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NONTRIVIAL PHOTON STATISTICS WITH LOW RESOLUTION-THRESHOLD PHOTON COUNTERS

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We address the reconstruction of photon statistics using realistic photodetectors having low quantum efficiency and finite resolution as photon counters. Using a maximum-likelihood method, based on measurements taken at different quantum efficiencies, we experimentally reconstruct the nontrivial photon statistics of the state obtained by mixing at a beam-splitter two phase-averaged coherent states. The effect of using different resolution thresholds is also discussed.

Keywords: Photodetection; photon statistics; state reconstruction.

The photon statistics, ρ_n, of optical states provides fundamental information on the nature of any optical field and finds relevant applications in foundations of quantum mechanics, quantum information and quantum metrology. Here we address the reconstruction of the photon statistics by realistic photodetectors, whose performances are degraded by two parameters: a nonunit quantum efficiency η (not all the photons are detected) and a finite resolution M (different numbers of detected photons lead to a discriminable output only up to M detected photons). The output of a photodetector consists of current pulses containing charge values, whose statistics is a generalized Bernoulli convolution of the actual photon distribution. The explicit inversion of such a convolution is possible only for $\eta > 0.5$ (see Ref. 1) and is inherently inefficient, as it requires a large data sample to provide a reliable result. On the other hand, maximum-likelihood (ML) methods assisted by measurements taken at different quantum efficiencies have been proved to be both effective

and statistically reliable[2−6] for reconstructing ρ_n, even starting from on/off detection (resolution $M = 1$) and without any *a priori* information on the state under investigation. It can be shown that detectors with $M = 1$ are generally sufficient to provide a reliable reconstruction of the photon statistics for single-peaked distributions, whereas detectors with higher M value, which we also call resolution threshold, do not lead to further improvements.[7]

The aim of the present paper is twofold: first, we experimentally demonstrate that the method may be successfully applied to the reconstruction of non-single-peaked photon distributions. Second, we show that in this case even on/off detectors are sufficient to obtain a reliable reconstruction. In particular, we present the reconstruction of the photon statistics of the state observed at one output arm of a beam splitter fed by two phase-averaged coherent states. The corresponding photon distribution is not single-peaked and has a nontrivial shape, thus being a relevant test of the method though involving only semiclassical light states.

The probability of obtaining m photocounts at the output of a photodetector with quantum efficiency η is given by

$$p_\eta(m) = \sum_{n=m}^{\infty} A_\nu(m, n)\varrho_n \,, \tag{1}$$

where $\varrho_n = \langle n|\varrho|n\rangle$ is the actual photon statistics and $A_\eta(m, n) = \binom{n}{m}(1-\eta)^{n-m}\eta^m$. If the resolution threshold is equal to M then we have M independent outcomes with probabilities

$$q_\eta^m = p_\eta(m), \quad m = 0, \dots, M-1; \quad q_\eta^M = \sum_{m=M}^{\infty} p_\eta(m) = 1 - \sum_{m=0}^{M-1} q_\eta^m \,. \tag{2}$$

By collecting the statistics of the q_η^m values for a suitably large set of known efficiency values η_ν ($1 < \nu < K$), we get sufficient information to reconstruct the whole photon distribution ρ_n. In fact, by assuming $\rho_n \approx 0$ for $n > N$ we may rewrite Eq. (1) as a finite sum over n from m to N and we have that Eqs. (2) represent a $K \times M$ linear statistical model in the N unknowns ϱ_n. We solve the model by maximizing the likelihood functional $\mathsf{L} = \prod_{\nu=1}^{K} \prod_{m=0}^{M-1} (q_\nu^m)^{n_\nu^m}$, where n_ν^m is the number of events "m photocounts" obtained with quantum efficiency η_ν. Under the conditions $\rho_n \geq 0$ and $\sum_n \rho_n = 1$, the solution can be well approximated by using the expectation-maximization algorithm, which leads to the iterative solution

$$\varrho_n^{(i+1)} = \varrho_n^{(i)} \left(\sum_{\nu=1}^{K} \sum_{m=0}^{M} A_\nu(m, n) \right)^{-1} \sum_{\nu=1}^{K} \sum_{m=0}^{M} A_\nu(m, n) \frac{f_\nu^m}{q_\nu^m[\{\varrho_n^{(i)}\}]} \,, \tag{3}$$

f_ν^m being the experimental frequency of the event "m photocounts" with quantum efficiency η_ν. Equation (3) provides a solution once an initial distribution is chosen: it can be shown that the choice of the initial distribution does not alter the quality of reconstruction, though it may affect the convergence properties of the algorithm.

As a measure of convergence we use the total absolute error at the ith iteration $\varepsilon^{(i)} = \sum_{\nu,m} |f_\nu^m - q_\nu^m[\{\varrho_n^{(i)}\}]|$ and stop the algorithm as soon as $\varepsilon^{(i)}$ goes below

a given level. The total error measures the distance of the probabilities $p_\nu[\{\varrho_n^{(i)}\}]$, as calculated at the ith iteration, from the actual experimental frequencies f_ν^m. As a measure of accuracy we adopt the fidelity $G^{(i)} = \sum_n \sqrt{\varrho_n \, \varrho_n^{(i)}}$ between the reconstructed distribution and the expected one.

