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Optimized phase detection

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Abstract

We analyze the detection of the phase of a single mode of the field with respect to a local oscillator, with the aim of optimizing the sensitivity as a function of the radiation intensity. We found that ideal scaling of sensitivity versus input energy is achieved for the open double homodyne detector in the limit of unit quantum efficiency.

1. Introduction

Despite the problem of phase detection having a long history, only recently it has been applied to actually feasible phase-detectors (see for example Refs. [1,2], and references therein). The relevance of the topic lies in the possibility of interferometric detection of minute variations of environmental parameters through the induced phase shift on the incident light beam. The optimization of such measurements versus the radiation intensity is of interest, because back-action effects due to radiation pressure pose severe limits to the energy carried by the input field.

The most appropriate approach to describe phase detection in the quantum domain is the quantum estimation theory of Helstrom [3]. The detection apparatus is described by means of a probability operator measure (POM) $d\mu(\phi)$, which provides the probability distribution of the phase outcomes for the field in the state ρ according to the formula

$$dP(\phi) = \mathrm{Tr}[\rho\, d\mu(\phi)] \ . \tag{1}$$

Actually, every POM corresponds to a conventional observable which acts on a larger Hilbert space describing also the apparatus degrees of freedom. At a purely abstract level the quantum estimation theory provides the following POM for an ideal apparatus,

$$d\mu(\phi) = \frac{d\phi}{2\pi} \sum_{n,m=0}^{\infty} \exp[i(n-m)\phi] |n\rangle\langle m| \ . \tag{2}$$

The best r.m.s. sensitivity $\delta\phi$ in this case is achieved by weakly squeezed states [2], giving the ultimate quantum limit

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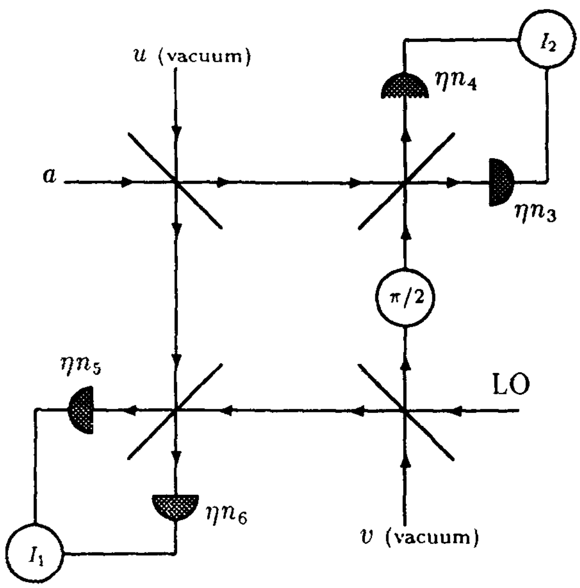

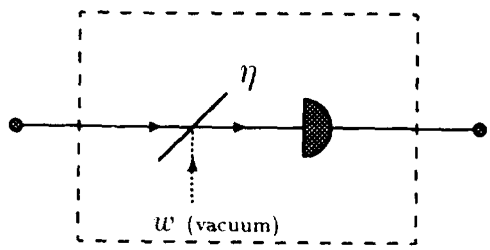

Fig. 1. Outline of the double homodyne detector (closed scheme).

Fig. 2. Equivalent scheme for a photodetector with quantum efficiency $\eta < 1$.

$$\delta\phi \simeq \frac{1.36}{\bar{n}} . \tag{3}$$

The ideal apparatus does not correspond to any feasible detection scheme. Therefore, the problem of optimizing phase detection is twofold. On the one hand, feasible detection schemes have to be devised, and their instrumental POM evaluated. On the other hand, optimal states have to be sought, in order to establish the ultimate sensitivity of each considered scheme. We consider that in any feasible detection, the phase corresponds to the polar angle between two measured photocurrents, say $\mathcal{I}_1$ and $\mathcal{I}_2$, which, in turn, trace two conjugated field-quadratures, say $a_0 = \frac{1}{2}(a^\dagger + a)$ and $a_{\pi/2} = \frac{1}{2}\mathrm{i}(a^\dagger - a)$. This is a consequence of the circular topology of the phase itself which, despite being a single bounded real parameter, nonetheless needs the specification of a pair of real quantities [4]. Detection of a couple of quadratures can be achieved by means of a double homodyne detector. The outcomes of the detector are points distributed in the complex plane $\alpha \equiv \mathcal{I}_1 + \mathrm{i}\mathcal{I}_2 \equiv \rho\,\mathrm{e}^{\mathrm{i}\phi}$ and the marginal distribution integrated over radius is the desired phase probability.

