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Minimum decoherence cat-like states in Gaussian noisy channels

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Abstract

We address the evolution of cat-like states in general Gaussian noisy channels, by considering superpositions of coherent and squeezed coherent states coupled to an arbitrarily squeezed bath. The phase space dynamics is solved and decoherence is studied, keeping track of the purity of the evolving state. The influence of the choice of the state and channel parameters on purity is discussed and optimal working regimes that minimize the decoherence rate are determined. In particular, we show that squeezing the bath to protect a non-squeezed cat state against decoherence is equivalent to orthogonally squeezing the initial cat state while letting the bath be phase insensitive.

Keywords: decoherence, noisy channels, Schrödinger cats

1. Introduction

In Schrödinger's original paper [1], a bipartite entangled state of the form

$$|\psi\rangle \propto |A\rangle|-\rangle + |B\rangle|+\rangle,$$

where $|A\rangle$ and $|B\rangle$ denote the 'alive' and 'dead' states of a cat and $|\pm\rangle$ two orthogonal states of a microscopic system, was suggested to illustrate the counterintuitive consequences of quantum mechanics in a macroscopic setting. More generally, in the literature, any single-system coherent superposition $|\psi\rangle \propto |\psi_-\rangle + |\psi_+\rangle$ of two pure quantum states $|\psi_\pm\rangle$ that are mesoscopically distinguishable is often referred to as a *Schrödinger cat* or a *cat-like* state. The interest in the study of such superpositions, possibly involving states of a microscopic system as well, stems from both theoretical and experimental considerations. Actually, a cat state is one of the simplest and most fundamental configurations allowing one to probe the archetypal aspects of quantum theory: the superposition principle and quantum entanglement.

As far as quantum optical systems are concerned, the possibility of realizing superposition states of the radiation field, first envisioned by Yurke and Stoler [2], has been extensively investigated in more recent years [3]. However, pure state superpositions are in general corrupted by the interaction with the environment. Therefore, cat-like states that are available for experiments are usually mixed states that have suffered a partial decoherence, and it is crucial to know whether and to what extent superpositions can survive the environmental noise. The theme of decoherence of cat-like states spurred relevant theoretical works [4–8], especially aimed to select schemes of quantum control and feedback stabilizing coherent superpositions against decoherence [7]. Recent promising experimental results and perspectives continue to maintain a widespread interest in this subject [9].

In the present paper, we study the non-unitary evolution of coherent and squeezed coherent single-mode Schrödinger cat-like states in generic Gaussian noisy channels, namely, either thermal or squeezed thermal baths of harmonic oscillators [10, 11]. In particular, we will focus our attention on the evolution of the purity (or linear entropy) of the states, showing how the quantum superposition is corrupted by the interaction with a noisy environment and how to optimize the state and channel parameters to minimize the decoherence rate.

The paper is structured as follows. In section 2 we introduce notations and evaluate the Wigner function of generalized cat-like states. In section 3 the time evolution in Gaussian noisy channels is studied, whereas in section 4 the dependence of decoherence and purity on the parameters of the initial state and of the channel is discussed in detail. In section 5 optimized regimes to minimize decoherence are discussed and, finally, section 6 provides some concluding remarks.

2. Cat-like states

The simplest example of a Schrödinger cat state of a single-mode radiation field is the following normalized superposition of coherent states:

$$|\alpha_0,\theta\rangle \equiv \frac{|\alpha_0\rangle + \mathrm{e}^{\mathrm{i}\theta}|-\alpha_0\rangle}{\sqrt{2+2\cos(\theta)\mathrm{e}^{-2|\alpha_0|^2}}}. \quad (1)$$

