

Non-Gaussian states by conditional measurements

Marco G Genoni^{1,2}, Federica A Beduini², Alessia Allevi^{1,4},
Maria Bondani^{3,4}, Stefano Olivares^{1,2} and Matteo G A Paris^{2,1}

¹ CNISM, U.d.R. Milano Università, I-20133 Milano, Italy

² Dipartimento di Fisica, Università degli Studi di Milano, I-20133 Milano, Italy

³ National Laboratory for Ultrafast and Ultraintense Optical Science, CNR-INFM, I-22100 Como, Italy

⁴ CNISM, U.d.R. Como, I-22100 Como, Italy

E-mail: matteo.paris@fisica.unimi.it

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Abstract

We address realistic schemes for the generation of non-Gaussian states of light based on conditional intensity measurements performed on correlated bipartite states. We consider both quantum and classically correlated states and different kinds of detection, comparing the resulting non-Gaussianity parameters upon varying the input energy and the detection efficiency. We found that quantum correlations generally lead to higher non-Gaussianity, at least in the low-energy regime. An experimental implementation feasible with current technology is also suggested.

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(Some figures in this article are in colour only in the electronic version.)

1. Introduction

Continuous variable (CV) quantum information has been developed with Gaussian states and operations, which allow the realization of fundamental protocols as teleportation, cloning and dense coding, and still play a relevant role in CV quantum information processing. Recently, the non-Gaussian sector of the Hilbert space has also received attention for its potential application in entanglement distillation and entanglement swapping for long-distance quantum communication. In turn, using non-Gaussian states and operations, the teleportation [1–3] and cloning [4] of quantum states may be improved. In order to quantify the amount of non-Gaussianity (nG) of a state, measures have been proposed recently based on the *distances* between the quantum state under investigation and a reference Gaussian state [5, 6].

Gaussian states are generated using linear and bilinear interactions in optical materials, whereas the generation of non-Gaussian states requires higher nonlinearities than those exhibited by a Kerr medium. An alternative approach is to exploit the effective nonlinearity induced by conditional measurements. In fact, if a measurement is performed on a portion of a composite system, the other component is conditionally prepared according to the outcome of the

measurement and the resulting dynamics may be highly nonlinear. The rate of success in getting a certain state is equal to the probability of obtaining a certain outcome and may be higher than the nonlinear efficiency, thus making conditional schemes possibly convenient even when a corresponding Hamiltonian process exists.

In this paper, we focus on preparation schemes feasible with current technology and address the generation of non-Gaussian states of light by conditional intensity measurements on the twin-beam state (TWB) generated by parametric down-conversion and on classically correlated states generated by mixing a thermal beam with the vacuum in a beam splitter (BS). In particular, we consider different kinds of conditional measurements: ideal photodetection, inconclusive photodetection and photodetection performed with a finite quantum efficiency η and no dark counts. We compare the resulting nG varying all the involved parameters and, in particular, the number of photons of the initial state.

Let us consider a CV system made of d bosonic modes described by the mode operators a_k , $k = 1, \dots, d$, with commutation relations $[a_k, a_j^\dagger] = \delta_{kj}$. A quantum state ϱ of d bosonic modes is fully described by its characteristic function $\chi[\varrho](\lambda) = \text{Tr}[\varrho D(\lambda)]$, where $D(\lambda) = \bigotimes_{k=1}^d D_k(\lambda_k)$ is the d -mode displacement operator, with

$\lambda = (\lambda_1, \dots, \lambda_d)^T$, $\lambda_k \in \mathbb{C}$, and where $D_k(\lambda_k) = \exp\{\lambda_k a_k^\dagger - \lambda_k^* a_k\}$ is the single-mode displacement operator. The canonical operators are given by $q_k = (a_k + a_k^\dagger)/\sqrt{2}$ and $p_k = (a_k - a_k^\dagger)/\sqrt{2}i$ with commutation relationships given by $[q_j, p_k] = i\delta_{jk}$. Upon introducing the vector $\mathbf{R} = (q_1, p_1, \dots, q_d, p_d)^T$, we have the vector of mean values $\mathbf{X} \equiv \mathbf{X}[\varrho]$, $X_j = \langle R_j \rangle$, and the covariance matrix (CM) $\sigma \equiv \sigma[\varrho]$ of a quantum state, $\sigma_{kj} = \frac{1}{2}(\langle R_k R_j + R_j R_k \rangle - \langle R_j \rangle \langle R_k \rangle)$, where $\langle O \rangle = \text{Tr}[\varrho O]$. A quantum state ϱ_G is said to be Gaussian if its characteristic function is Gaussian, that is, $\chi[\varrho_G](\Lambda) = \exp\{-\frac{1}{2}\Lambda^T \sigma \Lambda + \mathbf{X}^T \Lambda\}$, where $\Lambda = (\text{Re}\lambda_1, \text{Im}\lambda_1, \dots, \text{Re}\lambda_d, \text{Im}\lambda_d)^T$. A Gaussian state is fully determined by CM and \mathbf{X} .

