

On the Reconstruction of Diagonal Elements of Density Matrix of Quantum Optical States by on/off Detectors

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Abstract. We discuss a scheme for reconstructing experimentally the diagonal elements of a density matrix representing quantum optical state. Applications to PDC heralded photons, multi-thermal and attenuated coherent states are illustrated and discussed in more detail.

1. Introduction

The reconstruction of diagonal elements of density matrix of a quantum optical state, i.e. of its photon statistics, is fundamental for various applications ranging from quantum information [1] to the foundations of quantum mechanics [2] and quantum optics [3]. Nevertheless, photo-detectors well suited for this purpose are not available at the moment. Indeed, the choice of a detector with internal gain suitable for the measurement is not trivial when the flux of the photons to be counted is such that more than one photon is detected in the time-window of the measurement, which is set by the detector pulse-response, or by an electronic gate on the detector output, or by the duration of the light pulse. One needs a congruous linearity in the internal current amplification process: each one of the single electrons produced by different photons in the primary step of the detection process (either ionization or promotion to a conduction band) must experience

the same average gain and this gain must have sufficiently low spread. The fulfillment of both requirements is necessary for the charge integral of the output current pulse to be proportional to the number of detected photons. There exist few examples of photo-detectors that can operate as photon counters and each one has some drawback. Among these, PhotoMultiplier Tubes (PMT's) [4] and hybrid photodetectors [5] present low quantum efficiency, since the detection starts with the emission of an electron from a photocathode. Solid state detectors with internal gain, in which the nature of the primary detection process ensures higher efficiency, are still under development. Highly efficient thermal photon counters have also been used, though their operating conditions are still too extreme (cryogenic conditions) to be suited for common use [6, 7].

The advent of quantum tomography did provide an alternative method to measure photon number distributions [8]. However, this scheme, which was experimentally applied to several quantum states [9], needs the implementation of homodyne detection, which in turn requires the appropriate mode matching of the signal with a suitable local oscillator at a beam splitter. Such mode matching, in general not an extremely simple scheme to be realized, is a particularly challenging task in the case of pulsed optical fields.

On the other hand, the photodetectors usually employed in quantum optics, such as Avalanche PhotoDiodes (APD's) operating in the Geiger mode [10, 7], seem to be, by definition, useless as photon counters. They are solid state devices, which present the highest quantum efficiency and the greatest stability of internal gain. However, they have an obvious drawback that the breakdown current is independent of the number of detected photons, which in turn cannot be determined. The outcome of these APD's is either "off" (no photons detected) or "on" *i.e.* a "click", indicating the detection of one or more photons. Actually, such an outcome can be provided by any photodetector (PMT, hybrid photodetector, cryogenic thermal detector) for which the charge contained in dark pulses is definitely below the output current pulses corresponding to the detection of at least one photon. Note that for most high-gain PMT's the anodic pulses corresponding to no photon detected can be easily discriminated by a threshold from those corresponding to the detection of one or more photons.

Nevertheless, the possibility of reconstructing photon distribution through on/off detection at different efficiencies has been analyzed recently [11] and its statistical reliability investigated in some detail [12]. In addition, the case of few photons and small values of η [13] has been addressed. The first experimental application of the method of [13, 12] has been presented in [17] showing very interesting potentialities of this scheme. In short, the method used in [17] is based on the measurement of on/off detection frequencies for a certain optical field when varying the quantum efficiency of the system, *i.e.* in practice by interposing calibrated neutral filters on the optical path. The beauty of this scheme with respect to alternative ones resides indeed in the extreme simplicity that can allow an extensive application of it to test optical fields in various applications.

With the purpose to refine these initial studies, in this paper we present a detailed analysis of the possibility of experimental reconstruction of the diagonal

elements of quantum optical state density matrix and of the precise estimation of their uncertainties. Various examples will be considered, as PDC heralded photons, multi-thermal and attenuated coherent states.