Such a cat-like state is referred to as 'even' for $\theta = 0$ and 'odd' for $\theta = \pi$ [12]. We denote by $\varrho_{\alpha_0,\theta}$ the corresponding density matrix, whose symmetrically ordered characteristic function is given by

$$\begin{aligned}\chi_{\alpha_0,\theta}(\eta) &\equiv \mathrm{Tr}[\varrho_{\alpha_0,\theta}D(\eta)]\\ &= \frac{1}{\pi}\int \langle\alpha|\varrho_{\alpha_0,\theta}|\eta+\alpha\rangle \mathrm{e}^{(\eta\alpha^*-\eta^*\alpha)/2}\,\mathrm{d}^2\alpha\\ &= \left(2+2\cos(\theta)\mathrm{e}^{-2|\alpha_0|^2}\right)^{-1}\mathrm{e}^{-|\eta|^2/2}[2\cosh(\alpha_0^*\eta-\alpha_0\eta^*)\\ &\quad + \mathrm{e}^{-2|\alpha_0|^2}2\cosh(\alpha_0^*\eta+\alpha_0\eta^*+\mathrm{i}\theta)],\end{aligned} \quad (2)$$

where $D(\eta) = \exp(\eta a^\dagger - \eta^* a)$ denotes the displacement operator. The corresponding Wigner function is defined as

$$W_{\alpha_0,\theta}(\alpha) \equiv \frac{1}{\pi^2}\int \mathrm{e}^{\eta^*\alpha-\eta\alpha^*}\chi_{\alpha_0,\theta}(\eta)\,\mathrm{d}^2\eta. \quad (3)$$

From now on, let us move to quadrature variables x and p, defined through $\alpha = (x+\mathrm{i}p)/\sqrt{2}$. By defining

$$\tilde{\boldsymbol{\sigma}} \equiv \begin{pmatrix}\frac{1}{2} & 0\\ 0 & \frac{1}{2}\end{pmatrix}, \qquad X \equiv \begin{pmatrix}x\\ p\end{pmatrix}, \quad (4)$$

one can write the Wigner function $W_{\alpha_0,\theta}(x,p)$ as follows:

$$\begin{aligned}W_{\alpha_0,\theta}(x,p) &= \left(2\pi\left(1+\cos(\theta)\mathrm{e}^{-(x_0^2+p_0^2)}\right)\sqrt{\det\tilde{\boldsymbol{\sigma}}}\right)^{-1}\\ &\quad\times\left[\mathrm{e}^{-\frac{1}{2}\left(X^T-(x_0,p_0)\right)\tilde{\boldsymbol{\sigma}}^{-1}\left(X-\binom{x_0}{p_0}\right)} + \mathrm{e}^{-\frac{1}{2}\left(X^T+(x_0,p_0)\right)\tilde{\boldsymbol{\sigma}}^{-1}\left(X+\binom{x_0}{p_0}\right)}\right.\\ &\quad\left.+\,\mathrm{e}^{-(x_0^2+p_0^2)}\left(\mathrm{e}^{-\frac{1}{2}\left(X^T-\mathrm{i}(-p_0,x_0)\right)\tilde{\boldsymbol{\sigma}}^{-1}\left(X-\mathrm{i}\binom{-p_0}{x_0}\right)+\mathrm{i}\theta} + \mathrm{c.c.}\right)\right].\end{aligned} \quad (5)$$

The first two Gaussian terms are related to the projectors $|\alpha_0\rangle\langle\alpha_0|$ and $|-\alpha_0\rangle\langle-\alpha_0|$: they are the Wigner functions of the two coherent states $|\alpha_0\rangle$ and $|-\alpha_0\rangle$. The remaining two terms correspond to non-diagonal operators and are responsible for the interference effects which characterize a coherent superposition.

We now move to the study of a 'squeezed cat', defined as the superposition of two squeezed coherent states. Let us introduce the operator b by means of a Bogolubov transformation

$$b \equiv \mu a + \nu a^\dagger, \qquad \text{with } |\mu|^2 - |\nu|^2 = 1 \quad (6)$$

and the states $|\beta\rangle$ as its eigenvectors: $b|\beta\rangle = \beta|\beta\rangle$. Such states are known in the literature as 'two-photon coherent states' and are indeed squeezed coherent states, according to the following well known relation [13]:

$$|\beta\rangle = D(\alpha)S(r_0,\varphi_0)|0\rangle, \quad (7)$$

with the requirements

$$\alpha = \mu\beta - \nu\beta^*, \qquad \cosh r_0 = \mu, \qquad \mathrm{e}^{\mathrm{i}2\varphi_0}\sinh r_0 = \nu, \quad (8)$$

and the squeezing operator defined as $S(r,\varphi) = \exp\left(\frac{1}{2}r\mathrm{e}^{-\mathrm{i}2\varphi}a^2 - \frac{1}{2}r\mathrm{e}^{\mathrm{i}2\varphi}a^{\dagger 2}\right)$.

