Joint generation of identical squeezed states

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Abstract

An interaction scheme involving a parametric amplifier and beam splitters is suggested for the generation of two *identical* squeezed states in two initially unexcited spatial modes of the radiation field.

1. Introduction

As it was shown by Wootters and Zurek [1], the linear structure underlying quantum mechanics prohibits duplication of a generic state vector. That is, starting from a physical state described by $|\psi\rangle \otimes |\psi_i\rangle \otimes |A\rangle$, where $|\psi_i\rangle$ is the initial state of an idler mode and $|A\rangle$ the state of apparatus and environment, there are no physical interactions leading to an outgoing state described by $|\psi\rangle \otimes |\psi\rangle \otimes |B\rangle$. Actually, what would be really violated by duplication is the unitarity of quantum mechanics [2] which forbids duplication of states not belonging to an orthogonal set. In quantum optics this means that, among customary representations, only number states could, in principle, be duplicated [3,4].

The above *no-cloning* theorem is a precise and indisputable statement. However, it does not prevent one to devise some interaction scheme which creates two identical states starting from the vacuum state. In formula,

$$|0\rangle \otimes |0\rangle \xrightarrow{\hat{U}_\psi} |\psi\rangle \otimes |\psi\rangle . \qquad (1)$$

Indeed, we are going here to present an interaction scheme for generating two identical squeezed states in two different, initially unexcited, spatial modes of the radiation field. This will be an intriguing and concrete example in order to clarify the meaning of the no-cloning theorem. The latter, in fact, definitely does not forbid the synthesis of identical states. It deals with the duplication of a fixed initial state explicitly preventing its cloning not accompanied by its destruction.

From a more practical point of view, the synthesis of two identical copies of a squeezed state is welcome as it provides the basis for a high-sensitive interferometric scheme recently suggested in Ref. [5]. Moreover, it allows the direct sampling of the Wigner function by a modified eight-port homodyne detection scheme [6–9]. In both cases it is crucial to have at one's disposal two identical squeezed states *at the same time*, such that a repeated serial production cannot be helpful. As we will see, here the two identical squeezed states are jointly produced from a common classical source, thus they are also phase-matched.

In the next section we describe the present interaction scheme in details, whereas some concluding remarks are given in Section 3.
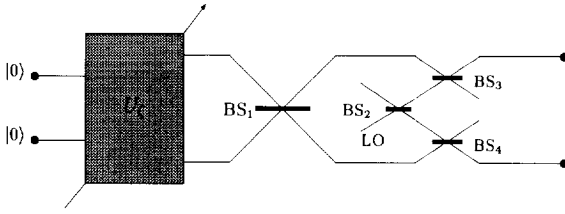
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Fig. 1. Schematic diagram of the interaction scheme for generating two identical squeezed states. The gray box represents a parametric amplifier along with its classical pump. BS_1 and BS_2 are balanced beam splitters whereas BS_3 and BS_4 are nearly-unit transmissivity beam splitters. LO denotes a local oscillator, namely a very intense laser beam.

2. From the vacuum to two identical squeezed states

A schematic diagram of the interaction scheme we will deal with is depicted in Fig. 1. At the first stage a parametric amplifier is employed for exciting the vacuum to a two-mode squeezed vacuum. The two modes are then impinged on a beam splitter which, for a particular choice of transmissivity, disentangles the squeezing leading to a two-single-mode squeezed vacuum. Finally, these two states can be displaced by means of a nearly-unit transmissivity beam splitter.

2.1. Parametric amplifier

A three-wave mixing device consists of a nonlinear medium showing second order susceptibility $\chi^{(2)}$. The interaction Hamiltonian can be written as

$$\hat{H}_I \propto \chi^{(2)}(a^\dagger b^\dagger c + abc^\dagger) , \qquad (2)$$

where the resonance condition $\omega_c = \omega_a + \omega_b$ is assumed. We consider a and b as two different spatial modes of the field at the same frequency $\omega_a = \omega_b = \omega$. Within the parametric approximation, namely by considering c as an undepleted classical pump, we can substitute the boson operator with the corresponding classical amplitude $c \longrightarrow |\gamma| e^{i\phi_\gamma}$, such that the evolution operator in the interaction picture can be written as

$$\hat{U}(\zeta) = \exp\left(\zeta a^\dagger b^\dagger - \bar{\zeta} ab\right) , \qquad (3)$$

with

$$\zeta = ig\Delta t |\gamma| e^{i\phi_\gamma} . \qquad (4)$$

In Eq. (4), g denotes the coupling constant, Δt the interaction time, whereas γ is the classical amplitude of the pump. The optical device described by Eq. (3) is known as a parametric amplifier and received both theoretical and experimental attention [10,11].

