction time.

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