We consider the following superposition:

$$|\beta_0,\theta\rangle \equiv \frac{|\beta_0\rangle + \mathrm{e}^{\mathrm{i}\theta}|-\beta_0\rangle}{\sqrt{2+2\cos(\theta)\mathrm{e}^{-2|\beta_0|^2}}}, \quad (9)$$

where the states $|\mp\beta_0\rangle$ are eigenstates of b: such a state will be referred to as to a 'squeezed cat' state.

The Wigner representation of the state $|\mp\beta_0\rangle$ can be easily found by recalling that a two-photon coherent state $|\beta_0\rangle$ may also be written as

$$|\beta_0\rangle = S(r_0,\varphi_0)D(\beta_0)|0\rangle,$$

with the squeezing parameters r_0 and φ_0 determined by equation (8). This means that one can promptly derive the Wigner function $W_{\beta_0,\theta}$ of a squeezed cat state by simply replacing α_0 with β_0 in equation (3) and then applying a squeezing transformation. In the following we will set $\varphi_0 = 0$, without loss of generality, as a reference choice for phase space rotation.

The squeezing transformation implemented by $S(r_0,0)$ corresponds, in terms of the phase-space variables $X = \binom{x}{p}$, to the map

$$X \to \mathbf{R}^{-1}X, \qquad \text{with } \mathbf{R} = \mathrm{diag}(\mathrm{e}^{r_0},\mathrm{e}^{-r_0}). \quad (10)$$

Applying such a transformation to the coherent cat Wigner function eventually yields

$$\begin{aligned}W_{\beta_0,\theta}(x,p) &= \frac{1}{2\pi\left(1+\cos(\theta)\mathrm{e}^{-(x_0^2+p_0^2)}\right)\sqrt{\det\boldsymbol{\sigma}_0}}\\ &\quad\times\left[\mathrm{e}^{-\frac{1}{2}\left(X^T-(x_0,p_0)\mathbf{R}\right)\boldsymbol{\sigma}_0^{-1}\left(X-\mathbf{R}\binom{x_0}{p_0}\right)}\right.\\ &\quad+\,\mathrm{e}^{-\frac{1}{2}\left(X^T+(x_0,p_0)\mathbf{R}\right)\boldsymbol{\sigma}_0^{-1}\left(X+\mathbf{R}\binom{x_0}{p_0}\right)}\\ &\quad\left.+\,\mathrm{e}^{-(x_0^2+p_0^2)}\left(\mathrm{e}^{-\frac{1}{2}\left(X^T-\mathrm{i}(-p_0,x_0)\mathbf{R}\right)\boldsymbol{\sigma}_0^{-1}\left(X-\mathrm{i}\mathbf{R}\binom{-p_0}{x_0}\right)+\mathrm{i}\theta} + \mathrm{c.c.}\right)\right],\end{aligned} \quad (11)$$

with

$$\boldsymbol{\sigma}_0 \equiv \mathbf{R}\tilde{\boldsymbol{\sigma}}\mathbf{R}. \quad (12)$$

Of course, for $r_0 = 0$ and $\beta_0 = \alpha_0$, one recovers equation (3) for a coherent cat.

3. Time evolution in noisy channels

We now consider the evolution in time of an initial squeezed cat state put in a noisy channel, in the presence of damping and/or pumping toward an asymptotic squeezed thermal state. The system is governed, in the interaction picture, by the following master equation [13]:

$$\begin{aligned}\dot{\varrho} &= \frac{\Gamma}{2}NL[a^\dagger]\varrho + \frac{\Gamma}{2}(N+1)L[a]\varrho - \frac{\Gamma}{2}(M^*D[a]\varrho\\ &\quad + MD[a^\dagger]\varrho),\end{aligned} \quad (13)$$

where the dot stands for the time derivative, the Lindblad superoperators are defined by $L[O]\varrho \equiv 2O\varrho O^\dagger - O^\dagger O\varrho -$

$\varrho O^\dagger O$ and $D[O]\varrho \equiv 2O\varrho O - OO\varrho - \varrho OO$, and M is the correlation function of the bath (which is usually referred to as the squeezing of the bath). It is in general a complex number $M \equiv M_1 + \mathrm{i}M_2$, while N is a phenomenological parameter related to the purity of the asymptotic state. Positivity of the density matrix imposes the constraint $|M|^2 \leqslant N(N+1)$. At thermal equilibrium, i.e. for $M = 0$, N coincides with the average number of thermal photons in the bath.

