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GENERATION AND MEASUREMENT OF NONCLASSICAL STATES BY QUANTUM FOCK FILTER*

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We study a novel optical setup which selects a specific Fock component from a generic input state. The device allows to synthesize number states and superpositions of few number states, and to measure the photon distribution and the density matrix of a generic signal.

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1 Introduction

In the last two decades in Quantum Optics there has been a fast and exciting development. Many experiments involving the optical field fall into the quantum domain, and many quantum mechanical *gedanken* experiments are now performed in optical laboratories. Basic concepts of quantum mechanics such as the reduction postulate, the uncertainty relation, the Schödinger cat and nonlocal phenomena play a major role and their effects can be directly observed. Besides this fundamental interest, the field receives attention also in view of future applications, which are mainly motivated by the potential improvement offered by Quantum Mechanics to the manipulation and the transmission of information. Such recent developments have renewed the interest for two fundamental themes in Quantum Optics, namely the *generation* and the *measurement* of nonclassical states of the radiation field.

In this paper, we address both these aspects and suggest a novel all-optical device which is able to select a fixed Fock component from a traveling wave initially prepared in a generic (possibly mixed) state. The scheme, which we named *Fock Filter*, consists of a ring cavity coupled to the signal through a cross-Kerr medium. At the output of the device, the signal and the cavity modes are strongly entangled, so that a successful photodetection of the cavity modes reduces the signal into a state with the desired number of photons. The proposed setup has two main applications. On one hand, it can be used to synthesize a generic number state $|n\rangle$ or a superposition of few number states, say $|\psi\rangle \propto \alpha|n_1\rangle + \beta|n_2\rangle$, starting from a coherent source. On the other hand, it allows to measure the photon distribution $P(n) = \langle n|\hat{\nu}|n\rangle$ and the density matrix $\hat{\nu}_{nk} = \langle n|\hat{\nu}|k\rangle$ of a generic input signal $\hat{\nu}$.

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2 The Fock Filter

The device we propose is schematically depicted in Fig. 1. It consists of an active ring cavity coupled to the signal by a cross-Kerr medium. The cavity has a high quality factor, namely it is built by low transmissivity (denoted by τ) beam splitters. The cross-Kerr interaction couples the cavity mode d to the traveling signal mode c_1, according to the unitary evolution $\hat{U}_K = \exp(-i\chi t d^\dagger d c_1^\dagger c_1)$. In this way, the cavity mode experiences a phase-shift which depends on the quantum state of the signal mode. A further tunable phase-shift ψ is also inserted in the cavity path. A port of the cavity (mode a_1) is fed by a strong coherent probe, whereas the second port (mode a_2) is left vacuum. At the output of the cavity, the mode b_1 is simply absorbed, whereas the mode b_2 is monitored by an avalanche photodetector. As we will see in the following, we only need to know whether or not any photon is present, namely to perform an ON/OFF photodetection.

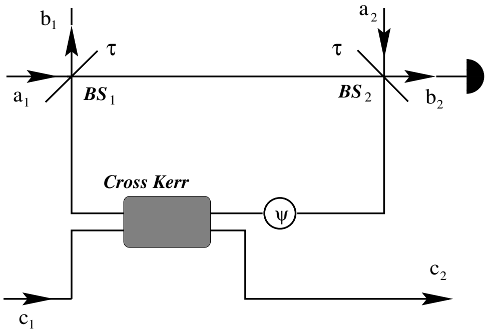

Fig. 1: Schematic diagram of the Fock Filter. BS_1 and BS_2 denote low transmissivity beam splitters, and ψ a tunable phase shift. The cavity input modes a_1 and a_2 are in a coherent state and in the vacuum respectively. The box denotes the cross Kerr medium that couples the cavity mode with the signal mode. A successful detection at the photodiode D filters the desired Fock component from the input signal.