2. Theoretical Methods

The statistics of the “no-click” and “click” events from an on/off detector, assuming no dark counts, is given by

$$p_0(\eta) = \sum_n (1 - \eta)^n \varrho_n, \quad (1)$$

where $\varrho_n = \langle n | \varrho | n \rangle$ is the photon distribution of the quantum state with density matrix ϱ and η is the quantum efficiency of the detector, *i.e.* the probability of a single photon to be revealed. At first sight the statistics of an on/off detector appears to provide quite a scarce piece of information about the state under investigation. However, if the statistics about $p_0(\eta)$ is collected for a suitably large set of efficiency values then the information is sufficient to reconstruct the whole photon distribution ϱ_n of the signal, upon a suitable truncation of the Hilbert space.

The procedure consists in measuring a given signal by on/off detection using different values η_ν , $\nu = 1, \dots, K$, of the quantum efficiency. The information provided by experimental data is contained in the collection of frequencies $f_\nu = f_0(\eta_\nu) = n_{0\nu}/n_\nu$ where $n_{0\nu}$ is the number of “no click” events and n_ν the total number of runs with quantum efficiency η_ν . Then we consider expression (1) as a statistical model for the parameters ϱ_n to be solved by maximum-likelihood (ML) estimation. Upon defining $p_\nu \equiv p_0(\eta_\nu)$ and $A_{\nu n} = (1 - \eta_\nu)^n$ we rewrite expression (1) as $p_\nu = \sum_n A_{\nu n} \varrho_n$. Since the model is linear and the parameters to be estimated are positive (LINPOS problem), the solution can be obtained by using the Expectation-Maximization algorithm (EM) [16]. By imposing the restriction $\sum_n \varrho_n = 1$, we obtain the iterative solution

$$\varrho_n^{(i+1)} = \varrho_n^{(i)} \sum_{\nu=1}^K \frac{A_{\nu n}}{\sum_m A_{\nu m} p_\nu[\{\varrho_n^{(i)}\}]} \frac{f_\nu}{p_\nu[\{\varrho_n^{(i)}\}]}, \quad (2)$$

where $p_\nu[\{\varrho_n^{(i)}\}]$ are the probabilities p_ν , as calculated by using the reconstructed distribution $\{\varrho_n^{(i)}\}$ at the i -th iteration. As a measure of convergence we use the total absolute error at the i -th iteration

$$\varepsilon^{(i)} = \sum_{\nu=0}^K \left| f_\nu - p_\nu[\{\varrho_n^{(i)}\}] \right|,$$

and stop the algorithm as soon as $\varepsilon^{(i)}$ goes below a given level. The total error measures the distance of the probabilities $p_\nu[\{\varrho_n^{(i)}\}]$, as calculated at the i -th iteration, from the actual experimental frequencies. As a measure of accuracy we

adopt the fidelity $G^{(i)} = \sum_n \sqrt{\varrho_n \varrho_n^{(i)}}$ between the reconstructed distribution and the theoretical one.

3. Confidence Intervals

Our reconstruction of the photon statistics is based on the ML algorithm, which by an iterative method reaches the ϱ_n best reproducing the experimental data, *i.e.* the statistics of ‘no-clicks’ f_ν . The confidence interval on the ML determinations may be thus evaluated according to the following argument. Due to the fact that ML methods are unbiased at convergence we have $p_\nu\{\varrho_n^\infty\} \equiv f_\nu$. If in practice we stop at a given iteration L we have fluctuations in the reconstructed $\{\varrho_n\}$ and, in turn, $p_\nu\{\varrho_n^L\} \neq f_\nu$. The fluctuations in the single component ϱ_n lead to

$$p_\nu\{\varrho_n^L\} = f_\nu + \frac{\partial p_\nu}{\partial \varrho_n} (\varrho_n - \varrho_n^*), \quad (3)$$

where we have denoted by ϱ_n^* the *true* value of the distribution. As a consequence we may write $\delta\varrho_n^\nu = |p_\nu\{\varrho_n^L\} - f_\nu|/A_{\nu n}$ and, averaging over different values of the quantum efficiency,

$$\delta\varrho_n \simeq \frac{1}{K} \sum_{\nu=1}^K \frac{|p_\nu\{\varrho_n^L\} - f_\nu|}{A_{\nu n}}. \quad (4)$$

4. Experimental Data

In the following we describe the reconstruction of photon statistics of several optical states.