As is well known, equation (13) can be transformed into a linear Fokker–Planck equation for the Wigner function of the system [13]. Moreover, the Gaussian solutions of such an equation have been thoroughly analysed in previous works [5, 14]. The initial condition we consider here is described by the Wigner function $W_{\beta_0,\theta}$ of equation (12), which is just a linear combination of Gaussian terms. Therefore, its evolution in the noisy channel can be followed straightforwardly by exploiting the general results derived in [14]: each Gaussian term evolves independently and it suffices to follow the time dependence of its first and second statistical moments.

Let $\sigma_{ij} \equiv \frac{1}{2}\langle \hat{x}_i\hat{x}_j + \hat{x}_j\hat{x}_i\rangle - \langle\hat{x}_i\rangle\langle\hat{x}_j\rangle$ and $X_{0i} \equiv \langle\hat{x}_i\rangle$ be, respectively, the covariance matrix and the vector of the first moments of a Gaussian state ($\hat{x}_1, \hat{x}_2 = \hat{x}, \hat{p}$ being the quadrature phase operators). Then, the time-evolution of $\boldsymbol{\sigma}(t)$ and $X_0(t)$ in the squeezed thermal channel is described by the following equation [14]:

$$X_0(t) = \mathrm{e}^{-\frac{\Gamma}{2}t} X_0(0), \tag{14}$$

$$\boldsymbol{\sigma}(t) = \boldsymbol{\sigma}_\infty(1 - \mathrm{e}^{-\Gamma t}) + \boldsymbol{\sigma}(0)\mathrm{e}^{-\Gamma t}, \tag{15}$$

$$\text{with } \boldsymbol{\sigma}_\infty = \begin{pmatrix} \frac{(2N+1)+2M_1}{2} & M_2 \\ M_2 & \frac{(2N+1)-2M_1}{2} \end{pmatrix}. \tag{16}$$

The time-dependent solution for the Wigner function $W_{\beta_0,\theta}(t)$ of an initial squeezed cat is thus readily found and reads

$$\begin{aligned} W_{\beta_0,\theta}(x, p) &= \left(2\pi\left(1 + \cos(\theta)\mathrm{e}^{-(x_0^2+p_0^2)}\right)\sqrt{\det\boldsymbol{\sigma}(t)}\right)^{-1} \\ &\times \Big[\mathrm{e}^{-\frac{1}{2}(X^T - \mathrm{e}^{-\frac{\Gamma}{2}t}(x_0,p_0)\mathbf{R})\boldsymbol{\sigma}(t)^{-1}\left(X - \mathrm{e}^{-\frac{\Gamma}{2}t}\mathbf{R}\binom{x_0}{p_0}\right)} + \mathrm{e}^{-(x_0^2+p_0^2)} \\ &\times \left(\mathrm{e}^{-\frac{1}{2}\left(X^T - \mathrm{i}\mathrm{e}^{-\frac{\Gamma}{2}t}(-p_0,x_0)\mathbf{R}\right)\boldsymbol{\sigma}(t)^{-1}\left(X - \mathrm{i}\mathrm{e}^{-\frac{\Gamma}{2}t}\mathbf{R}\binom{-p_0}{x_0}\right)+\mathrm{i}\theta} + \mathrm{c.c.}\right) \\ &+ \mathrm{e}^{-\frac{1}{2}\left(X^T + \mathrm{e}^{-\frac{\Gamma}{2}t}(x_0,p_0)\mathbf{R}\right)\boldsymbol{\sigma}(t)^{-1}\left(X + \mathrm{e}^{-\frac{\Gamma}{2}t}\mathbf{R}\binom{x_0}{p_0}\right)}\Big], \end{aligned} \tag{17}$$