For a generic CV quantum state ϱ , a measure of nG based on the quantum relative entropy has been introduced in [6] as $\delta[\varrho] = S(\tau) - S(\varrho)$, where $S(\varrho) = -\text{Tr}[\varrho \log \varrho]$ is the von Neumann entropy of a quantum state ϱ , and τ is the Gaussian state with the same CM and \mathbf{X} as ϱ . In this paper, we will deal with single-mode non-Gaussian states that can be written as diagonal mixtures of Fock states $\varrho = \sum_{n=0}^{\infty} p_n |n\rangle\langle n|$, for which the reference Gaussian state is the single-mode thermal state $\nu_N = 1/(1+N_{\text{ph}}) \sum_{n=0}^{\infty} (N_{\text{ph}}/(1+N_{\text{ph}}))^n |n\rangle\langle n|$, where $N_{\text{ph}} = \text{Tr}[\varrho a^\dagger a]$ is the mean photon number. The von Neumann entropy for these states can be easily obtained and therefore the nG can be written as

$$\delta[\varrho] = N_{\text{ph}} \log \left(\frac{N_{\text{ph}} + 1}{N_{\text{ph}}} \right) + \log(1 + N_{\text{ph}}) + \sum_{n=0}^{\infty} p_n \log p_n. \quad (1)$$

2. Non-Gaussian states by conditional measurements

We want to study the quantum states generated by conditional measurements on quantum and classically correlated states. In particular, as the initial states we consider (i) an entangled TWB, obtained from (spontaneous) parametric down-conversion (SPDC) in second-order nonlinear crystals and (ii) a thermal state mixed with the vacuum state in a BS of transmissivity T . The TWB state can be written as $|\Lambda\rangle = \sqrt{1-|\lambda|^2} \sum_n \lambda^n |n\rangle_1 \otimes |n\rangle_2$, where $|n\rangle_j$ denotes the Fock number state in the Hilbert space of the j th mode. The parameter $|\lambda| < 1$ may be taken as real without loss of generality and the mean number of photons for each mode is given by $N = \text{Tr}[|\Lambda\rangle\langle\Lambda| \hat{n}_1 \otimes \mathbb{1}] = \text{Tr}[|\Lambda\rangle\langle\Lambda| \mathbb{1} \otimes \hat{n}_2] = \lambda^2/(1-\lambda^2)$. The output state from a BS of transmissivity T mixing a thermal state ν_{2N} and a vacuum is described by a density operator

$$R = \sum_{stpq} T^{(s+t)/2} (1-T)^{(p+q)/2} \nu_{p+s,t+q} \times \sqrt{\binom{p+s}{s} \binom{q+t}{q}} |s\rangle\langle t| \otimes |p\rangle\langle q|, \quad (2)$$

where $\nu_{h,k} = \delta_{h,k} (1+2N)^{-2} [2N/(1+2N)]^k$ and $2N$ is the mean photon number in R . In the next subsection, we will derive the states generated from $|X\rangle$ and R by means of different conditional measurements, that is,

$$\varrho_{\Lambda,i} = \frac{\text{Tr}_2[|\Lambda\rangle\langle\Lambda| \mathbb{1} \otimes M^{(i)}]}{\text{Tr}[|\Lambda\rangle\langle\Lambda| \mathbb{1} \otimes M^{(i)}]},$$

$$\varrho_{R,i} = \frac{\text{Tr}_2[R \mathbb{1} \otimes M^{(i)}]}{\text{Tr}[R \mathbb{1} \otimes M^{(i)}]},$$

where i denotes the kind of measurement we are performing and $M^{(i)}$ is the operator of the corresponding probability operator valued measure (POVM). In particular, we will consider (i) ideal photodetection, (ii) (ideal) inconclusive photodetection and (iii) photodetection with a non-unit quantum efficiency. We will then evaluate their nG parameter and describe their photon statistics. In particular, we will give the mean photon number N_{ph} , the variance σ_{ph}^2 and the corresponding Fano factor defined as $F_{\text{ph}} = \sigma_{\text{ph}}^2/N_{\text{ph}}$.