We begin by considering Parametric Down Conversion (PDC) heralded photons.

In our setup, a pair of correlated photons is generated by pumping a β -Barium-Borate (BBO) crystal with a continuous wave (cw) Argon ion laser beam (351 nm) in collinear geometry. The crystal has been cut for producing type II PDC, *i.e.* photons with orthogonal polarisation. After having separated the photons of the pair by means of a polarizing beam splitter, the detection of one of the two by a silicon avalanche photodiode detector (SPCM-AQR-15, Perkin Elmer) was used as an indication of the presence of the second photon in the other channel, namely a window of 4.9 ns was opened for detection in arm 2 in correspondence to the detection of a photon in arm 1. The arm 2 “heralded photon” was then measured by a silicon avalanche photodiode detector (SPCM-AQR-15, Perkin Elmer) preceded by an iris and an interference filter (IF) at 702 nm, 4 nm Full Width Half Maximum (FWHM), inserted with the purpose of reducing the stray light. More in detail, the output of the first detector started a ramp on a Time to Amplitude Converter (TAC) that received the second detector signal as stop. The TAC output was then addressed to a multichannel analyser (MCA) and a window of 4.9 ns was set by selecting only the respective channels of MCA.

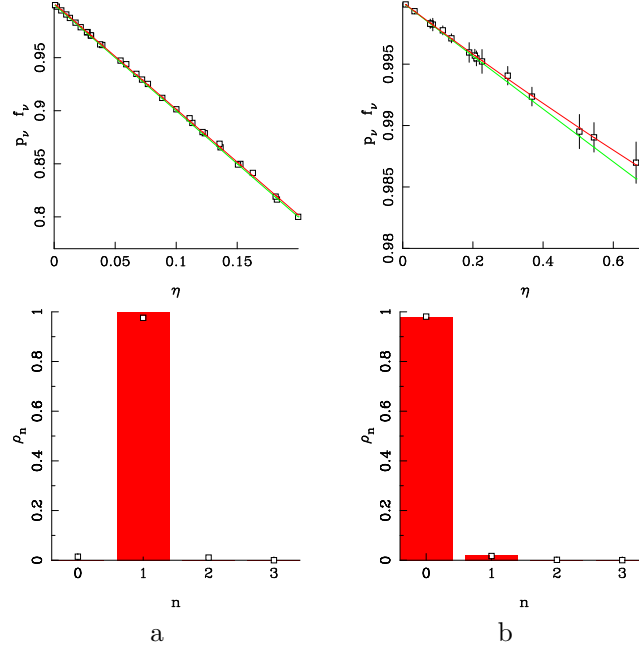


Fig. 1: The reconstruction of statistics of quantum optical states. In the highest part: Experimental frequencies f_ν of no-click events compared with reconstructed ones (red curve) and theoretical model for the state (black curve). In the lowest part, reconstructed probabilities compared with data. a) PDC heralded photon (produced in type II PDC cw-regime). b) Strongly attenuated coherent state (He-Ne laser beam)

The quantum efficiency of the arm 2 detection apparatus (including IF and iris) was measured to be 20% by using the PDC calibration scheme (see [18]). Lower quantum efficiencies were simulated by inserting calibrated neutral optical filters on the optical path.

A comparison of the observed frequencies f_ν with the theoretical curve $(1 - \eta_\nu)$ is presented in the inset of Fig. 1. In the figure evaluated uncertainties are also shown.