with $\boldsymbol{\sigma}(t)$ given by equations (15) and (16). The first moments of each Gaussian term are exponentially damped in the channel. Any initial cat state is attracted toward an asymptotic centred squeezed thermal state with Wigner function

$$W_\infty(x, p) = \frac{\mathrm{e}^{-\frac{1}{2}X^T\boldsymbol{\sigma}_\infty^{-1}X}}{\pi\sqrt{\det\boldsymbol{\sigma}_\infty}}. \tag{18}$$

This state, like all asymptotic quantities, is a property of the channel and does not depend on the initial state.

4. Decoherence of an initial cat state

In order to quantify decoherence of the state caused by environmental noise, we consider the loss of purity. The degree of purity of a continuous variable quantum state ϱ can be effectively characterized either by its Von Neumann entropy $S_\mathrm{V} \equiv -\mathrm{Tr}(\varrho\ln\varrho)$ or by its linear entropy S_l

$$S_\mathrm{l} \equiv 1 - \mathrm{Tr}(\varrho^2) \equiv 1 - \mu = 1 - \frac{\pi}{2}\int_{\mathbb{R}^2} W^2\,\mathrm{d}x\,\mathrm{d}p. \tag{19}$$

In the following we will adopt linear entropy, which can be conveniently evaluated. The quantity $\mu = \mathrm{Tr}\varrho^2$, conjugate to S_l, will be referred to as the 'purity' from now on.

For an initial pure cat state ($\mu = 1$) at time $t = 0$, the asymptotic value μ_∞ can be obtained from equation (18) by straightforward integration [14]

$$\mu_\infty = \frac{1}{2\sqrt{\det\boldsymbol{\sigma}_\infty}} = \frac{1}{\sqrt{(2N+1)^2 - 4|M|^2}}. \tag{20}$$

At finite times the purity of a decohering cat can be determined by integrating the function $W_{\beta_0,\theta}(x, p)$ given by equation (17), according to equation (19). The integration can be promptly performed with the help of the following thumb rule:

$$\begin{aligned} &\text{if} && X_1 \equiv \begin{pmatrix} x - x_1 \\ p - p_1 \end{pmatrix}, \qquad X_2 \equiv \begin{pmatrix} x - x_2 \\ p - p_2 \end{pmatrix} \\ &\text{and} && \bar{X} \equiv \begin{pmatrix} x_1 - x_2 \\ p_1 - p_2 \end{pmatrix}, \\ &\text{then} && \int \mathrm{e}^{-\frac{1}{2}X_1^T\boldsymbol{\sigma}^{-1}X_1}\mathrm{e}^{-\frac{1}{2}X_2^T\boldsymbol{\sigma}^{-1}X_2}\,\mathrm{d}x\,\mathrm{d}p \\ &&& = \pi\sqrt{\det\boldsymbol{\sigma}}\,\mathrm{e}^{-\frac{1}{4}\bar{X}^T\boldsymbol{\sigma}^{-1}\bar{X}}. \end{aligned} \tag{21}$$

Exploiting the above rule one eventually has

$$\begin{aligned} \mu_{\beta_0,\theta}(t) &= \left(8\left(1 + \cos(\theta)\mathrm{e}^{-(x_0^2+p_0^2)}\right)^2\sqrt{\det\boldsymbol{\sigma}(t)}\right)^{-1} \\ &\times \Big[2\left(1 + \mathrm{e}^{-\mathrm{e}^{-\Gamma t}X_0^T\mathbf{S}(t)X_0}\right) + 2\mathrm{e}^{-2(x_0^2+p_0^2)} \\ &\times \left(\cos(2\theta) + \mathrm{e}^{\mathrm{e}^{-\Gamma t}X_0^T\mathbf{T}(t)X_0}\right) + 4\mathrm{e}^{-(x_0^2+p_0^2)}\cos(\theta) \\ &\times \left(\mathrm{e}^{-\mathrm{e}^{-\Gamma t}(x_0+\mathrm{i}p_0)^2A(t)} + \mathrm{c.c.}\right)\Big], \end{aligned} \tag{22}$$

with

$$A(t) \equiv \frac{S_{xx}(t) - S_{pp}(t) - 2\mathrm{i}S_{xp}(t)}{4} \tag{23}$$

and

$$\mathbf{S}(t) \equiv \mathbf{R}\boldsymbol{\sigma}(t)^{-1}\mathbf{R}, \qquad \mathbf{T}(t) \equiv (\det\boldsymbol{\sigma})^{-1}\mathbf{S}(t)^{-1}. \tag{24}$$