By means of the following two-boson Schwinger realization of the $SU(1,1)$ algebra [12],

$$[\hat{K}_+, \hat{K}_-] = 2\hat{K}_3 , \quad [\hat{K}_3, \hat{K}_\pm] = \pm\hat{K}_\pm ,$$
$$\hat{K}_+ = a^\dagger b^\dagger , \quad \hat{K}_- = ab ,$$
$$K_3 = \tfrac{1}{2}(b^\dagger b + a^\dagger a + 1) , \qquad (5)$$

the evolution operator in Eq. (3) can be written as a $SU(1,1)$ displacement operator. The $SU(1,1)$ Baker–Haussdorff formula reads

$$\exp\left(\zeta\hat{K}_+ - \bar{\zeta}K_-\right) = \exp\left(\beta_0\hat{K}_+\right)\exp\left(\beta_1\hat{K}_3\right)$$
$$\times \exp\left(-\bar{\beta}_0\hat{K}_-\right) , \qquad (6)$$

where

$$\beta_0 = \frac{\zeta}{|\zeta|}\tanh|\zeta| , \quad \beta_1 = \log(1 - |\beta_0|^2) . \qquad (7)$$

By means of Eq. (6) the action of the parametric amplifier on the electromagnetic vacuum can be easily evaluated,

$$\hat{U}(\zeta)|00\rangle = \sqrt{1 - |\beta_0|^2}\sum_n \beta_0^n|n,n\rangle . \qquad (8)$$

The state in Eq. (8) is a two-mode entangled state. It is usually termed two-mode squeezed vacuum.

2.2. Beam splitter

The beam splitter represents the most simple optical device to couple two modes of the field. It is realized by a linear medium showing only first order $\chi^{(1)}$ susceptibility. The interaction Hamiltonian is written as

$$\hat{H}_I \propto \chi^{(1)}(a^\dagger b + ab^\dagger) , \qquad (9)$$

whereas the evolution operator (in the interaction picture) is expressed as

$$\hat{V}(\lambda) = \exp\left(\lambda a^\dagger b - \bar{\lambda}ab^\dagger\right) , \qquad (10)$$

$$\lambda = i\arctan\sqrt{\frac{1-\tau}{\tau}} , \qquad (11)$$

where τ represents the transmissivity of the beam splitter. The evolution operator (10) has the form of a SU(2) displacement operator when using the following Schwinger two-boson representation of SU(2) algebra [12],

$$[\hat{J}_+, \hat{J}_-] = 2\hat{J}_3, \quad [\hat{J}_3, \hat{J}_\pm] = \pm\hat{J}_\pm, \quad \hat{J}_+ = ab^\dagger,$$
$$\hat{J}_- = a^\dagger b, \quad J_3 = \tfrac{1}{2}(b^\dagger b - a^\dagger a). \tag{12}$$

The Baker–Haussdorff formula reads

$$\hat{V}(\lambda) = \exp(\beta_0 a^\dagger b)\exp[\tfrac{1}{2}\beta_1(a^\dagger a - b^\dagger b)]$$
$$\times \exp(-\bar{\beta}_0 ab^\dagger), \tag{13}$$

where

$$\beta_1 = -\log\tau, \quad \beta_0 = i\sqrt{\frac{1-\tau}{\tau}}. \tag{14}$$

It is worth noticing that the special case of a balanced beam splitter, namely $\tau = 1/2$, corresponds to the evolution operator $\hat{V}(i\pi/4)$. We also notice that a beam splitter is a passive device, i.e. no energy is added to the interacting modes. Therefore, the vacuum state is invariant under the action of $\hat{V}(\lambda)$ for any value of λ,

