Joint multipartite photon statistics by on/off detection

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We demonstrate a method to reconstruct the joint photon statistics of two or more modes of radiation by using on/off photodetection performed at different quantum efficiencies. The two-mode case is discussed in detail, and experimental results are presented for the bipartite states obtained after a beam splitter fed by a single photon state or a thermal state. © 2006 Optical Society of America

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The reconstruction of the joint photon distribution of two or more correlated modes of radiation plays a crucial role in fundamental quantum optics[1] and finds relevant applications in quantum communication,[2,3] imaging, and spectroscopy. Nevertheless, photodetectors suited for this purpose are currently not available, because the few existing examples[4] still suffer from limitations. On the other hand, reconstruction by quantum tomography[5] is not an easily implemented technique suited for a widespread use.

Recently,[4] a maximum-likelihood (ML) method based on on/off detection performed at different quantum efficiencies[6,7] was developed and demonstrated for reconstructing the photon distribution of single-mode states. The results are also reliable and accurate for relatively low quantum efficiency of the detector. Since for many applications multipartite states are needed, in this Letter we extend our previous results to this case as well. In particular, the bipartite case (easily extendable to multipartite) will be discussed in detail. Examples of experimental reconstructions for bipartite states are presented to test and assess our method.

The statistics of on/off detection performed with quantum efficiency η on a single-mode state ϱ is given by $p_{0\eta} = \Sigma_n A_{\eta n} \varrho_n [p_{1\eta} = 1 - p_{0\eta}]$, where $\varrho_n = \langle n|\varrho|n\rangle$ is the photon distribution (diagonal matrix elements) of the state and $A_{\eta n} = (1-\eta)^n$. By performing independent on/off photodetection on two (spatially separated) modes of radiation, globally described by the two-mode density matrix ϱ, the joint on/off statistics are given by $p_{00\eta} = \Sigma_{nk} A_{\eta n} A_{\eta k} \varrho_{nk}$, $p_{01\eta} = \Sigma_{nk} A_{\eta n} (1-A_{\eta k}) \varrho_{nk}$, $p_{10\eta} = \Sigma_{nk} (1-A_{\eta n}) A_{\eta k} \varrho_{nk}$, and, of course, $p_{11\eta} = 1 - p_{00\eta} - p_{10\eta} - p_{01\eta}$, where $\varrho_{nk} = \langle\langle nk|\varrho|nk\rangle\rangle$ ($|nk\rangle\rangle = |n\rangle \otimes |k\rangle$) is the joint photon distribution of the two modes. Once the value of the quantum efficiency is known, the above equations provide a relation between the statistics of clicks and the actual statistics of photons. At first sight this represents a scarce piece of information about the state under investigation. However, if the on/off statistics are collected for a suitably large set of efficiency val-

ues, then the information is enough to reconstruct the joint photon distribution of the bipartite state. We adopt the following strategy: by placing in front of the detector K filters with different transmissions, we may perform the detection with K different values η_ν, $\nu = 1, \ldots, K$, ranging from $\eta_1 = \eta_{\min}$ to a maximum value $\eta_K = \eta_{\max}$ equal to the nominal quantum efficiency of the detector. Upon writing $\mathbf{g} = (p_{00\eta_1}, \ldots, p_{00\eta_K}, p_{01\eta_1}, \ldots, p_{01\eta_K}, p_{10\eta_1}, \ldots, p_{10\eta_K})$ and $\mathbf{q} = (\varrho_{00}, \varrho_{01}, \varrho_{10}, \ldots)$, according to $\varrho_{nk} \to q_p$ with $p = 1 + k + n(1+N)$, i.e., $k = (p-1)\mathrm{mod}(1+N)$ and $n = (p-1-k)/(1+N)$, we can summarize the on/off statistics as

$$g_\mu = \sum_p B_{\mu p} q_p, \qquad \mu = 1, \ldots, 3K, \quad p = 1, \ldots, (1+N)^2,$$

(1)

where we have introduced the matrix $\mathbf{B}$

$$B_{\mu p} = \begin{cases} A_{\mu n} A_{\mu k} & \mu = 1, \ldots, K \\ A_{\mu n}(1 - A_{\mu k}) & \mu = K+1, \ldots, 2K \\ (1 - A_{\mu n}) A_{\mu k} & \mu = 2K+1, \ldots, 3K \end{cases}.$$