The photon distribution has been reconstructed using $K = 34$ different values of the quantum efficiency from $\eta_\nu \simeq 0$ to $\eta_\nu \simeq 20\%$ with $n_\nu = 10^7$ runs for each η_ν . Results at iteration $i = 10^7$ are shown in the figure. As expected the PDC heralded photon state largely agrees with a single photon Fock state. The estimated relative uncertainty for the ρ_1 element is 0.1 %, confirming the precision of the method. One can notice that also a small two photon component and a vacuum one are observed. The ρ_2 contribution is expected, by estimating the probability that a second photon randomly enters the detection window, to be 1.85% of ρ_1 , in agreement within uncertainty with what observed. A non zero ρ_0

is also expected due to background. This quantity can be evaluated to correspond to $(2.7 \pm 0.2)\%$ by measuring the counts when the polarization of the pump beam is rotated to avoid generation of parametric fluorescence. Also this estimate is in good agreement with the reconstructed ρ_0 .

As an example of application of the reconstructed diagonal elements of the density matrix, a clear test of non-classical nature of this optical field can then be performed by using the evaluated probabilities of finding n photons, p_n , to estimate the Klyshko [23] parameter $K_n = (n+1) \frac{p_{n-1}p_{n+1}}{np_n^2}$, which is always larger than unity for classical fields. In our case we obtain $K_1 = (3.2 \pm 0.4) \cdot 10^{-4}$.

A further reconstruction of diagonal elements of density matrix has concerned a strongly attenuated coherent state. This state has been produced by a He-Ne laser beam attenuated to photon-counting regime by insertion of neutral filters. Also in this case the same silicon avalanche photodiode detector was used. The distribution has been reconstructed with $K = 15$ different values of the quantum efficiency from $\eta_\nu \simeq 0$ to $\eta_\nu \simeq 66\%$ with $n_\nu = 10^7$ runs for each η_ν . It agrees well with what is expected for a coherent state with average number of photons $|\alpha|^2 \simeq 0.02$. In the inset of the Fig. 1 the frequencies f_ν as a function of η_ν are compared with the theoretical prediction $p_\nu = \exp\{-\eta_\nu|\alpha|^2\} \simeq 1 - \eta_\nu|\alpha|^2$. Notice that in this case we do not have IF or irises in front of the detector and all the other attenuations can be included in the generation of the state: thus the highest quantum efficiency is taken to be 66% as declared by the manufacturer's data-sheet for the photodetector.

Finally, as the last example, we present here our preliminary data on a single branch of PDC in pulsed regime, which is expected to correspond to a multi-thermal state [3].

The state has been generated by pumping a $1 \times 1 \times 1$ mm type I BBO crystal by a beam of a Q-switched triplicated (to 355 nm) Neodimium-Yag laser with pulses of 5 ns, power up to 100 mJ per pulse and 10 Hz repetition rate. Since this source was built with the purpose of having a high spectral selection (as a step toward a source useful for realising a quantum memory and quantum logical gates based on Kerr effect [24]) a monochromator with a 0.2 nm selection (obtained by an entrance 0.025 mm slit) was inserted in the optical path. More specifically, after having identified the precise direction of PDC emission at 780.2 nm by injecting into the crystal a laser beam locked on this wave length at the angle to observe stimulated emission, the monochromator was alligned on this beam (after a lens) and followed by an objective coupling of the transmitted PDC to a fiber addressed to photo-detector. The photo-detector was then gated by a signal coming from the laser in order to be opened in coincidence with the pulse arrival. On/off measurements corresponding to this window were then used for the statistics reconstruction, whose results are shown in the figure.

Also in this case uncertainties in the evaluation of a diagonal element of density matrix were computed according to the discussion in Sect. 3. As it is shown in Fig. 2 the reconstructed statistics of our preliminary data agrees (within uncertainties) with the one expected for a multi-thermal case. Nevertheless the fit seems

to favour a relatively small number of modes (smaller than 10), whilst the number of modes dictated by the ratio of the pulse duration (5 ns) and the coherence time for the 0.2 nm spectral selection is rather large. If one simply fixes the number of modes to 500 or more, the fit becomes worse (the reduced χ^2 increases from 0.5 to about 10).