2.1. Ideal photodetection

The POVM of an ideal photon-resolving detector is given by projectors on the Fock number basis: $P_m = |m\rangle\langle m|$. By taking the TWB $|\Lambda\rangle$ as the initial state, the output state after measuring m photons is simply the corresponding Fock state $\varrho_{\Lambda,1} = |m\rangle\langle m|$. The photon statistics of $\varrho_{\Lambda,1}$ is trivial, since $N_{\text{ph}} = m$ and both σ_{ph}^2 and F_{ph} are null. The nG of the output state can be easily evaluated by using equation (1) and it monotonically depends on the number of detected photons m .

In the case of the classically correlated state R with N mean photons in each mode, performing an ideal photodetection on one mode leaves the other mode in the following conditional state:

$$\varrho_{R,1} = \left(\frac{1+2N(1-T)}{1+2N} \right)^{m+1} \sum_{s=0}^{\infty} \binom{m+s}{m} \left(\frac{2NT}{1+2N} \right)^s |s\rangle\langle s|.$$

In this case, the mean number of photons, the variance and the Fano factor of the output state are non-trivial but can be evaluated, obtaining the following results:

$$N_{\text{ph}} = \frac{2(1+m)NT}{1+2N(1-T)}, \quad \sigma_{\text{ph}}^2 = \frac{2(1+m)NT(1+2N)}{(1+2N(1-T))^2},$$

$$F_{\text{ph}} = \frac{1+2N}{1+2N(1-T)}.$$

The corresponding nG $\delta[\varrho_{R,1}]$ can be evaluated numerically and is plotted in figure 1 along with the nG of the previous case. As one can observe from the plot, the state generated by the classically correlated state is less non-Gaussian than that generated by TWB. Its nG increases with the number of detected photons m , with the initial mean number of photons N and with the transmissivity of the BS T . Although increasing with N , we have numerical evidence that the asymptotic value for $N \rightarrow \infty$ is below the nG of the Fock states $|m\rangle\langle m|$ obtained from the TWB.

2.2. Inconclusive photodetection

The ideal inconclusive photodetection is described by the POVM operator $\Pi_{\text{off}} = |0\rangle\langle 0|$ if no photons are detected and $\Pi_{\text{on}} = \mathbb{1} - |0\rangle\langle 0|$ if one or more photons are detected. Let us first consider the state obtained when one or more photons are detected on one mode of the TWB $|\Lambda\rangle$. The state that is generated can be written in the Fock basis as

$$\varrho_{\Lambda,2} = \frac{1-\lambda^2}{\lambda^2} \sum_{s=1}^{\infty} \lambda^{2s} |s\rangle\langle s|.$$