Equation (22) shows that $\mu_{\beta_0,\theta}$ is a decreasing function of the initial parameters x_0 and p_0. This should be expected: the 'bigger' the cat is, the faster it decoheres. For $\beta_0 = 0$ the superposition disappears and the cat state reduces to a centred squeezed state; the latter, whose evolution in noisy channels has been studied in [14], decoheres more slowly than the corresponding cat state. The numerical analysis shows that the phase θ has little effect on the behaviour of purity at large times: in fact, as is evident from equation (22), all the terms involving θ are suppressed by Gaussian terms of the form $\exp(-x_0^2 - p_0^2)$. In figure 1 the behaviour of the purity over the full temporal range up to the asymptotic regime is shown for various choices of the parameters of the channel and of the initial state.

One feature which is most evident in all instances is the fast initial fall of the purity. Although the minimum value of the purity attained in such a steep descent can vary, the

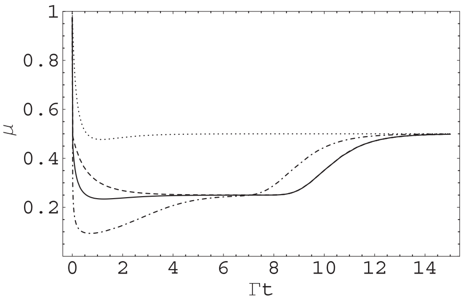

Figure 1. Evolution of the purity for different initial cat states and channels. In all cases the asymptotic purity of the bath is $\mu_\infty = 0.5$. The dotted curve shows the behaviour of an initial cat with $x_0 = p_0 = 1$ and $r_0 = 0$ in a non-squeezed channel. The dashed and the continuous curves refer to an initial cat with $x_0 = p_0 = 100$ and $r_0 = 0$ evolving, respectively, in a non-squeezed channel and in a squeezed channel with $M = 2 + 2\mathrm{i}$. The dot–dashed curve refers to an initial cat state with $x_0 = p_0 = 10$ and $r_0 = 2$, evolving in a non-squeezed channel.

temporal scale in which the minimum is reached is always of the order of Γ^{-1}, which is indeed the only time characterizing the losses in the channel. Besides, it can be seen that the general behaviour of purity in squeezed baths is almost the same as in non-squeezed ones. One can also see that the value of the first minimum of the purity depends drastically on the squeezing parameter r_0, decreasing with greater r_0, while, for a given squeezing parameter, an increase in the parameters x_0 and p_0 delays the reaching of the asymptotic purity (see figure 1).

5. Optimal regimes

Optimal regimes with minimized decoherence can be determined by maximizing the purity at any given time for fixed values of the parameters of the channel and of the initial state. Notice that, as we have shown in the previous section, a cat-like state decoheres on a timescale of the order of the photon lifetime Γ^{-1}, regardless of the choice of the parameters of both the Gaussian reservoir and the initial pure cat state. This fact is a manifestation of a fundamental feature of quantum mechanics: once a single photon is added or lost, all the information contained in a coherent superposition 'leaks out to the environment' and is therefore lost as well, together with the possibility of detecting such a coherent behaviour by means of interferometry [15, 16]. A simple, meaningful example in this respect is just a coherent even cat $|\alpha_0, 0\rangle$ subjected to damping. Under the loss of a photon, such a state jumps into $a|\alpha_0, 0\rangle \propto |\alpha_0, \pi\rangle$, which is an odd cat and has opposite interference terms. Therefore, as soon as the probability of losing a photon reaches 0.5, the original superposition turns into an incoherent mixture of an even and an odd cat, whose interference terms cancel out each other exactly [16].