The probability operator measure (POM) describing such a detection scheme is given by the the two-value operator $\hat{\Pi}_0 \doteq \sum_{k=0}^{\infty}(1-\eta)^k|k\rangle\langle k|$ and $\hat{\Pi}_1 \doteq \hat{\mathbb{I}} - \hat{\Pi}_0$ where η is the quantum efficiency of the photodetector. The modes transformation of the cavity is expressed by [1]

$$\left\{ \begin{array}{l} b_1 = \kappa(\varphi)a_1 + e^{i\varphi}\sigma(\varphi)a_2 \\ b_2 = \sigma(\varphi)a_1 + \kappa(\varphi)a_2 \end{array} \right. , \tag{1}$$

where the phase-dependent transmissivity σ and reflectivity κ amplitude are given by

$$\kappa(\varphi) \doteq \frac{\sqrt{1-\tau}(e^{i\varphi}-1)}{1-[1-\tau]e^{i\varphi}} \qquad \sigma(\varphi) \doteq \frac{\tau}{1-[1-\tau]e^{i\varphi}}, \tag{2}$$

with $|\kappa(\varphi)|^2 + |\sigma(\varphi)|^2 = 1$. The phase φ is the total phase-shift experienced by the cavity mode, namely the sum of the shift due to the Kerr interaction and the tunable shift ψ. For the signal in a number state $|n\rangle$ the total phase shift imposed to the cavity mode is $\varphi \equiv \varphi_n = \psi - \chi n t$. In order to simplify notation, we write $\sigma_n \doteq \sigma(\varphi_n)$ and analogously $\kappa_n \doteq \kappa(\varphi_n)$. The input state of the device can be written as $\hat{\varrho}_{in} = |\alpha\rangle\langle\alpha| \otimes |0\rangle\langle 0| \otimes \hat{\nu}_{in}$, namely a generic state $\hat{\nu}_{in}$ for the signal mode and a strong coherent state $|\alpha\rangle$ for the probe, the second port of the cavity being left unexcited. The output state can be easily found in the Schrödinger picture as

$$\hat{\varrho}_{out} = \sum_{n,m=0}^{\infty} \nu_{nm}\, |\kappa_n\alpha\rangle\langle\kappa_m\alpha| \otimes |\sigma_n\alpha\rangle\langle\sigma_m\alpha| \otimes |n\rangle\langle m| \ . \tag{3}$$

The measurement scheme consists in detecting whether (detector D on) or not (detector D off) any photon is present at the output of the cavity. The corresponding probabilities are given by

$$P_1 = \mathrm{T}r[\hat{\Pi}_1\,\hat{\varrho}_{out}] = \sum_{n=0}^{\infty} \nu_{nn} \left(1 - e^{-\eta|\alpha|^2\,|\sigma_n|^2}\right) \qquad P_0 = 1 - P_1 \ , \tag{4}$$

whereas the output signal conditioned by the photodetection reads

$$\hat{\nu}_{out}(\mathsf{ON}) \;=\; \frac{e^{-|\alpha|^2}}{P_1} \sum_{n,m=0}^{\infty} \nu_{nm}\, e^{|\alpha|^2\left[\kappa_n\kappa_m^* + \sigma_n\sigma_m^*\right]} \left(1 - e^{-\eta|\alpha|^2\sigma_n\sigma_m^*}\right) |n\rangle\langle m| \ . \tag{5}$$

The filtering properties of the device are due to the strong dependence of the cavity transmissivity on the internal phase-shift. The overall transmissivity function writes as

$$|\sigma_n|^2 = \left[1 + 4\frac{1-\tau}{\tau^2}\sin^2\frac{\psi - \chi n t}{2}\right]^{-1} \ , \tag{6}$$

and exhibits a periodic structure sharply peaked at $n = n^* + 2\pi j/(\chi t)$ with $n^* = \psi/(\chi t)$ and $j \in \mathbb{Z}$. In the peaks, it has unit height and width of the order of the beam splitter transmissivity τ (typically $\tau \sim 1\% - 0.01\%$). The value n^* can be adjusted to an arbitrary integer by tuning the phase-shift ψ as a multiple of χt, whereas multiple resonances are avoided by using relatively small values of the nonlinearity χt, so that the values of n for $j \neq 0$ correspond to vanishing matrix elements $\nu_{ni} \simeq 0\ \forall i$. In this way the cavity is set at resonance only by a single Fock component $|n^*\rangle$ of the signal, which is *filtered* at the output in the case of successful photodetection. In the next section we will analyze this process in more detail.

