Innovative method to investigate how the spatial correlation of the pump beam affects the purity of polarization entangled states

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We present an innovative method to address the relation between the purity of type-I polarization entangled states and the spatial properties of the pump laser beam. Our all-optical apparatus is based on a spatial light modulator, and it offers unprecedented control on the spatial phase function of the entangled states. In this way, we demonstrate quantitatively the relation between the purity of the generated state and the spatial field correlation function of the pump beam. © 2012 Optical Society of America

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Spontaneous parametric downconversion (SPDC) is a crucial process in the development of quantum technology, and represents one of the most effective sources of entangled photon pairs and of single photons [1,2]. For these reasons, the spatial and the spectral properties of the downconverted beams have been extensively analyzed as a function of the coherence properties of the pump beam [3–6]. Less attention has been paid to the effect of the spatial properties of the pump on the purity of polarization entangled states, especially those generated with type-I parametric downconversion, since this may be revealed only by an accurate control of the phase profile of the output beams. In this Letter, we exploit an all-optical innovative method based on a spatial light modulator (SLM) to gain an unprecedented control on the spatial phase function of the generated entangled states and demonstrate experimentally the relation between the purity of the generated state and the field correlation function of the pump beam.

The downconverted state at the output of the two crystals, assuming that the spectra of the pump and of the parametric downconversion are quasi-monochromatic, may be written as

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \iint \mathrm{d}\theta_s \mathrm{d}\theta_i \,\mathrm{sinc}\left(\frac{1}{2}\Delta k_\parallel L\right) F(\Delta k_\perp) \times [|H,\theta_s\rangle|H,\theta_i\rangle + e^{i\Phi(\theta_s,\theta_i)}|V,\theta_s\rangle|V,\theta_i\rangle], \quad (1)$$

where L is the crystal length and $|P,\theta\rangle$ denote a single-photon state with polarization $P = H$, V emitted at angle θ. $\Delta k_\parallel$ and $\Delta k_\perp$ are the shifts with respect to the phase-matching condition of the longitudinal and transverse momentum of the two photons. The sinc function comes from the integration along the longitudinal coordinate inside the crystals, and the function F from the integration over the transverse coordinate: denoting by $A_p(x)$ the complex amplitude of the pump, we have $F(\Delta k_\perp) = \int \mathrm{d}x A_p(x) e^{i\Delta k_\perp x}$. The phase term $\Phi(\theta_s,\theta_i)$ arises from the optical path of the two photons generated in the first crystal inside the second crystal, and from the spatial walk-off between the H and the V beams of the downconversion outside the crystals [7,8]. In general, we may write $\Phi(\theta_s,\theta_i) = \phi(\theta_s) + \phi(\theta_i) + \Phi_a$ where, up to first order, we have $\phi(\theta) \simeq n^e kL/\cos[(\theta_0+\theta)/n^e] - kL \tan[(\theta_0+\theta)/n^e]\sin(\theta_0+\theta) \simeq \frac{1}{2}\phi_0 + \alpha_0\theta$, where n^e is the extraordinary index of refraction in the second crystal, $k = 2\pi/\lambda$, θ_0 is the central angle, and L is the crystal length. The term Φ_a represents the additional phase possibly added by any external optical component, e.g., the SLM. Since we employ noncollinear SPDC, we can address the two variables θ_s and θ_i independently by the different region of the SLM. The shifts $\Delta k_\parallel$ and $\Delta k_\perp$ are given by

$$\begin{aligned}\Delta k_\parallel &= k_p - k_s \cos[(\theta_0+\theta_s)/n^o] - k_i \cos((\theta_0+\theta_i)/n^o) \\ &\times \simeq k\theta_0(\theta_s+\theta_i) = k\theta_0\theta_+, \\ \Delta k_\perp &= k_s \sin[(\theta_0+\theta_s)/n^o] - k_i \sin[(\theta_0+\theta_i)/n^o] \\ &\times \simeq k(\theta_s-\theta_i) = k\theta_-, \end{aligned} \quad (2)$$

where $\theta_+ = \theta_s + \theta_i$ and $\theta_- = \theta_s - \theta_i$, and n^o is the ordinary index of refraction. Using the new variables, the overall phase function rewrites as

$$\Phi(\theta_-,\theta_+) = \phi_0 + \alpha_0\theta_+ + \Phi_a.$$

The purity of the state, which in this case equals the visibility, may be written as

$$p = \iint \mathrm{d}\theta_+ \mathrm{d}\theta_- |\mathrm{sinc}(\gamma\theta_+)|^2 |F(k\theta_-)|^2 \cos\Phi(\theta_+,\theta_-),$$

where $\gamma = \frac{1}{2}k\theta_0 L$. The normalization condition is given by $\iint \mathrm{d}\theta_+ \mathrm{d}\theta_- |\mathrm{sinc}(\gamma\theta_+)|^2 |F(k\theta_-)|^2 = 1$. In other words, the overall polarization state is given by $\rho_p = p|\psi_+\rangle\langle\psi_+| + (1-p)\rho_{\mathrm{mix}}$ where $|\psi_+\rangle = (|H\rangle|H\rangle + |V\rangle|V\rangle)/\sqrt{2}$ is a maximally entangled Bell state, and ρ_{mix} is the corresponding mixture.