In Section 2 we analyze the *joint* measurement of conjugated quadratures, which is experimentally achieved by a double homodyne detector in the *closed* scheme (see Fig. 1). For a quantum efficiency η of the photodetectors this scheme measures the Wigner function[1] $W_{1-2\eta^{-1}}(\alpha,\bar{\alpha})$, and thus in the limit of $\eta = 1$ it corresponds to a measurement of the Husimi distribution function $W_{-1}(\alpha,\bar{\alpha})$ [5]. This distribution is positive definite, and hence its marginal distribution defines a genuine phase probability for any quantum state of radiation. However, phase sensitivity is nonideal, and is bounded by $\delta\phi \simeq \bar{n}^{-2/3}$. For $\eta < 1$ the phase distribution becomes broader, with a corresponding degradation of sensitivity. The latter is again described by the same power law dependence, however with a greater multiplicative constant factor. The optimal states are weakly squeezed coherent states, with a squeezing fraction which is an increasing function of η.

[1] The generalized Wigner distribution function $W_s(\alpha,\bar{\alpha})$ in the phase space is defined as $W_s(\alpha,\bar{\alpha}) = \int \mathrm{d}^2\lambda \exp(\alpha\bar{\lambda} - \bar{\alpha}\lambda + \frac{1}{2}s|\lambda|^2)\, \mathrm{Tr}[\rho \exp(\lambda a^\dagger - \bar{\lambda} a)]/\pi^2$.

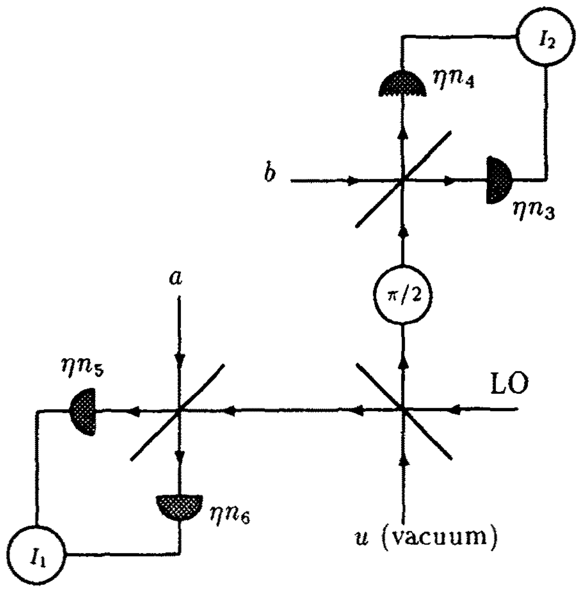

Fig. 3. Outline of the double homodyne detector (open scheme).

In Section 3 the double-homodyne scheme is considered in its *open* version, corresponding to *independent* measurements of the two conjugated quadratures (see Fig. 3). The apparatus now detects the Wigner function $W_{1-\eta^{-1}}(\alpha, \bar{\alpha})$, but only when the inputs are in a coherent or squeezed state. The phase sensitivity is improved, corresponding to the bound $\delta\phi \simeq \bar{n}^{-1}$ for $\eta = 1$. Hence, the open scheme reaches the ideal sensitivity for suited squeezed states and unit quantum efficiency. For nonunit quantum efficiency the detector measures a smoothed phase probability, which however, remains sharper than the corresponding one that is achieved by the closed scheme for the same η. Again the optimal states are weakly squeezed, with squeezing fraction increasing versus η. For $\eta \leqslant 0.5$ the open scheme detects the same smoothed Wigner functions as in the closed scheme, and the same results are recovered.

2. Double homodyne: closed scheme

The double balanced homodyne detector provides a way for simultaneously measuring a couple of quadratures for one mode of the field. The schematic diagram of the set-up is reported in Fig. 1. There are four 50–50 beam splitters and four photocounters, whereas a $\pi/2$ phase shifter is inserted in one arm. The mode carrying the measured phase is a, and a stable reference for the phase is provided by the local oscillator (LO) which is synchronous with a and is prepared in a highly excited coherent state $|z\rangle$. We consider the general case of nonunit quantum efficiency $\eta < 1$ at photodetectors [1], which can be simply represented as an additional beam splitter of transmissivity η in front of each detector (see Fig 2). Each experimental event consists of a simultaneous detection of the two difference-photocurrents $I_1 = \eta(n_6 - n_5)$ and $I_2 = \eta(n_4 - n_3)$, whose "reduced" versions $\mathcal{I}_1 = I_1/\eta|z|$ and $\mathcal{I}_2 = I_2/\eta|z|$ trace a pair of field-quadratures. The reduced currents are expressed in terms of input field as follows,