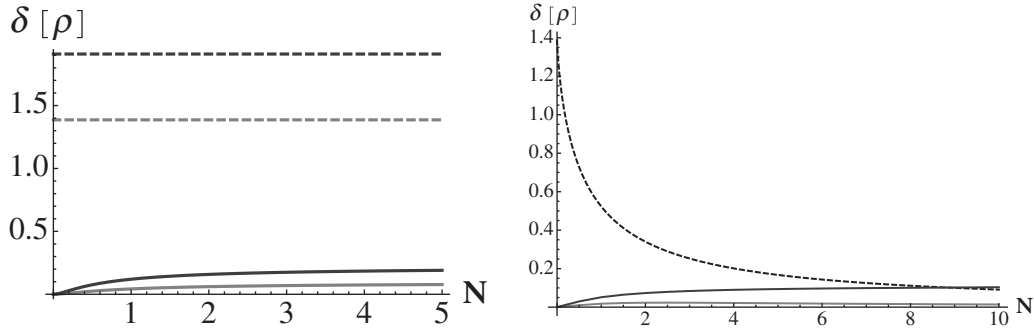


Figure 1. Left: non-Gaussianity of quantum states generated by performing an ideal photodetection on one arm of a TWB $|\Lambda\rangle$ (dashed lines) or of the classically correlated state R (solid lines) obtained with a BS with transmissivity $T = 0.9$, as a function of the initial average number of photons N and for different values of detected photons m . From lighter to darker grey: $m = \{1, 2\}$. Right: non-Gaussianity of quantum states generated by inconclusive photodetection on a TWB $|\Lambda\rangle$ (black dashed line) or on the quantum state R (solid lines) for different values of the BS transmissivity T (from lighter to dark grey: $T = \{0.5, 0.99\}$) as a function of the initial number of photons N .

Note that for $\lambda \rightarrow 0$ ($N \rightarrow 0$) we obtain the Fock state $|1\rangle\langle 1|$, while for $\lambda \rightarrow 1$ ($N \rightarrow \infty$), the state is not physical. The photon statistics of $\varrho_{R,2}$ can be summarized in terms of the initial number of photons N using the parameters $N_{\text{ph}} = 1 + N$, $\sigma_{\text{ph}}^2 = N(1 + N)$ and $F_{\text{ph}} = N$. The corresponding nG $\delta[\varrho_{\Lambda,2}]$ can be evaluated numerically and is plotted in figure 1. It is apparent from the plot that $\delta[\varrho_{\Lambda,2}]$ is a monotonically decreasing function of the initial number of photons N , starting from the nG of the Fock state $|1\rangle\langle 1|$ and approaching zero for $N \rightarrow \infty$.

Let us consider now the conditional state generated from the classically correlated state R . It is diagonal in the Fock basis and can be written as

$$\varrho_{R,2} = \frac{1 + 2N(1 - T)}{2N(1 - T)} \sum_{s=0}^{\infty} \left(\frac{2NT}{1 + 2N} \right)^s \times \left[\frac{1}{1 + 2NT} \left(\frac{1 + 2N}{1 + 2NT} \right)^s - \frac{1}{1 + 2N} \right] |s\rangle\langle s|.$$

The average photon number, its variance and the Fano factor are

$$N_{\text{ph}} = \frac{4NT(1 + N(1 - T))}{1 + 2N(1 - T)},$$

$$\sigma_{\text{ph}}^2 = \frac{4NT(1 + N(3 - T + 4TN^2(1 - T)^2 + 2N(1 - T^2)))}{(1 + 2N(1 - T))^2},$$

$$F_{\text{ph}} = 2(1 + NT) - \frac{2(1 + N)}{1 + N(1 - T)} + \frac{1 + 2N}{1 + 2N(1 - T)}.$$

The nG $\delta[\varrho_{R,2}]$ can be numerically evaluated and is plotted in figure 1. In this case, it does not have a monotonic behaviour as a function of N , while it is still an increasing function of the transmissivity T . In the low-energy regime the state generated starting with a TWB is definitely more non-Gaussian than that generated by means of R . By increasing the initial mean number of photons, since $\delta[\varrho_{R,1}]$ approaches zero, we observe a region where the classical correlated state gives birth to a more non-Gaussian state than the quantum one.

2.3. Inefficient photodetection

The POVM of a photon-number-resolving detector with quantum efficiency η and no dark counts is given by the Bernoullian convolution of the ideal number projectors $P_l = |l\rangle\langle l|$, and thus, for m detected photons, it is described by the operator $\Pi_m = \eta^m \sum_{l=m}^{\infty} (1 - \eta)^{l-m} \binom{l}{m} P_l$. The state generated by detecting m photons on one arm of a TWB can thus be written as

$$\varrho_{\Lambda,3} = \frac{1 - \lambda^2(1 - \eta)}{(\lambda^2(1 - \eta))^m} \sum_{l=m}^{\infty} \binom{l}{m} (\lambda^2(1 - \eta))^l |l\rangle\langle l|.$$

Here we summarize the photon statistics of this state in terms of the previously introduced quantities, by substituting λ with N :

$$N_{\text{ph}} = \frac{(1 + m)(1 + N)}{1 + N\eta} - 1, \quad \sigma_{\text{ph}}^2 = \frac{(1 + m)(1 - \eta)N(1 + N)}{(1 + N\eta)^2},$$

$$F_{\text{ph}} = \frac{N(1 + N)(1 + m)(1 - \eta)}{(m(1 + N) + N(1 - \eta))(1 + N\eta)}.$$