We consider a class of semiclassical states obtained by superimposing two phase-averaged coherent states at a beam splitter. The joint photon distribution at the input is $p_{in}(n_1, n_2) = e^{-N_1 - N_2} N_1^{n_1} N_2^{n_2}/(n_1! n_2!)$, where N_1 and N_2 are the average photon numbers of the input states. As the input states are diagonal in the number representation the output distribution can be written as[8]

$$p_{\text{out}}(n_1, n_2) = \sum_{l=0}^{n_1+n_2} \Gamma_{l,n1,n2} \, p_{in}(l, n_1 + n_2 - l) , \tag{4}$$

where $\Gamma_{l,n_1,n_2} = R_{l,n_1,n_2}^2$ with

$$R_{l,n_1,n_2} = \sum_{k=0}^{l} \left[\binom{n_1}{k} \binom{n_2}{l-k} \binom{l}{k} \binom{n_1+n_2-l}{n_1-k} \tau^{n_2+2k-l} (1-\tau)^{n_1+l-2k} \right]^{1/2} . \tag{5}$$

The marginal distribution at one arm is of course given by

$$\varrho_{n_1} = \sum_{n_2=0}^{\infty} p_{\text{out}}(n_1, n_2) . \tag{6}$$

If the mean photon number at the input is known, one may assume ϱ_{n_1} as the expected statistics at the output, and use it to evaluate fidelity of the reconstruction.

The experimental measurement was performed with a hybrid photodetector (Hamamatsu H8236-40) as the detector and a diode laser (634 nm, operated at 43 kHz, 50 ns pulse width) as the light source. The maximum efficiency of the detection chain was $\eta_{\max} = 0.27$. As the detector is able to resolve up to three photoelectron peaks, we used it as device with variable resolution threshold in order to compare the results of the reconstruction algorithm at different M values. In Fig. 1, we show the distribution obtained at the output of a beam splitter ($\tau = 0.5$) mixing two phase-averaged coherent states with $N_1 = 4.8$ and $N_2 = 3.6$ photons, respectively. Note that in this case a detector with $M = 1$ is sufficient to reconstruct the distribution.

The number of iterations required to achieve the reconstruction depends on the resolution threshold M. In fact by plotting the values of $\varepsilon^{(i)}$ [see Fig. 2(a)], we see that the total error reaches the minimum at three different iteration numbers ($i = 35,000$ for $M = 1$, $i = 2,000$ for $M = 2$ and $i = 300$ for $M = 3$). The plot of the fidelity [see Fig. 2(b)] shows that the number of iterations needed for $G^{(i)}$ to approach unity is of the same order of that evaluated from $\varepsilon^{(i)}$.

In conclusion, we used a ML method based on repeated measurements taken at different quantum efficiencies and experimentally reconstructed the nontrivial photon statistics of the state obtained by mixing at a beam-splitter two phase-averaged
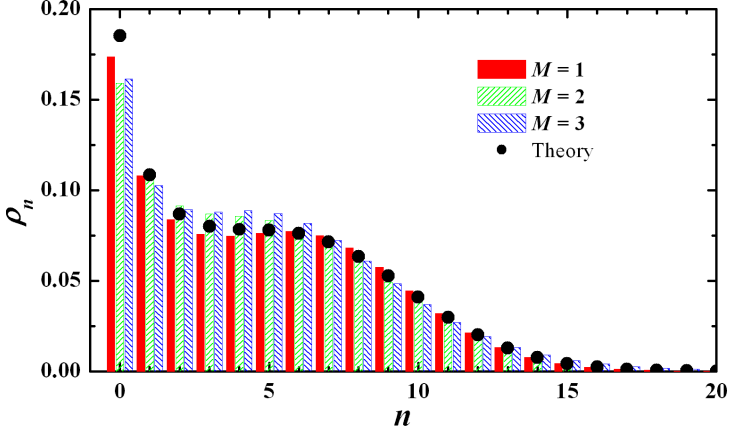
Fig. 1.   Reconstruction of the photon statistics at one output arm of a beam splitter for three different resolution thresholds compared to theoretical prediction from Eqs. (4) and (6).
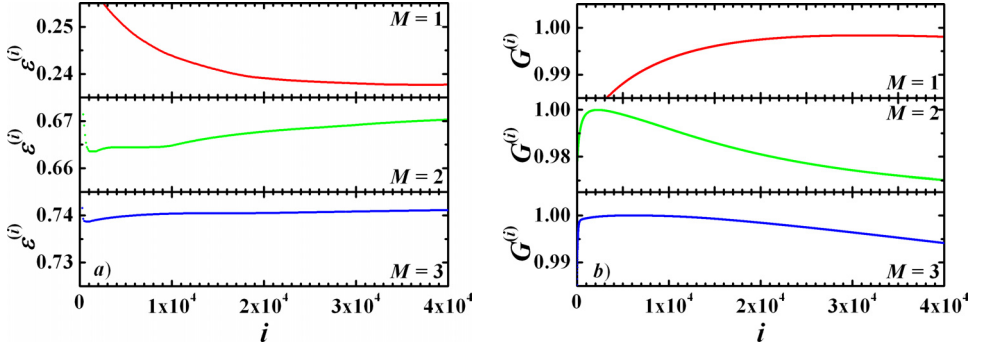

Fig. 2.   (a) Total absolute error $\varepsilon^{(i)}$ and (b) Fidelity $G^{(i)}$ as a function of the iteration number i.

coherent states. In this way, we have demonstrated that the ML method may be successfully applied to the reconstruction of non-single-peaked photon distributions and that even on/off detectors may be enough to obtain a reliable reconstruction. Our result, though involving only semiclassical states of light, is a further relevant test of the method and demonstrates the interesting potentialities of this technique.

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