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Two cases are of special interest: if $\Phi_a = -\phi_0$ the purity does not depend on F and we obtain the case that is usually described in the literature [7,8]. On the other hand, upon imposing $\Phi_a = -\phi_0 - \alpha_0\theta_+ + \beta\theta_-$ one obtains $\Phi = \beta\theta_-$; i.e., the purity is now a function of F. In this second case, using the Wiener–Khinchin theorem, we have

$$p = \int \mathrm{d}\theta_- |F(k\theta_-)|^2 \cos\beta\theta_- \propto \left\langle A_p^*\left(x + \frac{\beta}{k}\right) A_p(x) \right\rangle_x,$$

i.e., the purity of the state is proportional to the spatial field correlation function of the pump beam.

The experimental setup is shown in Fig. 1. A linearly polarized cw 405 nm diode laser (Newport *LQC*405–40*P*) passes through two cylindrical lenses, which compensate beam astigmatism, then a spatial filter composed by two lenses and a pin-hole in the Fourier plane to obtain a Gaussian profile by removing the multimode spatial structure of the laser pump, and finally a telescopic system prepares a beam with the proper beam radius and divergence. A couple of 1 mm beta-barium borate crystals, cut for type-I downconversion, with optical axis aligned in perpendicular planes, are used as a source of polarization and momentum entangled photon pairs with $\theta_0 = 3°$. In order to match the above theoretical model, we use a compensation crystal on the pump, which removes the delay time between the vertical and horizontal polarization [8,9], and put a 10 nm interference filter on the signal path, in order to reduce the spectral width of the generated radiation. An SLM, which is a liquid crystal phase mask (64 × 10 mm) divided in 640 horizontal pixels, each $d = 100$ μm wide, is set before the detectors in order to introduce the spatial phase function at 310 mm from the generating crystals [7]. We also place a window of 5 mm in front of the couplers of the detectors. A cylindrical lens is placed immediately after the two generating crystals, whereas a camera sets a the focal distance (1 m) to obtain the square modulus of the Fourier transform of the pump.

In order to complete the theoretical model, we have to take into account the fact that the spatial coupling is not flat, but rather has a Gaussian profile with an FWHM of about 5 mm. We thus insert this function when tracing out the spatial degrees of freedom in order to obtain the polarization state and its purity. In addition, there are some elements that introduce decoherence not compensable with the SLM; these are the gaps between the pixels of the SLM (3 μm), the imperfect compensation of the delay time, the spectral effects of the parametric downconversion, and the imperfect superposition between the amplitudes generated by the two crystals. In order to include these effects in the model, we write the output state as the mixed state $\rho_{\mathrm{tot}} = m\rho_p + (1 - m)\rho_{\mathrm{mix}} = mp|\psi_+\rangle\langle\psi_+| + (1 - mp)\rho_{\mathrm{mix}}$. The overall purity is thus given by $p_{\mathrm{tot}} = mp$, where m depends only slightly on the characteristic of the pump.

In order to measure the visibility, we insert two polarizers after the SLM (see Fig. 1) set at the angles $45°/-45°$ for the minimum, and $45°/45°$ for the maximum. In Fig. 2, we report the typical behavior of the visibility for $\beta = 0$, measured as a function of the parameter α, which itself governs the phase function $\Phi_a = -\phi_0 - \alpha\theta_+$ imposed by the SLM (the visibility without compensation is low because we deliberately use a large collection angle). In this configuration the overall phase function is given by $\Phi = (\alpha_0 - \alpha)\theta_+$ and thus it is possible to tune Φ_a and find the optimal value $\alpha = \alpha_0$, which removes the initial phase function and maximizes the purity. We find that this value is in good agreement with the expected theoretical value. The beam has a spot of 220 μm: looking at the Fourier transform, we see that we are not exactly dealing with a single-mode Gaussian profile. However, this is not a problem since to fit data we use the square modulus of the Fourier transform obtained with the method of the cylindrical lens.