$$\hat{V}(\lambda)|00\rangle = |00\rangle. \tag{15}$$

2.3. Disentangling squeezing

We now want to analyze the combined action, on the vacuum, of the parametric amplifier and the first of the beam splitters in Fig. 1. This will be a two-mode state given by

$$|\psi_{ab}\rangle = \hat{V}(\lambda)\hat{U}(\zeta)|00\rangle. \tag{16}$$

Taking advantage of the vacuum invariance (15), we write Eq. (16) as

$$|\psi_{ab}\rangle = \hat{V}(\lambda)\hat{U}(\zeta)\hat{V}^\dagger(\lambda)|00\rangle, \tag{17}$$

and proceed with the evaluation of $\hat{V}(\lambda)\hat{U}(\zeta)\hat{V}^\dagger(\lambda)$. The latter can be written as

$$\hat{V}(\lambda)\hat{U}(\zeta)\hat{V}^\dagger(\lambda) = \sum_{n=0}^{\infty} e^{\hat{A}}\frac{\hat{B}^n}{n!}e^{-\hat{A}}$$
$$= \sum_{n=0}^{\infty} \frac{(e^{\hat{A}}\hat{B}e^{-\hat{A}})^n}{n!}, \tag{18}$$

where we have introduced the notation

$$\hat{A} = \lambda a^\dagger b - \bar{\lambda}ab^\dagger,$$
$$\hat{B} = \zeta a^\dagger b^\dagger - \bar{\zeta}ab. \tag{19}$$

Eq. (18) can be evaluated using the operator relation

$$e^{\hat{A}}\hat{B}e^{-\hat{A}} = \hat{B} + [\hat{A},\hat{B}] + \tfrac{1}{2}[\hat{A},[\hat{A},\hat{B}]] + \ldots$$
$$+ \frac{1}{n!}\underbrace{[\hat{A},[\hat{A},\ldots,[\hat{A},\hat{B}]]]}_{n \text{ times}} + \ldots. \tag{20}$$

Upon the further position

$$\hat{K} = \zeta\lambda a^{\dagger 2} - \bar{\zeta}\bar{\lambda}a^2 + \bar{\zeta}\lambda b^2 - \zeta\bar{\lambda}b^{\dagger 2}, \tag{21}$$

we have

$$\underbrace{[\hat{A},[\hat{A},\ldots,[\hat{A},\hat{B}]]]}_{n \text{ times}} = (-4|\tau|^2)^{(n-1)/2}\hat{K} \quad n \text{ odd},$$
$$= (-4|\tau|^2)^{n/2}\hat{B} \quad n \text{ even}. \tag{22}$$

We thus arrive at the expression

$$e^{\hat{A}}\hat{B}e^{-\hat{A}} = \left(\sum_{k=0}^{\infty}(-1)^k\frac{(4|\lambda|^2)^k}{(2k)!}\right)\hat{B}$$
$$+ \left(\sum_{k=0}^{\infty}(-1)^k\frac{(4|\lambda|^2)^k}{(2k+1)!}\right)\hat{K}$$
$$= \cos(2|\lambda|)\hat{B} + \frac{\sin(2|\lambda|)}{2|\lambda|}\hat{K}. \tag{23}$$

Let us now consider the special case of a balanced beam splitter. As we noted above, this corresponds to $\lambda = i\pi/4$, so that we have

$$e^{\hat{A}}\hat{B}e^{-\hat{A}} = \frac{2}{\pi}\hat{K}, \tag{24}$$

and

$$\hat{V}(i\pi/4)\hat{U}(\zeta)\hat{V}^\dagger(i\pi/4) = \exp[(2/\pi)\hat{K}]$$
$$= \hat{S}_a(i\pi\zeta/2) \otimes \hat{S}_b(i\pi\zeta/2), \tag{25}$$

where $\hat{S}(\rho) = \exp[(\rho/2)a^{\dagger 2} - (\bar{\rho}/2)a^2]$ is the squeezing operator.