$$\begin{aligned}\mathcal{I}_1 &= a_\phi + u_\phi + \sqrt{(1-\eta)/\eta}(w_{1\phi} - w_{2\phi}),\\ \mathcal{I}_2 &= a_{\phi+\pi/2} + u_{\phi+\pi/2} + \sqrt{(1-\eta)/\eta}(w_{3\phi+\pi/2} - w_{4\phi+\pi/2}),\end{aligned} \tag{4}$$

where w_i $(i = 1, 4)$ represent the additional vacuum modes simulating the efficiency of the ith photodetector. The POM of the apparatus is obtained upon tracing over the probe Hilbert space $\mathcal{H}_P$ (including all vacuum modes plus the highly excited coherent LO), thus obtaining the operator which acts on the system space $\mathcal{H}_S$ only,

$$d\mu_\eta^C(\alpha, \bar\alpha) = \frac{d^2\alpha}{\pi} \int_{-\infty}^{\infty} \frac{d\mu}{2\pi} \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} \mathrm{Tr}_P\{\rho_P \otimes 1_S \exp[i\mu(\mathcal{I}_1 - \mathrm{Re}\,\alpha) + i\nu(\mathcal{I}_2 - \mathrm{Im}\,\alpha)]\} . \tag{5}$$

After some calculations, and changing variables, $\lambda = \frac{1}{2}(i\mu - \nu)$, Eq. (5) can be rewritten as follows,

$$d\mu_\eta^C(\alpha, \bar\alpha) = \frac{d^2\alpha}{\pi} \int_{\mathbb{C}} \frac{d^2\lambda}{\pi} \exp[\alpha\bar\lambda - \bar\alpha\lambda + \tfrac{1}{2}(1 - 2\eta^{-1})|\lambda|^2] D(\lambda) . \tag{6}$$

In Eq. (6) $D(\lambda) = \exp(\lambda a^\dagger - \bar\lambda a)$ is the displacement operator. For unit quantum efficiency Eq. (6) becomes

$$d\mu_1^C(\alpha, \bar\alpha) = \frac{d^2\alpha}{\pi} |\alpha\rangle\langle\alpha| , \tag{7}$$

whereas for $\eta < 1$ one obtains the Gaussian convolution of the POM for $\eta = 1$,

$$\begin{aligned} d\mu_\eta^C(\alpha, \bar\alpha) &= \frac{d^2\alpha}{\pi} \frac{\eta}{1-\eta} \int_{\mathbb{C}} \frac{d^2\lambda}{\pi} \exp\left(-\frac{\eta}{1-\eta}|\alpha - \lambda|^2\right) |\lambda\rangle\langle\lambda| \\ &= \frac{d^2\alpha}{\pi} \eta \exp\left[-|\alpha|^2 \frac{\eta}{1-\eta} \exp\left(\frac{\eta}{1-\eta}\bar\alpha a\right)\right] (1-\eta)^{a^\dagger a} \exp\left(\frac{\eta}{1-\eta}\alpha a^\dagger\right) . \end{aligned} \tag{8}$$

After tracing the detector POM with the density matrix of the detected field, one obtains the probability distribution of the outcomes of the apparatus,

$$dP_\eta^C(\alpha, \bar\alpha) = \mathrm{Tr}[\rho\, d\mu_\eta^C(\alpha, \bar\alpha)] = W_{1-2\eta^{-1}}(\alpha, \bar\alpha) \frac{d^2\alpha}{\pi} . \tag{9}$$

The phase-POM is just the marginal POM integrated over the modulus $|\alpha|$, namely

$$\begin{aligned} d\mu_\eta^C(\phi) &= \frac{d\phi}{2\pi} \int_0^\infty d\rho\, \rho \int_{\mathbb{C}} \frac{d^2\lambda}{\pi} \exp[\rho e^{i\phi}\bar\lambda - \rho e^{-i\phi}\lambda + \tfrac{1}{2}(1 - 2\eta^{-1})|\lambda|^2] D(\lambda) \\ &= \frac{d\phi}{2\pi} \sum_{n,m=0}^{\infty} \exp[i(n-m)\phi] \eta^{(n+m)/2} \frac{\Gamma(\frac{1}{2}(n+m)+1)}{n!m!} a^{\dagger n} (1-\eta)^{a^\dagger a} a^m , \end{aligned} \tag{10}$$

which corresponds to the marginal phase distribution

$$dP_\eta^C(\phi) = \frac{d\phi}{2\pi} \int_0^\infty d\rho\, \rho W_{1-2\eta^{-1}}(\rho e^{i\phi}, \rho e^{-i\phi}) \frac{d^2\alpha}{\pi} . \tag{11}$$