The nG can be evaluated numerically and it is plotted in figure 2 for different values of the parameter. We observe that, as expected, nG is a monotonically increasing function of the number of detected photons m . Moreover, we observe that it decreases with the initial mean photon number N , while it increases with increasing detector efficiency. By choosing $\eta = 1$, we trivially obtain the results of the ideal photodetection. For $N \rightarrow \infty$, the state generated by measuring a TWB turns out to be the corresponding (normalized) POVM operator, that is, $\varrho_{\Lambda,3} = \eta \Pi_m$. The asymptotic values approached for $N \rightarrow \infty$ correspond to the nG of this particular state and increases with increasing η . Finally, we consider the classically correlated state R . Also in this case, we can obtain the state generated by measuring m photons in one arm, with efficiency η :

$$\varrho_{R,3} = \left(\frac{1 + 2N\eta(1 - T)}{1 + 2N(T + \eta(1 - T))} \right)^{m+1} \times \sum_{s=0}^{\infty} \binom{m+s}{m} \left(\frac{2NT}{1 + 2N(T + \eta(1 - T))} \right)^s |s\rangle\langle s|.$$

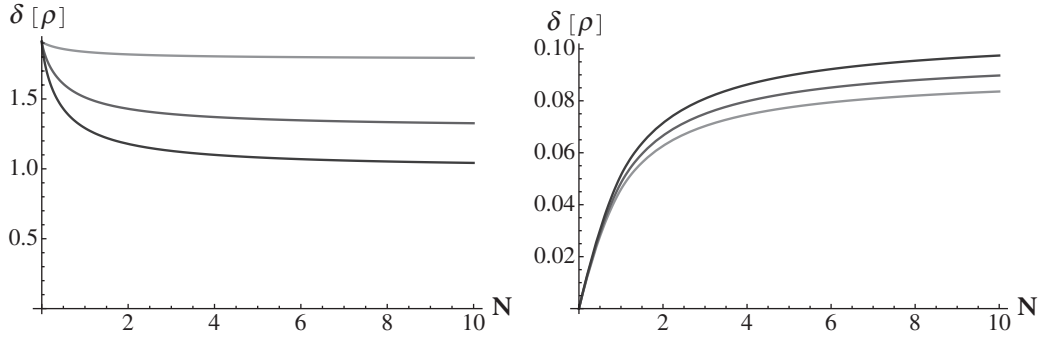


Figure 2. NG of conditional states generated by performing an inefficient photodetection on one mode of a TWB $|\Lambda\rangle$ (left figure) or of the classical correlated state R (right figure) obtained by a BS with transmissivity $T = 0.5$, as a function of the initial average number of photons N , for $m = 2$ photons detected and for different values of the efficiency η . From darker to lighter grey: $\eta = \{0.8, 0.9, 0.99\}$. As one can observe on comparing the two plots, the nG of states generated from TWB is higher than the one obtained from thermal states.

The mean number of photons, its variance and the Fano factor are

$$N_{\text{ph}} = \frac{2(1+m)NT}{1+2N\eta(1-T)},$$

$$\sigma_{\text{ph}}^2 = \frac{2(1+m)NT(1+2N(T+\eta(1-T)))}{(1+2N\eta(1-T))^2},$$

$$F_{\text{ph}} = 1 + \frac{2NT}{1+2N\eta(1-T)}.$$

Again the nG $\delta[\varrho_{R,3}]$ has been numerically evaluated and is plotted in figure 2. As in the ideal case, the nG obtained from a classically correlated state takes lower values than those from TWB. Moreover, we again observe that a higher nG is obtained at higher transmissivity (as in the previous cases) and, unlike the TWB case, at lower detection efficiency and higher values of the initial number of photons N . For $N \rightarrow \infty$ the asymptotic state can be written as

$$\varrho_{R,3} = \left(\frac{\eta(1-T)}{T+\eta(1-T)} \right)^{m+1} \times \sum_{s=0}^{\infty} \binom{m+s}{m} \left(\frac{T}{T+\eta(1-T)} \right)^s |s\rangle\langle s|. \quad (3)$$

Again we have numerical evidence that for $N \rightarrow \infty$ the nG obtained for the classical correlated state is below those obtained by measuring the TWB state.

