Non collinear configuration for dichromatic squeezing

Alessandra Andreoni and Maria Bondani
Unitá INFM di Como and Dipartimento di Scienze Chimiche Fisiche e Matematiche
Universitá dell'Insubria, via Valleggio 11, I-22100 Como, Italia
G. Mauro D'Ariano and Matteo G. A. Paris
Quantum Optics Group, Unitá INFM and Dipartimento di Fisica 'Alessandro Volta'
Universitá di Pavia, via Bassi 6, I-27100 Pavia, Italia

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Abstract

We propose a non collinear experimental scheme for the joint generation of two amplitude-squeezed beams at the frequencies ω_1 and ω_2, fundamental and second harmonics of a Nd:YAG laser pulse. The scheme consists of two successive steps, both involving second-order nonlinear interactions in $\beta - \mathrm{BaB_2O_4}$ nonlinear crystals. One of the output beams show subPoissonian photon statistics, and this allows us to use photodetection instead of homodyne detection for diagnostics.

In this paper we describe an experimental realization of the scheme for the generation of dichromatic squeezing recently proposed in [1]. The experiment requires two non collinear interactions in two (second order) nonlinear $\beta - \mathrm{BaB_2O_4}$ crystals at the frequencies $\omega_3 = \omega_1 + \omega_2$, ω_1 being the fundamental, ω_2 the second harmonics and ω_3 the third harmonics of a Nd:YAG laser. Our setup is schematically depicted in Fig. 1. The first crystal operates as a nondegenerate optical parametric amplifier (NOPA), which couples the three modes a_j at frequencies ω_j, $j = 1, 2, 3$. The NOPA is pumped at frequency ω_3, whereas a weak seeding beam excites the mode at frequency ω_1. In the rotating wave approximation, the Hamiltonian of the NOPA under phase-matching conditions can be written as $\hat{H}_1 \propto a_1 a_2 a_3^\dagger + a_1^\dagger a_2^\dagger a_3$, and therefore, within the parametric approximation, the unitary evolution (interaction picture) is given by

$$\hat{U}_\lambda = \exp\left[\lambda a_1^\dagger a_2^\dagger - \lambda^* a_1 a_2\right] \ , \tag{1}$$

where the coupling λ is given by $\lambda = -i\tau\gamma$, being τ a rescaled interaction time and γ the complex amplitude of the pump, that we will take as real in the following. If the seed beam at frequency ω_1 is a coherent state of amplitude

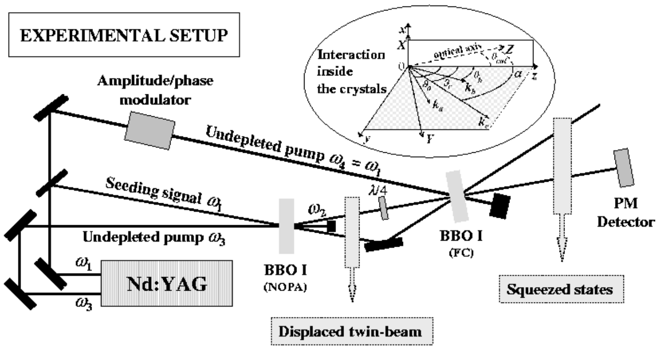

Figure 1: Experimental setup for noncollinear dichromatic squeezing generation. The cut angles of the crystals are supposed to be equal to the respective tuning angles, so that the two extraordinary fields (the one at ω_3 in the first crystal, NOPA, and the one at ω_2 in the second crystals, FC) enter the crystals orthogonally. The inset shows the detail of the noncollinear interaction inside the two crystals ($a \to 1$, $b \to 2$, $c \to 3$, in the NOPA while $a \to 1$, $b \to 4$, $c \to 2$ in the FC).

α_1 at the output of the first crystal we have $|\psi_1\rangle = \hat{D}_1(\mu\alpha_1) \otimes \hat{D}_2(\nu\alpha_1^*)\,|\psi_{\mathtt{twb}}\rangle$ where $\mu = \cosh|\lambda|$, $\nu = \sinh|\lambda|$ and $|\psi_{\mathtt{twb}}\rangle$ denotes the so-called twin-beam

$$|\psi_{\mathtt{twb}}\rangle \quad = \quad \hat{U}_\lambda\,|\mathbf{0}\rangle = \sqrt{1-|\chi|^2}\,\sum_{n=0}^{\infty}\chi^n\,|n\rangle_1 \otimes |n\rangle_2\ , \tag{2}$$

with $\chi = \arg(\lambda)\tanh|\lambda|$. The expression in (2) represents the maximally entangled state of the two out-coming modes a_1 and a_2.