After having found the optimal $\alpha \simeq \alpha_0$ to maximize the purity, we then exploit the SLM to impose the phase function $\Phi_a = -\phi_0 - \alpha_0\theta_+ + \beta\theta_-$. In Fig. 3 we report the behavior of the visibility as a function of the parameter β (right column), together with the spatial profile of the pump (left column) and its Fourier transform (middle column). We consider three relevant examples: in the first row we report the results obtained with a pin-hole in the spatial filter, in order to obtain a quasi-single-mode Gaussian profile with a spot of 220 μm on the crystal plane (the same configuration as Fig. 2). The visibility has a Gaussian shape, in excellent agreement with the theoretical model. In the second case, the spatial width is similar to the previous case but, as it is apparent from the Fourier transform,

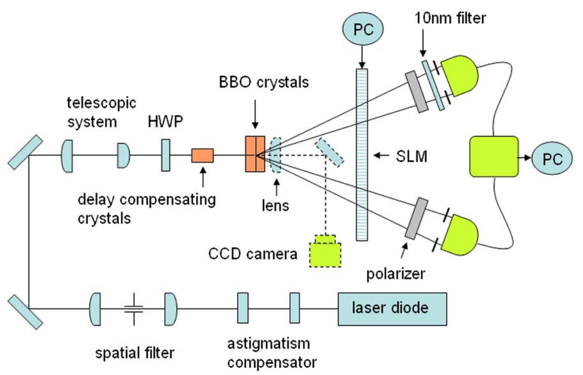

Fig. 1. (Color online) Schematic diagram of the experimental setup. The dashed part is used to measure the Fourier transform of the pump beam, and it is not present during measurements on the PDC output.

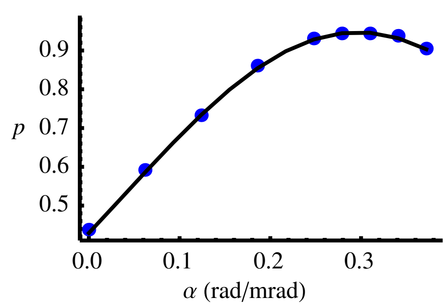

Fig. 2. (Color online) Purity (visibility) as a function of α for a beam with spot of 220 μm. The phase function imposed by the SLM is given by $\Phi_a = -\phi_0 - \alpha\theta_+$. Error bars on the experimental values are within the points. The solid black line is the theoretical prediction.

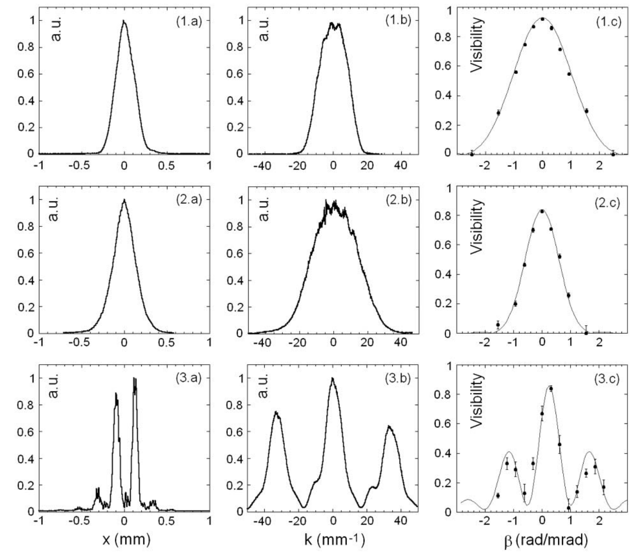

Fig. 3. Spatial field correlations of the pump beam and purity of the entangled output in three relevant cases: (first row) collimated pump beam of 220 μm, (second row) divergent pump beam of 220 μm, and (third row) pump beam with two peaks. We report the spatial profile of the pump (left column), its Fourier transform (right column), and the visibility as a function of β (right column). The phase function imposed by the SLM is given by $\Phi_a = -\phi_0 - \alpha_0\theta_+ + \beta\theta_-$. Error bars on the experimental values are within the points. The solid black lines are the theoretical predictions (which include the effects of the walk-off in the two-peaks case).

the beam is divergent (by few milliradian). Now the visibility is a Gaussian function with a smaller width. In the last example, we place a grid with a step of 100 μm in front of the two generating crystals. In the spatial profile, we obtain two peaks, and this corresponds to a revival in the visibility after a collapse. In fact, the pump degrees of freedom represent a noisy environment for the polarization ones, and thus upon modifying the spatial pump profile we are modifying its noise properties. The correlation properties of the pump induce spatial memory effects for the polarization degrees of freedom [10,11], which correspond to a non-Markovian dynamics. The small shift in the bottom right picture (about 0.2 rad/mrad) is probably due to an imperfect compensation (of about 30 μm) of the spatial walk-off between the H and V polarization of the pump beam.

In conclusion, we have demonstrated the quantitative relation between the purity of type-I polarization entangled states and the spatial properties of the pump. In order to obtain this result, we exploited the unprecedented control of the spatial phase function of the generated states that is achievable by the use of a SLM on noncollinear SPDC. Our method may be used for entanglement engineering [12] and purification [7], and it paves the way for investigating fundamental effects in non-Markovian open systems [13].

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