In Eq. (25) the two squeezing operators act on only one mode. This means that a balanced beam splitter is able to disentangle squeezing from a parametric amplifier. The output state at this stage would be represented by a two-identical single-mode squeezed vacuum. In the next subsection we will show how to displace them, in order to generate two identical squeezed states.

2.4. Displacing states

The displacement properties of a beam splitter for a coherent state have been known for a long time. Recently it has been shown [13] that it can be used to displace any quantum state of radiation. The signal mode is impinged onto a beam splitter whose second port (idler) is fed by a very intense laser beam (local oscillator). At the same time the transmissivity of the beam splitter is supposed to be very close to unity. That is, only a little mixing of the input state with the idler is allowed. However, the latter beam is very intense. The evolution of the signal mode can be obtained by a partial trace of the evolution operator (10) over the idler mode. The latter is described by a coherent state $|z\rangle$, thus we have

$$\mathrm{Tr}\left\{\hat{1}\otimes|z\rangle\langle z|\hat{V}(\lambda)\right\}\stackrel{|z|\to\infty,\,\tau\to1}{=}\hat{D}(\beta)\,,\qquad(26)$$

with $\hat{D}(\beta)=\exp(\beta a^{\dagger}-\bar{\beta}a)$ the displacement operator of amplitude $\beta=-iz\sqrt{1-\tau}$.

In the present scheme the local oscillator of both the beam splitters BS$_3$ and BS$_3$ comes from a common source (see Fig. 1). This is to assure phase matching and balance of amplitudes. The classical pump of the parametric amplifier can nicely provide this common source. It is also assumed that the two beam splitters have the same transmissivity $\tau_3=\tau_4=\tau$.

Following the scheme of Fig. 1, we have that after the beam splitters BS$_3$ and BS$_3$ the state of the two modes is described by

$$\hat{V}(\lambda_3)\hat{V}(\lambda_4)\hat{V}\left(\lambda_1\equiv i\pi/4\right)\hat{U}(\zeta)|0\rangle|0\rangle$$

$$=\hat{V}(\lambda_3)|0,i\pi\zeta/2\rangle_a\hat{V}(\lambda_4)|0,i\pi\zeta/2\rangle_b$$

$$=|\beta,i\pi\zeta/2\rangle_a|\beta,i\pi\zeta/2\rangle_b\,.\qquad(27)$$

In Eq. (27) $|\beta,\zeta\rangle=\hat{D}(\beta)\hat{S}(\zeta)|0\rangle$ denotes a squeezed state with coherent amplitude β and squeezing parameter ζ, β is given by $\beta=-iz/\sqrt{2}\sqrt{1-\tau}$.

Two identical squeezed states have been indeed synthesized in two different spatial modes of the field.

3. Summary and some remarks

In this paper we have suggested an interaction scheme to excite two different spatial modes of the radiation field to identical squeezed states. First the vacuum has to be excited to a two-mode squeezed vacuum by a parametric amplifier. Then, the squeezing is disentangled by a balanced beam splitter, and finally the resulting two-single-mode squeezed vacuum is displaced by a nearly-unit transmissivity beam splitter whose second port is fed by a highly excited coherent state. The squeezing parameter can be tuned by varying the amplitude and phase of the classical pump, whereas the coherent amplitude is controlled by the intensity of the local oscillator.

The final result of two identical squeezed states relies on the capacity to carefully control two parameters of the whole device, namely the transmissivity of the two balanced beam splitters BS$_1$ and BS$_2$. In particular, the disentanglement properties of BS$_1$ would be destroyed if its transmissivity deviates from the value $\tau_1=1/2$. On the other hand, any fluctuation of the transmissivity of BS$_2$ results in an unbalanced splitting of the local oscillator, thus leading to a different displacement of the two states.

One can argue that a scheme for the generation of a single squeezed state could be enough, as one can generate any number of copies of it by repeated preparations. However, this cannot be used to obtain two identical squeezed states at the same time. Also, a parallel production of the states by duplication of the preparation apparatus cannot be useful due to fluctuations, acting in different manners in the different devices. On the contrary, the situation considered here is qualitatively different, as the two identical squeezed states are jointly generated from the same common classical source, namely the pump of the parametric amplifier.

Acknowledgement

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