3 Synthesis of number states

Let us now consider a cavity with a high quality factor (i.e. $\tau \ll 1$) adjusted to select the Fock component n^* by tuning $\psi = \chi t n^*$. In this case, the detection probability and the conditional output states of Eqs. (4) and (5) rewrite as follows

$$P_1 \simeq (1 - e^{-\eta|\alpha|^2})\nu_{n^*n^*} + \frac{\eta|\alpha|^2\tau^2}{(\chi t)^2} \sum_{p\neq n^*} \frac{\nu_{pp}}{(n^* - p)^2} \ , \tag{7}$$

and

$$\hat{\nu}_{out}(\mathsf{ON}) \simeq \frac{1}{\sqrt{\mathcal{N}}}\left[|n^*\rangle\langle n^*| + \frac{\eta|\alpha|^2\tau^2}{(\chi t)^2} \sum_{n,k\neq n^*} \frac{\nu_{nk}}{(n^*-n)(n^*-k)}|n\rangle\langle k|\right] \ , \tag{8}$$

where $\mathcal{N} = 1 + (\eta|\alpha|^2\tau^2)/(\chi t)^2 \sum_{p\neq n^*} \nu_{pp}/(p - n^*)^2$ is a normalization constant. Both equations are valid when small values of the nonlinearity are involved, namely when a single Fock component sets the cavity into resonance. The physical meaning of Eqs. (7) and (8) is apparent: when the cavity is "good enough" to appreciate the phase-shift imposed by the passage of the desired Fock component [i.e. when $\tau \ll (\chi t)$] the detection probability equals the probability of having n^* photons in the signal $P_1 \simeq \nu_{n^*n^*}$, and the conditional output state approaches the corresponding number state $\hat{\nu}_{out}(\mathsf{ON}) \simeq |n^*\rangle\langle n^*|$, which is *synthesized* from the input signal. Eqs. (7) and (8) also illustrate the effect of nonunit quantum efficiency of the probe photodetector. If η is lower than 100%, the detection probability decreases, and thus also the synthesizing rate. However, the synthesized state is closer to the desired number state, namely the synthesizing quality is improved. In Fig. 2 the synthesis of the number state $|n^* \equiv 4\rangle$ is illustrated for decreasing values of the beam splitter transmissivity.

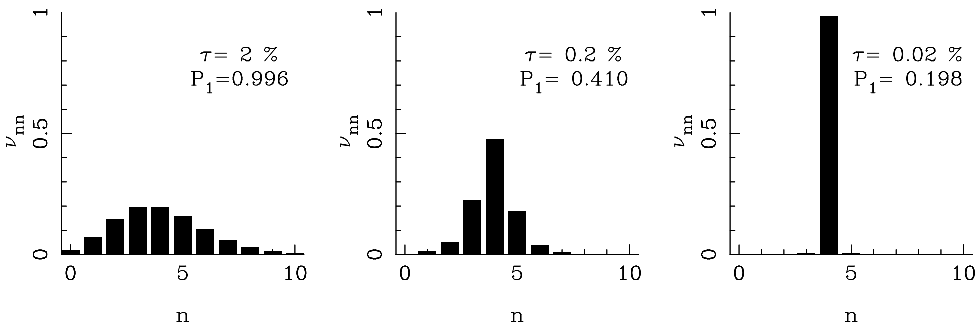

Fig. 2: The photon distribution of the conditional output state $\hat{\nu}_{out}(\mathsf{ON})$ for different values of the transmissivity τ, reported on each plot. The plots refer to the synthesis of the number state $|n^* \equiv 4\rangle$, with $\chi t = 0.01$, $\psi = 0.04$ and $\eta = 0.8$. Both the probe mode a_1 and the signal mode c_1 are excited in a coherent state, with real amplitude $\alpha = 20$ and $\beta = 2$ respectively. The probabilities of obtaining the three states (i.e. the detection probability at D) are also indicated on each plot. The last plot shows that a transmissivity $\tau = 0.02\%$ is sufficient for a reliable synthesis of the desired number state.