Actually, a more detailed analysis would reveal that decoherence times are even shorter than Γ^{-1}: for a coherent cat $|\alpha_0, \theta\rangle$ evolving in a thermal environment, coherence is lost at $t_{\mathrm{dec}} = \Gamma^{-1}/2|\alpha_0|^2$ [4]. In view of these considerations, we are interested in maximizing the purity in the time region $\Gamma t \lesssim 1/2|\beta_0|^2$.

A relevant question in such a context is the following: given an initial bath and an initial squeezing of the cat-like state, which is the optimal phase space direction of $\beta_0 \equiv |\beta_0|\mathrm{e}^{\mathrm{i}\xi}$ at fixed $|\beta_0|$? Note that the last condition can be seen as a constraint on the energy of the cat-like state. Obviously, for a non-squeezed cat in a thermal channel, the symmetry of the problem forbids the existence of a privileged direction.

Interesting issues come instead from the consideration of a squeezed cat in a thermal channel and a non-squeezed cat in a squeezed bath. For the moment, let us consider the instance of an even cat, i.e. of a cat with coherent phase $\theta = 0$. The dependence of the purity on β_0 is essentially contained in exponentials of quadratic forms, see equation (22). The algebraic analysis of such terms in the case of a squeezed cat in thermal baths and of a coherent cat in squeezed baths is quite easy (see appendix). If the squeezing is performed on the initial cat state, the coherence is better preserved if β_0 is chosen in the same phase space direction as the variance which is suppressed by squeezing. With our notation, this corresponds to $\xi = \varphi_0 + \pi/2$ (where φ_0 is the squeezing angle). In contrast, if the squeezing is performed on the bath in the direction φ_∞, the choice $\xi = \varphi_\infty$ turns out to be the best one to slow down the rate of decoherence. This somewhat counterintuitive situation is due to the existence of complex fringe patterns of a cat state in phase space, especially in the presence of squeezing. Actually, preserving quantum coherence is crucially related to the persistence of the interference fringes: reducing quantum fluctuations in a phase space direction protects the fringes from degradation and the cat state from decoherence. With the above expedient choices of the phase ξ, squeezing does actually improve the coherence of the superposition at small times with respect to dissipation in a phase insensitive setting. This result, obtained by the exact computation of the purity during the evolution, complies with the results of [5], in which the coherence of the evolving state was evaluated by analysing the interference patterns of homodyne detections.

Now, let us suppose that for an even cat state the phase ξ is optimally chosen and let us consider a channel with asymptotic purity μ_∞. It can be easily shown (see the appendix) that the purity of an initially squeezed cat (with squeezing r) in such a thermal channel equals, at any given time, that of a non-squeezed cat evolving in a squeezed channel with the same squeezing r. The same protection against decoherence provided by the squeezing of the bath can be obtained, in a thermal phase insensitive channel, by squeezing the initial even cat state of the same amount in an orthogonal direction.

We finally remark that, with an optimal phase setting, an optimal finite value of the squeezing parameter r does indeed exist (see figure 2). This fact can be best appreciated from equation (22): even if the exponential terms increase with increasing r, the factor $\det \boldsymbol{\sigma}^{-1/2}$ decreases with r. The value of r allowing the maximum slowing down of decoherence increases with increasing $|\beta_0|$.

We now briefly consider the instance $\theta = \pi/2$ [2, 17]: in such a case, the last term in equation (22) vanishes, so that the optimal choice for ξ can be easily found for any choice of r_0, r_∞ and φ_∞. To this end, it is sufficient to determine the eigenvector corresponding to the smallest eigenvalue of the matrix $\mathbf{S}(t)^{-1}$, which coincides with the analogous eigenvector of $\mathbf{R}^{-1}\boldsymbol{\sigma}_\infty\mathbf{R}^{-1}$ and does not depend on time. However, we once again stress that, as soon as one deals with mesoscopic cat states (so that $\exp(-|\beta_0|^2) \ll 1$), the dependence on the coherent phase θ is severely suppressed and all the considerations made for even cat states still hold, regardless of the choice of θ. Results for the evolution of purity at small times are summarized in figure 2.