We can define the gain of the device as

$$G_1 = \frac{\langle a_1^\dagger a_1\rangle_{out} - \langle a_1^\dagger a_1\rangle_{in}}{\langle a_1^\dagger a_1\rangle_{in}} = \sinh^2|\lambda|\left(1 + \frac{1}{\langle a_1^\dagger a_1\rangle_{in}}\right)\ . \tag{3}$$

The second term in (3) accounts for the parametric down-conversion of the vacuum, a genuine quantum effect, while the first term can be compared with the gain calculated in a classical analysis of the parametric amplifier. In fact, if we consider a vectorial and noncollinear system of Maxwell equations [2] in phase-matching conditions, we get the following expression for the gain in the first crystal:

$$\Gamma_1 = \sinh^2\left(\frac{2g_{eff}|a_3(0)|}{\sqrt{\cos\theta_1\cos\theta_2}}\frac{z}{2}\right)\ . \tag{4}$$

where, according to Fig. 1, θ_j are the angles between the propagation directions of the fields inside the crystal and the direction z of the normal to the entrance face and

$$g_{eff} = (d_{22}\cos\alpha + d_{31}\sin\alpha)\sqrt{\frac{2\hbar\omega_1\omega_2\omega_3\eta_0^3}{n_1 n_2 n_3}}\ , \tag{5}$$

α being the tuning angle, η_0 the vacuum impedance, n_j the refraction indexes at frequencies ω_j, and d_{22} and d_{31} the second-order nonlinear coefficients of the crystal [3]. Finally $|a_3(0)|^2 = I_3/\hbar\omega_3$ is the photon flux corresponding to an intensity I_3 of the pump.

Thus, by identifying the gain in (4) with the first term in (3), we obtain:

$$|\lambda| = \frac{g_{eff}|a_3(0)|}{\sqrt{\cos\theta_1\cos\theta_2}} z\ , \tag{6}$$

which establishes a relation between quantum and classical parameters.

The second crystal operates as a frequency converter (FC), and, in the limit of a classical undepleted pump at $\omega_4 = \omega_2 - \omega_1 \equiv \omega_1$, realizes an "effective" balanced beam splitter, which mixes the two incident fields at ω_1 and ω_2. As a consequence of mixing, the entanglement obtained in the first crystal is transformed into squeezing [4], so that the output consists of a pair of uncorrelated squeezed beams at frequencies ω_1 and ω_2 [1]. The Hamiltonian of the device, under phase-matching conditions and in the rotating wave approximation, can be written as follows $\hat{H}_2 \propto a_1 a_4 a_2^\dagger + a_1^\dagger a_4^\dagger a_2$ and the corresponding unitary evolution operator can be made equivalent to that of a balanced 50/50 beam splitter by an appropriate tuning of the effective interaction time (*i.e.* crystal length and pump intensity) $\hat{V}_{\pi/4} = \exp[-i\pi/4(a_1 a_2^\dagger + a_1^\dagger a_2)]$. The overall output state from the setup is thus given by

$$|\psi_2\rangle = \Big[\hat{D}_1(\beta_1)\otimes\hat{D}_2(\beta_2)\Big]\ \Big[\hat{S}_1(\lambda)\otimes\hat{S}_2(-\lambda)\Big]\ |\mathbf{0}\rangle \tag{7}$$

where

$$\beta_1 = \mu\frac{\alpha_1}{\sqrt{2}} + \nu\frac{\alpha_1^*}{\sqrt{2}} \qquad \beta_2 = \mu\frac{\alpha_1}{\sqrt{2}} - \nu\frac{\alpha_1^*}{\sqrt{2}} \tag{8}$$

are the final amplitudes of the fields at ω_1 and ω_2 respectively. The final result is thus a couple of squeezed state in the modes at the frequencies ω_1 and ω_2. The squeezing amplitudes $|\lambda|$ is the same for the two modes, whereas the squeezing phases are shifted by π to each other, which means that the two modes are squeezed in orthogonal directions. By varying the initial coherent amplitude α_1 one may tune the final amplitudes according to (8).