(2)

If the ϱ_{nk}'s are negligible for $n, k > N$, and the η_μ's are known, then Eq. (1) represents a finite statistical linear model for the positive unknown q_p. The ML solution of this linear positive problem is well approximated by the iterative algorithm[8]

$$q_p^{(i+1)} = q_p^{(i)} \left(\sum_{\mu=1}^{3K} B_{\mu p} \right)^{-1} \sum_{\mu=1}^{3K} B_{\mu p} \frac{h_\mu}{g_\mu[\{q_p^{(i)}\}]}.$$

(3)

In Eq. (3) $q_p^{(i)}$ denotes the pth element of reconstructed statistics at the ith step, $g_\mu[\{q_p^{(i)}\}]$ denotes the theoretical on/off probabilities as calculated from Eq. (1) at the ith step, whereas h_μ is the measured frequency of the events with quantum efficiency η_μ, i.e., $\mathbf{h} = (f_{00\eta_1}, \ldots, f_{00\eta_K}, f_{01\eta_1}, \ldots, f_{01\eta_K}, f_{10\eta_1}, \ldots, f_{10\eta_K})$, with $f_{ij\eta_\mu} = n_{ij\eta_\mu}/n_\mu$, n_μ being the total number of runs performed with $\eta = \eta_\mu$.

The convergence of the algorithm may be checked by the total error $\epsilon_i = (3K)^{-1}\Sigma_\mu |h_\mu - g_\mu[\{q_p^{(i)}\}]|$, which

measures the distance of the reconstructed statistics (of clicks) from the measured one: the algorithm is stopped when ϵ reaches its minimum or goes below a certain threshold value.

To test our method we have experimentally performed the reconstruction for different bipartite states. The schematic diagrams of the experimental setups are reported in Fig. 1. As a first test, we use our method to reconstruct the bipartite states obtained at the output of a beam splitter (BS) fed by the heralded single-photon state generated by parametric downconversion (PDC). In our setup a 0.2 W, 398 nm pulsed (with 200 fs pulses) laser beam, generated by the second harmonic of a Ti:sapphire beam at 796 nm, pumps a 5 mm $\times$ 5 mm $\times$ 5 mm type II β-barium borate (BBO) crystal. Upon detecting a photon in a branch of the degenerate PDC emission one triggers the presence of the correlated photon in the other direction. The heralded single photon is then impinged onto a BS with unexcited second port, thus generating a bipartite state of the form $|\psi\rangle = \sqrt{\tau}\,|01\rangle \pm \sqrt{1-\tau}\,|10\rangle$, τ being the transmissivity. After the BS both arms are measured by on/off detectors. All the detectors were silicon avalanche photodetectors (APD), whose quantum efficiency has been calibrated with the traditional PDC scheme.[9,10] The proper set of quantum efficiencies is obtained by inserting before the BS several Schott neutral filters (NFs) of different transmittance, evaluated by measuring the ratio between the counting rates with the filter inserted and without it. The data for the reconstruction have been taken by using $K=34$ values of η from $\eta_{\min}=0.015$ to $\eta_{\max}=0.325$. In correspondence to the detection of a photon in arm 1, a coincidence window has been opened on both detectors on arm 2. This is obtained by sending the output of the first detector as start to two time-to-amplitude converters (TACs) that receive the detector signal as stop. By centering a 20 ns window on the observed coincidence peak, spurious coincidences with PDC photons of other pulses are excluded (the repetition rate of the laser being 70 MHz). The TAC outputs are then addressed both to counters and to an AND logical gate for measuring coincidences between them. These outputs, together with one TAC valid start (giving us the total number of opened coincidence windows), allows one to evaluate the frequencies on/off h_μ needed for

reconstructing the joint photon statistics of the bipartite state. The background has been evaluated and subtracted by measuring the TAC and AND outputs out of the triggered window.