3. Experimental proposal

The experimental implementation of the optical states described in the previous sections, namely TWB states and thermal states, would allow us to verify the correctness of our model. Obviously, as the real detectors are endowed with a non-ideal quantum efficiency, here we consider only the case of imperfect detection presented in section 2. In order to match the requirements of the theoretical model, we plan to use two hybrid photodetectors (HPD; Hamamatsu) as the detectors, which are endowed with a partial photon-number-resolving capability and no dark counts [9].

First of all, we discuss the case of the generation of TWB states with a sizeable mean photon number. Such

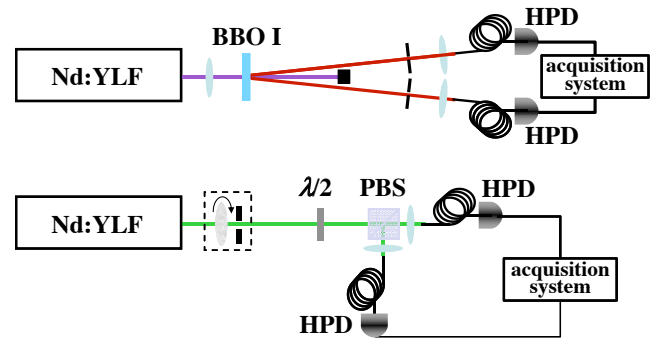


Figure 3. Proposed experimental setups. Upper panel: TWB state. Lower panel: classically correlated thermal state.

states can be obtained by pumping a second-order nonlinear crystal (either in type-I or type-II phase-matching) by means of a pulsed laser in a travelling-way configuration [7]. As sketched in the upper panel of figure 3, we can exploit either the third harmonics ($\lambda = 349$ nm, 4.4 ps pulse duration) or the fourth harmonics (at $\lambda = 262$ nm, 4 ps pulse duration) of a frequency-tripled Nd:YLF mode-locked laser amplified at 500 Hz to produce bright cones of SPDC in a type-I β -BaB₂O₄ crystal. The TWB state will be measured by selecting with suitable pinholes a pair of twin coherence areas on the signal and idler cones [8]. In order to make the quantum efficiencies of the two arms as equal as possible, we will operate at frequency degeneracy. Note that with both choices for the pump, the measured TWB wavelength will be in the visible spectrum range, where most photodetectors have the maximum quantum efficiency. Multi-mode fibres having a good transmissivity in the visible spectrum range will be employed to collect all the light passing the pinholes and to send it to the detectors. The study of the dependence of the nG factor on the overall quantum efficiency will be experimentally reproduced by inserting a variable neutral density filter on the arm of the detector performing conditional measurement. Unfortunately, depending on crystal length and phase-matching conditions, the TWB state produced by ps-pulsed lasers is intrinsically temporal multi-mode. For this reason, the single-mode theory previously developed should be extended to correctly describe the experimental situation.

The production of a single-mode pseudo-thermal state can be easily obtained by inserting a rotating ground glass

plate on the pathway of a coherent field, followed by a pinhole selecting a single coherence area in the far-field speckle pattern [10] (see the lower panel of figure 3). To match the maximum quantum efficiency of the detectors, we can use the second harmonics ($\lambda = 523$ nm, 5.4 ps pulse duration) of the Nd:YLF laser described above. The thermal light will then be split into two parts by means of a BS in order to perform conditional measurements. To make the setup more versatile and, in particular, to make it possible to study the dependence of the nG factor on the transmissivity of the BS, it is convenient to substitute the ordinary BS with a system composed of a half-wave plate ($\lambda/2$ in figure 3) and a polarizing cube BS. Again the measurements can be performed with a pair of HPD.

4. Conclusions

We have suggested an experimentally feasible scheme for the generation of non-Gaussian states of light by conditional intensity measurements on correlated bipartite states. We have considered both quantum and classically correlated states and evaluated the amount of resulting nG for different kinds of detection and input energy. Although the emergence of nG is not related to the quantum nature of the involved beams, we

found that quantum correlations generally lead to higher nG, especially in the low-energy regime.

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