The comparison between quantum and classical parameters in the second crystal can be made by solving the same system of vectorial and noncollinear Maxwell equations cited above [2] in phase-matching conditions, and comparing the coefficients of the classical mode-transformation with (**??**). We thus get:

$$\frac{2g|a_4(0)|}{\sqrt{\cos\theta_1\cos\theta_2}}\frac{z}{2} = \frac{\pi}{4}\ , \tag{9}$$

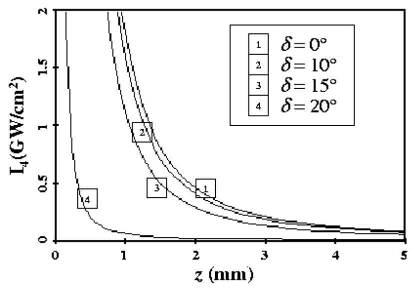

Figure 2: Intensity of the pump in the second crystal necessary to realize the dichromatic beam splitter, as a function of the thickness of the crystal (cut at $\theta_{cut} = 22.78^{\rm deg}$, optimal for collinear second harmonics generation) and for several angles $\delta = \theta_1 - \theta_2$ (see Fig. 1)

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where $g = \sqrt{d_{22}^2 + d_{31}^2}$ and $|a_4(0)|^2 = I_4/\hbar\omega_1$ is the photon flux corresponding to an intensity I_4 of the FC pump.

In Fig. 2 we show the intensity I_4 calculated after (9) as a function of the z-coordinate inside the crystal for several angles of noncollinearity $\delta = \theta_1 - \theta_2$.

The amount of squeezing S achievable at the output depends only on the gain of the parametric amplifier, *i.e.* on the value of λ. The squeezing S can be quantified by [1] $S = (1 + N) - \sqrt{N(N + 2)}$ where, in the case of one incident coherent state of amplitude α_1, we have

$$N = \frac{2G_1|\alpha_1|^2}{1 + |\alpha_1|^2}, \tag{10}$$

N being the number of photons in the twin-beam and G_1 the gain of the parametric amplifier as given in (3).

In Fig. 3 we show the squeezing (noise reduction expressed in dB) as a function of the first-crystal thickness, and of the intensity of the pump in the first step.

An alternative characterization of the nonclassicality of the radiation emerging from the system is obtained by studying the photon-statistics of the fields. In fact, it is easy to show that one of output squeezed states is also squeezed in the number of photons n, *i.e.* shows subPoissonian photon statistics, which, in terms of the Mandel's Q parameter, means

$$-1 \leq Q = \frac{\Delta n)^2 - \langle n \rangle}{\langle n \rangle} < 0 \ , \tag{11}$$

In terms of the parameters of our system, the subPoissonian field results to be the one at frequency ω_2, for which we can write

$$Q = [2\mu^2\nu^2 + \beta_2^2(\mu - \nu)^2]/(\nu^2 + \beta_2^2) - 1 \ , \tag{12}$$

which is displayed in Fig. 3.

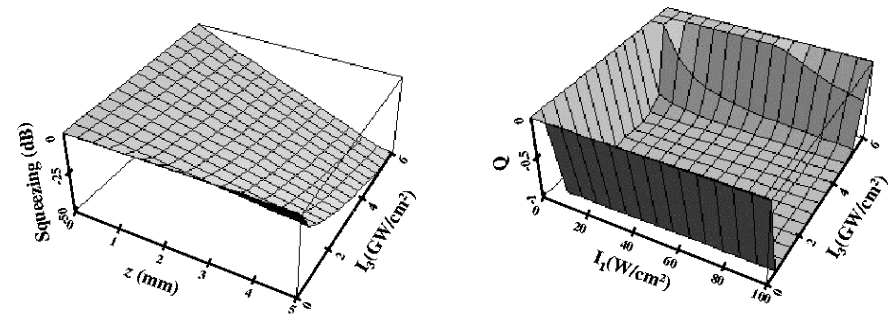

Figure 3: Left: squeezing as a function of the first-crystal thickness and of the intensity of the pump in the first step. Right: Mandel Q-factor for the beam at frequency ω_2 as a function of the intensity of the pump and the seeding beam in the first step. The crystal is supposed to be cut at $\theta_{cut} = 31.29^{\rm deg}$ (optimal for collinear third harmonics generation) and a non collinearity angle $\delta = \theta_1 - \theta_3 = 10^{\rm deg}$ (see Fig. 1) has been chosen.

In conclusion, we have described a non collinear experimental scheme for the joint generation of two amplitude-squeezed beams at the fundamental and second harmonics of a Nd:YAG laser pulse. Since one of the output beams show subPoissonian photon statistics, we suggest photodetection instead of homodyne detection for diagnostics. Notice that a detection based on photomultiplier tubes capable of demonstrating a Poissonian statistics at relatively low number of photons is also suitable to detect subPoissonian statistics with a Q parameter as low as that reported in Fig. 3.

Acknowledgement

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