To verify the method in different cases we considered four different alternatives given by the combination of a balanced ($\tau=0.5$) or unbalanced ($\tau=0.4$) BS with either large band, red glass filters (RG) with cutoff wavelength at 750 nm, or interference filters (IF), with peak wavelength at 796 nm and a 10 nm FWHM. The reconstructed statistics for these four situations are shown in Fig. 2. The uncertainties have been evaluated as described in Ref. 4. As is apparent from Fig. 2, the reconstructed state well corresponds to a single photon in one of the modes. Only the elements ϱ_{01}, ϱ_{10} are different (within uncertainties) from zero, and their ratio is the value expected by the ratio of the outputs ports of the BS (unity for the balanced one, 2/3 for the unbalanced one). As expected in this regime, no multiphoton component is observed; i.e., ϱ_{11}, ϱ_{20}, ϱ_{02}, etc. are zero within uncertainties. The small uncertainties also show that less unbalanced BS would be distinguishable.

As a second example we consider a single branch of PDC emission without triggering, which corresponds to a multithermal state with number of modes of the order of $\sim 10^3$. A bipartite state is generated by impinging this signal onto a BS with the second port unexcited. The output bipartite state is classically correlated (not entangled, but not factorizable) with the two partial traces corresponding to multithermal states. The expected on/off statistics are given by $p_{00\eta}=\mu^\mu(\mu+\eta N)^{-\mu}$, $p_{01\eta}=\mu^\mu[(\mu+\eta\tau N)^{-\mu}-(\mu+\eta N)^{-\mu}]$, and $p_{10\eta}=\mu^\mu[(\mu+\eta(1-\tau)N)^{-\mu}-(\mu+\eta N)^{-\mu}]$, respectively, where N is the average number of photons and μ the number of modes. In this case the state has been produced by pumping a 5 mm $\times$ 5 mm $\times$ 5 mm type I BBO crystal by a beam of a Q-switched triplicated (to 355 nm) Nd:YAG laser with pulses of 5 ns, power up to 200 mJ per pulse, and 10 Hz repetition rate. Because of the very high power of the pump beam, a state with a large number of photons is generated.

We have therefore attenuated (by 1 nm FWHM IF and neutral filters) the multithermal state before splitting and detection. Measurements at the different quantum efficiencies have been obtained by in-
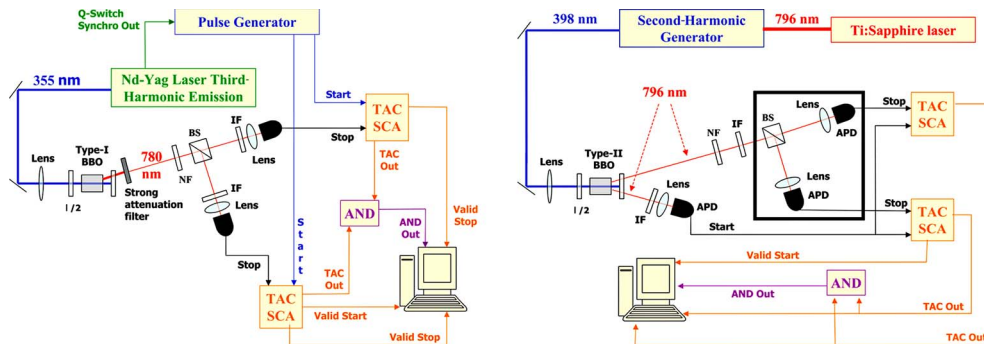