The Fock Filter behaves differently when larger nonlinearities or quite excited input signals are involved. In this case, the cavity may be set at resonance by several Fock components of the signal mode, corresponding to different integers that are multiple of χt. Let us consider, as an example, the situation in which two Fock components, corresponding to the values $n_1 \equiv n^*$ and $n_2 = n_1 + 2\pi/(\chi t)$, set the cavity into resonance. For $\tau \ll \chi t$ Eqs. (4) and (5) can be written as $P_1 \simeq \nu_{n_1 n_1} + \nu_{n_2 n_2}$ and $\hat{\nu}_{out}(\mathsf{ON}) \simeq 1/P_1 \Big[\nu_{n_1 n_1}|n_1\rangle\langle n_1| + \nu_{n_2 n_2}|n_2\rangle\langle n_2| + \nu_{n_1 n_2}|n_1\rangle\langle n_2| + \nu_{n_2 n_1}|n_2\rangle\langle n_1|\Big]$. If the input signal $\hat{\nu}$ is excited in a coherent state, one has $\nu_{n_1 n_1}\nu_{n_2 n_2} = \nu_{n_1 n_2}\nu_{n_2 n_1}$, and hence $\hat{\nu}_{out}(\mathsf{ON})$ is a pure state. Actually, by varying the amplitude of the input coherent signal any superposition of the form $|\psi\rangle \propto \alpha|n_1\rangle + \beta|n_2\rangle$ may be synthesized at the output. We just mention that this kind of superposition is the paradigm for the realization of an optical qubit.

4 State measurement

In this section we show how the Fock filter can be used to measure the photon distribution, and the whole density matrix, of a generic input signal. The method is based on Eq. (4), which shows that for moderate nonlinearities and low beam splitter transmissivity ($\tau \ll \chi t < 1$) the detection probability P_1 at the output of the cavity is equal to the diagonal matrix element of the signal $P_1 \simeq \nu_{n^* n^*}$ corresponding to the integer n^* selected by tuning the phase ψ. Therefore, by repeated preparations of the signal and by varying the cavity tuning $\psi = \chi t n^*$,

$n^* = 0, 1, 2, ...$ in order to span the whole excitation spectrum of the signal it is possible to record the photon number distribution of a generic input state. Actually, such a measurement may be implemented by a more efficient scheme using a set of cavities in cascade, each tuned on a different integer n_k. The input state of the k-th cavity is the output state from the $(k-1)$-th one and this allows to largely reduce the number of repeated preparations of the signal. At the k-th step, in the limit of good cavities, the detection probabilities approaches $P_1^{(k)} \simeq \nu^{(k)}_{n_k n_k}$ and $P_0^{(k)} \simeq 1 - \nu^{(k)}_{n_k n_k}$ where the density matrix $\hat{\nu}^{(k)}$ has been reduced according to the result of the detection at the $(k-1)$-th photodiode as in Eq. (5). For good cavities one has

$$\hat{\nu}^{(k)}_{out}(\mathsf{ON}) \simeq |n_k\rangle\langle n_k| \ , \qquad \hat{\nu}^{(k)}_{out}(\mathsf{OFF}) \simeq \sum_{p \neq n_k} \frac{\nu^{(k)}_{pp}}{P_0^{(k)}} |p\rangle\langle p| \ . \tag{9}$$

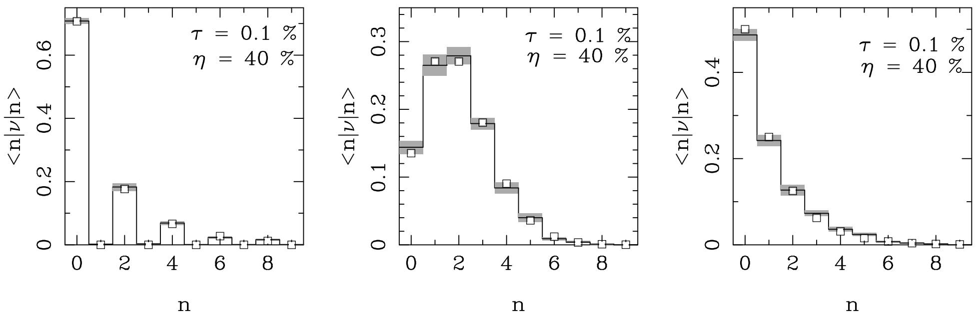

Fig. 3: Monte Carlo simulations of the detection of the photon number distribution by the Fock filter with $\chi t = 0.1$, $\tau = 0.1\%$ and $\eta = 40\%$. The distributions for a squeezed vacuum with $\langle a^\dagger a\rangle = 1$ average photons, a coherent state with $\langle a^\dagger a\rangle = 2$ average photons and a thermal state with $\langle a^\dagger a\rangle = 1$ average photons are reported from the left to the right together with the corresponding confidence interval. The empty squares indicates the theoretical values. In all cases a sample of 2000 data has been used.