The r.m.s. phase sensitivity is given by

$$\delta\phi_\eta^C = \left[\int_{-\pi}^{\pi} \phi^2\, dP_\eta^C(\phi) - \left(\int_{-\pi}^{\pi} \phi\, dP_\eta^C(\phi)\right)^2\right]^{1/2} , \tag{12}$$

and has to be optimized over all states with the constraints of fixed normalization and mean energy, namely

$$\mathrm{Tr}(\rho) = 1\,, \qquad \mathrm{Tr}(\rho n) = \bar{n}\,. \tag{13}$$

The case $\eta = 1$ has been already treated in Ref. [2]. More generally, for $\eta < 1$ the phase sensitivity is given by the closed formula (states with zero mean phase have been chosen for simplicity)

$$\delta\phi_\eta^{C2} = \tfrac{1}{3}\pi^2 + 2\sum_{n\neq m} A_{n,m}(\eta)\rho_{n,m}\,, \tag{14}$$

where $A_{n,m}(\eta)$ is the real symmetric matrix (see also Ref. [6])

$$A_{n,n+d}(\eta) = (-1)^d \frac{1}{d^2}\frac{\Gamma(n+\frac{1}{2}d+1)}{\sqrt{n!(n+d)!}}\sum_{s=0}^{n}\eta^{s+d/2}(1-\eta)^{n-s}\frac{\Gamma(s+\frac{1}{2}d+1)}{s!(s+d)!(n-s)!}\,. \tag{15}$$

For pure states $\rho_{n,m} = c_n c_m$ with $c_n \in \mathbb{R}^+$, the minimization of (14) with constraints (13) is carried out by the method of Lagrange multipliers, which reduces the problem to that of minimizing the function

$$F(\{c_n\};\lambda,\beta|\bar{n}) = \tfrac{1}{3}\pi^2 + 2\sum_{n\neq m} A_{n,m}c_n c_m + \lambda\Big(\sum_{n=0}^{\infty} c_n^2 - 1\Big) + \beta\Big(\sum_{n=0}^{\infty} n c_n^2 - \bar{n}\Big) \tag{16}$$

with respect to the coefficients $\{c_n\}$, λ and β being the Lagrange multipliers. The variational problem (16) is equivalent to the diagonalization of the quadratic symmetric form

$$[\mathbf{M}(\beta,\eta) + \lambda\mathbf{I}]\cdot\boldsymbol{c} = 0\,, \qquad \boldsymbol{c} \equiv (c_0, c_1, \ldots)\,, \tag{17}$$

where now the matrix $\mathbf{M}(\beta,\eta) = \{M_{nm}(\beta,\eta)\}$ is given by

$$M_{n,m}(\beta,\eta) = 2A_{nm}(\eta) + n\beta\delta_{nm}\,. \tag{18}$$

Eq. (17) can be solved numerically, after a suitable truncation of the Hilbert space dimension. One can see that the absolute minimum corresponds to the minimum eigenvalue $\lambda = \frac{1}{3}\pi^2$, whereas the average number $\bar{n}$ becomes a decreasing function of the running parameter $\beta \in [0, 1]$. In order to avoid the problem of nonvanishing tails of the number distribution near the border of the truncated Hilbert space, we have considered only average values $\bar{n} \ll \dim \mathcal{H}_s/2$.

The optimal states are weakly squeezed states, and the corresponding lower bound for phase sensitivity is given by

$$\delta\phi_\eta^{C} = \frac{K(\eta)}{\bar{n}^{0.65\pm0.01}}\,, \tag{19}$$

where $K(\eta)$ runs from 1.00 ± 0.01 to 1.46 ± 0.01 for η varying from $\eta = 1.00$ to $\eta = 0.5$. In Fig. 4 we report the phase sensitivity and the squeezing fraction of the corresponding optimal states as a function of the average energy for various values of η.