Fig. 1.   (Color online) Schematic diagrams of the experimental setups. Left, setup to reconstruct the joint photon statistics of a bipartite state obtained by splitting a multithermal state by a balanced beam splitter. Right, setup to reconstruct the joint photon statistics of the bipartite states obtained by splitting a PDC-II heralded single-photon state by balanced and unbalanced beam-splitters. SCA, single-channel analyzer. Other abbreviations are defined in text.
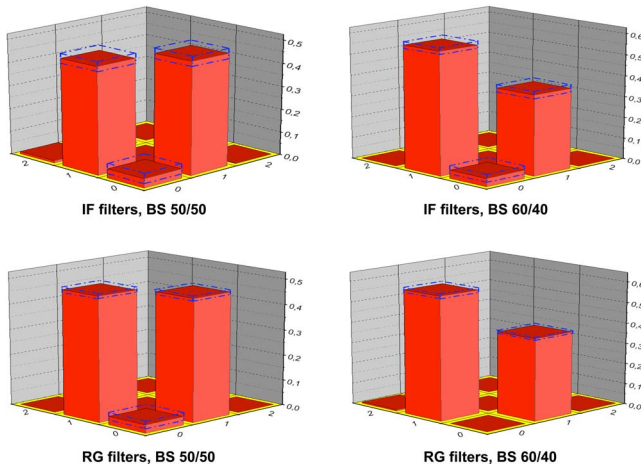
Fig. 2. (Color online) Reconstruction of the joint photon distribution of a bipartite state: PDC heralded photon state split by a BS (balanced and unbalanced). Dashed lines represent one standard deviation.
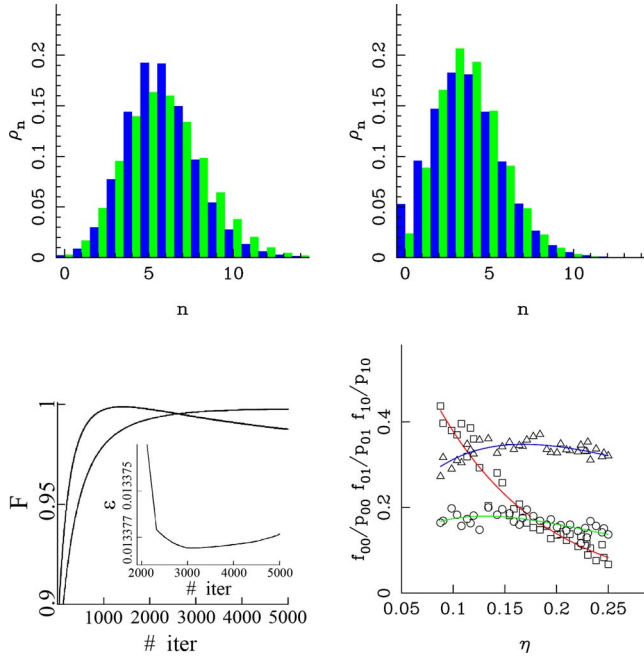

Fig. 3. (Color online) Reconstruction of the joint photon distribution of a bipartite state: attenuated PDC multithermal distribution split by a balanced BS. The two histograms correspond to reconstructed distributions (dark gray) compared with multithermal ones. Bottom left, fidelity F of the reconstructed marginal distributions to multithermal ones as a function of the number of the iterations of the algorithm. Inset, total error ϵ. Bottom right, measured frequencies f_{00}, f_{01}, f_{10} as function of the quantum efficiency compared with the expected ones (multithermal, solid curves).

serting (before the BS) Schott NFs, whose calibration has been obtained by measuring the power of a diode

laser (at the same wavelength of the used PDC emission) before and after the filters. The data for the reconstruction have been taken by using $K=35$ values of η from $\eta_{\min}=0.05$ to $\eta_{\max}=0.25$. The coincidence scheme has been realized by addressing two Q-switched triggered pulses to two TAC modules as starts, and the detectors outputs as stops. Then, having set properly a 20 ns coincidence window, we sent the two TAC outputs to an AND logic port, and the valid stops to counting modules (together with one TAC valid start and the AND output). The results of this reconstruction are shown in Fig. 3. Also in this case the comparison between theoretical expectations and reconstructed statistics is rather good. The fidelity $F=\Sigma_n\sqrt{\varrho_n\varrho_n^{mth}}$ of the reconstructed distribution to the expected multithermal $\{\varrho_n^{mth}\}$ is larger than 99% for both the marginals. Notice that the optimal number of iterations (i.e., leading to maximum average fidelity of the two marginals) corresponds to the minimum of ϵ, thus confirming the good convergence properties of the algorithm.

In conclusion, we demonstrated a method to reconstruct the joint photon statistics of two or more modes by using on/off photodetection. Experimental reconstruction has been presented for the bipartite states obtained after a beam splitter fed by a single photon state or a thermal state. Our results clearly show that the maximum-likelihood reconstruction based on on/off detection can be successfully applied to measure the joint photon statistics for multipartite systems.

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