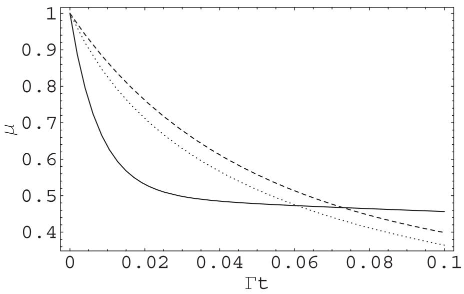

Figure 2. Comparison between the evolution of purity of an initial non-squeezed cat in a non-squeezed bath (continuous curve) and of initial cats in squeezed configurations with optimal phase choices. The dashed curve refers to an initial squeezed cat with $r_0 = 1$, whereas the dotted curve refers to an initial squeezed cat with $r_0 = 1.5$. In all instances $\mu_\infty = 0.5$, $|\beta_0|^2 = 16$, and $\theta = 0$. The decoherence time for the non-squeezed cat is $t_{\rm dec} \simeq 0.03\Gamma$, in good agreement with the initial decrease of purity. The choice $r_0 \simeq 1$ appears to be optimal for such a value of $|\beta_0|$.

6. Conclusions

The study of the decoherence of initial coherent and squeezed coherent cat states in arbitrary Gaussian reservoirs has been carefully carried out by determining the exact time evolution of the purity of the state. Optimal settings that minimize the rate of decoherence in relevant configurations have been determined.

In particular, we have shown that the same protection against decoherence granted by a squeezed bath can be achieved by squeezing the initial cat-like state. In view of the well known difficulties involved in the experimental realization of squeezed baths, even as effective descriptions of feedback schemes [7], this equivalence could provide a relevant alternative option for experimental purposes.

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Appendix

In this appendix we analytically single out the optimal phase space orientations of the cat state for the configurations discussed in section 5. The orientation of the cat state is determined by the angle $\xi = \arg(\beta_0)$. In the following we will set $\theta = 0$ and define, for ease of notation, $u \equiv (1 - \exp(-\Gamma t))/2\mu_\infty$ and $v \equiv \exp(-\Gamma t)/2$. The squeezing parameter r_∞ of the bath is determined by $\cosh(2r_\infty) = \sqrt{1 + 4\mu_\infty^2 |M|^2}$ [14].

We first consider a squeezed cat in a thermal channel, with $r_0 \neq 0 = r_\infty$. In this instance, one has

$$\mathbf{S}(t) = \mathrm{diag}\{e^{2r_0}(u + e^{2r_0} v)^{-1}, e^{-2r_0}(u + e^{-2r_0} v)^{-1}\},$$

and

$$A(t) = (2 \det \boldsymbol{\sigma}(t))^{-1} u \sinh(2r_0).$$

Substituting these expressions in equation (22), it is easy to see that, for fixed $2|\beta_0|^2 = x_0^2 + p_0^2$, all the exponential terms are maximized by the choice $x_0 = 0$, $p_0 = \sqrt{2}|\beta_0|$. This corresponds to $\xi = \pi/2 = \varphi_0 + \pi/2$.

Analogously, for a non-squeezed cat in a squeezed channel (with $r_0 = 0 \neq r_\infty$), one gets[4]

$$\mathbf{S}(t) = \mathrm{diag}\{e^{-2r_\infty}(u + e^{-2r_\infty} v)^{-1}, e^{2r_\infty}(u + e^{2r_\infty} v)^{-1}\},$$

and

$$A(t) = -(2 \det \boldsymbol{\sigma}(t))^{-1} u \sinh(2r_\infty).$$

Equation (22) shows that the choice $x_0 = \sqrt{2}|\beta_0|$, $p_0 = 0$ (corresponding to $\xi = 0 = \varphi_\infty$) is optimal in this case.

Finally, it is easy to verify that, adopting such optimal choices and putting $r_0 = r_\infty$, all the exponential terms entering equation (22) take the same values for a squeezed cat in a thermal channel and a non-squeezed cat in a squeezed channel. Since $\det \boldsymbol{\sigma}$ depends only on the difference $|r_0 - r_\infty|$ [14], this implies that the time evolutions of purity are identical in these two instances.

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