We checked the whole detection strategy with a Monte Carlo simulation. In Fig. 3 we show the photon distributions obtained for a squeezed vacuum, a coherent state and a thermal state at the input. Remarkably, the photon distributions are reliably determined using a relatively small sample of data and a low value for the quantum efficiency of the photodetectors.

The Fock filter, in conjunction with the unbalanced homodyning technique [2,3], allows also to measure the entire density matrix of the input signal. In order to achieve this goal, it is necessary to mix the input signal $\hat{\nu}$ with a strong coherent state $|z\rangle$ by using a high transmissivity beam splitter before the set of cavities (see Fig. 4). In the limit $(1-\tau) \ll 1$ and $|z| \gg 1$ the state entering the set of cavities is the displaced signal $\hat{\nu}_\gamma = \hat{D}(\gamma)\hat{\nu}\hat{D}^\dagger(\gamma)$, where $\hat{D}(\gamma) = \exp(\gamma a^\dagger - \bar{\gamma} a)$ is the displacement operator and $\gamma = z\sqrt{1-\tau}$. In this case, the Fock filter provides the photon distribution of the displaced state $P_\gamma(n) = \langle n|\hat{\nu}_\gamma|n\rangle$, which can be expressed in terms of the density matrix of the original signal $\hat{\nu}$ by the linear relation $P_\gamma(n) = \sum_{km}\langle k|\hat{\nu}|m\rangle A_{kmn}(\gamma)$ where $A_{kmn}(\gamma) = \langle n|\hat{D}(\gamma)|k\rangle\langle m|\hat{D}^\dagger(\gamma)|n\rangle$. By measuring the photon distribution for different values of the displacing amplitude γ, this relation can be inverted, leading to the reconstruction of the signal density matrix. In particular, Opatrný *et*

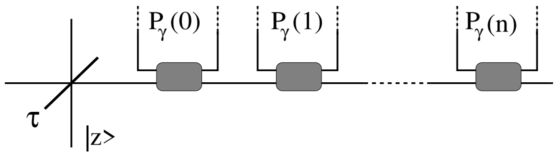

Fig. 4: Schematic diagram of the setup for measuring the density matrix with the Fock filter. The input state is displaced, and then enters the set of cavities in cascade, each tuned on a different Fock component.

al. [4] have shown that it is enough to measure $P_\gamma(n)$ for a fixed value of the modulus $|\gamma|$ and different values of the phase. In this case one has

$$P^{(s)}_{|\gamma|}(n) = \sum_{m=0}^{M-s} A_{m+s,m,n}(|\gamma|)\, \nu_{m+s,m}\ , \tag{10}$$

where M is the maximum Fock component excited in the signal, and $P^{(s)}_{|\gamma|}(n)$ is the Fourier transform of the photon distribution obtained by varying the phase $\varphi = \arg(\gamma)$ The linear system (10) is overdetermined and may be solved by least squares method. The solution represents the best estimate for the density matrix of the input signal.

5 Conclusions

In this paper, we have studied a novel all-optical device: the Fock filter, which is able to select the desired Fock component starting from a generic input state. The device may be used to synthesize number states and superpositions of few number states, as well as for measuring the photon distribution and the density matrix of a generic input signal. The feasibility of the proposed setup relies on the realization of cavities with a high quality factor, namely cavities built with low transmissivity beam splitters. Monte Carlo simulations have shown that transmissivities of the order of $\tau \sim 1\% - 0.01\%$ are needed, which corresponds to beam splitters currently available in optical labs. We conclude by pointing out that the quantum efficiency of the photodetectors is not a crucial parameter for the Fock filter: low quantum efficiency does not degrade the performances of the device, for both the generation and the measurement scheme.

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