The solution of this problem is postponed to a further data acquisition and to a more careful theoretical analysis. One possibility is that the method is reaching its limit when the average number of photons is small (smaller than 1 in our case) while the number of modes is very large. An alternative solution could be derived from the presence of a non-PDC background, e.g. the introduction into the fit of a strong poissonian background can lead to a good fit (reduced $\chi^2 \approx 3 \cdot 10^{-3}$).

For the sake of completeness, we have also considered the case of multithermal distribution with a very low level of photons. In this case the optical state was a single branch of PDC emission obtained by pumping a type II BBO crystal with a cw Argon ion laser beam (351 nm) of 0.3 W. The emission was selected by a 4 nm FWHM interference filter (which fixes the coherence time of the field) and addressed to an APD single photon detector where on/off counts were collected in a 20 ns window. The reconstructed statistics distribution agrees well, within uncertainties, with a multithermal state with a large number of modes (see Fig. 2), but at this low intensity level (0.064 photons on average) the distinction among different optical states as coherent, multithermal, thermal or poissonian becomes rather weak and no clear distinction emerges from data.

5. Conclusions

In summary, in this paper we have presented a detailed analysis of the possibility of experimental reconstruction of diagonal elements of a density matrix, with an accurate evaluation of uncertainties, corresponding to a quantum optical state by using on/off detectors.

The method that has been used [13, 12] is based on the measurement of on/off detection frequencies for a certain optical field when varying the quantum efficiency of the system, i.e. in practice by interposing calibrated neutral filters on the optical path.

Various examples have been considered, extending and deepening the analysis presented in [17]. In particular, we have considered PDC heralded photons, multi-thermal and attenuated coherent states. Estimation of uncertainties with this technique has been discussed in detail. The results of reconstruction look very good for weak coherent and heralded photon states, while further analysis is still necessary for clarifying the case of multithermal state, despite qualitative agreement.

Altogether our results further demonstrate the potentialities of this technique, whose main advantage with respect to alternative schemes resides in the extreme simplicity that allows an extensive application for testing of optical fields in a wide number of applications.

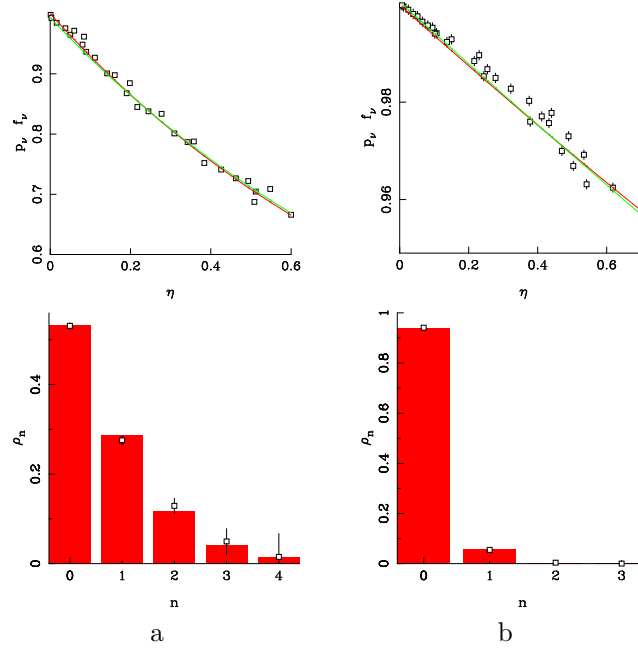


Fig. 2: The reconstruction of statistics of quantum optical states. In the highest part: Experimental frequencies f_ν of no-click events compared with the reconstructed ones (red curve) and theoretical model for the state (green curve). In the lowest part, the reconstructed probabilities compared with data. a) multithermal, single branch of PDC in nanosecond pulsed regime with 0.2 nm spectral selection. The curves correspond to 2 modes, 0.74 average photons. b) multithermal, single branch of PDC in cw regime with 4 nm spectral selection. The curves correspond to 10000 modes 0.064 average photons.

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