3. Double homodyne: open scheme

The double homodyne detector in its open scheme [7] is depicted in Fig. 3. The detector needs two identical input states, with a stable reference for the phase provided by a highly excited coherent local oscillator (LO). There are three beam splitters, four photodetectors, and a $\pi/2$ shifter inserted in one arm, such that the scheme

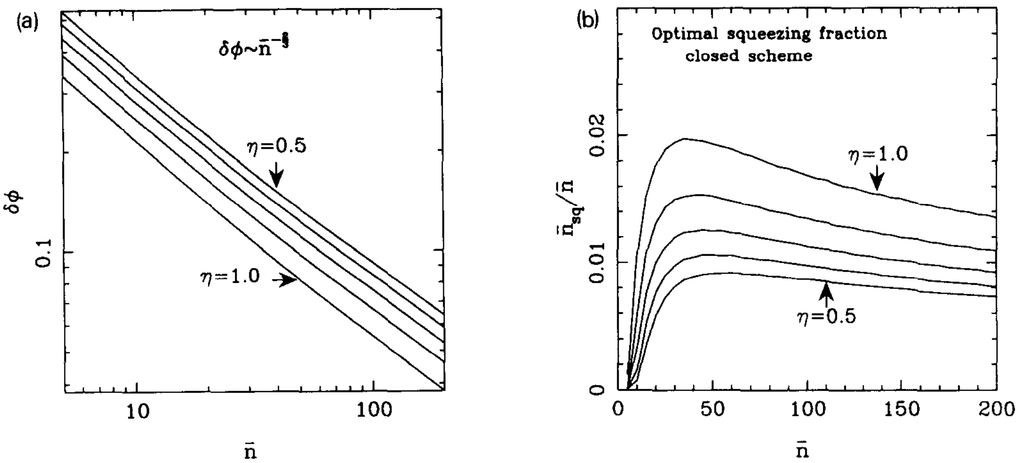

Fig. 4. Optimal phase sensitivity and corresponding fraction of squeezing photons (closed double homodyne detector) versus total average number of photons $\bar{n}$ for different values of η. (Notice that actually the log-log dependence of $\delta\phi_\eta^{\rm C}$ versus $\bar{n}$ is not perfectly linear: this small effect is due to truncation of the Hilbert space dimension. The power law in Eq. (19) corresponds to the maximum slope.)

becomes equivalent to two independent single homodyne measurements – one with the LO shifted by $\pi/2$ – each working on a separate input (or, in other words, the scheme corresponds to two independent measurements of conjugated quadratures which are successively performed on a single input prepared twice in the same state ρ). In this case the reduced photocurrents are

$$\mathcal{I}_1 = \frac{I_1}{2\eta|z|} = a_\phi + u_\phi + \sqrt{(1-\eta)/2\eta}\,(w_{1\phi} - w_{2\phi}) \,,$$
$$\mathcal{I}_2 = \frac{I_2}{2\eta|z|} = a_{\phi+\pi/2} + u_{\phi+\pi/2} + \sqrt{(1-\eta)/2\eta}\,(w_{3\phi+\pi/2} - w_{4\phi+\pi/2}) \,, \tag{20}$$

with the same meaning of the w_i as for Eq. (4). Due to the independence between photocurrents, the POM of the apparatus is calculated as follows,

$$d\mu_\eta^{\rm O}(\alpha,\bar{\alpha}) = \frac{d^2\alpha}{\pi}\int_{-\infty}^{\infty}\frac{d\mu}{2\pi}{\rm Tr_P}\{\rho_{\rm P}\otimes 1_{\rm S}\exp[i\mu(\mathcal{I}_1 - {\rm Re}\alpha)]\}\int_{-\infty}^{\infty}\frac{d\nu}{2\pi}{\rm Tr_P}\{\rho_{\rm P}\otimes 1_{\rm S}\exp[i\nu(\mathcal{I}_2 - {\rm Im}\,\alpha)]\} \tag{21}$$

and, following the lines of the previous section, one obtains

$$d\mu_\eta^{\rm O}(\alpha,\bar{\alpha}) = d^2\alpha\frac{2\eta}{1-\eta}\int_{\rm C}\frac{d^2\lambda}{\pi}\exp\left(-\frac{2\eta}{1-\eta}|\alpha-\lambda|^2\right)|\,{\rm Re}\,\lambda\rangle_0\,{}_0\langle{\rm Re}\,\lambda|\otimes|\,{\rm Im}\,\lambda\rangle_{\pi/2}\,{}_{\pi/2}\langle{\rm Im}\,\lambda| \,. \tag{22}$$

In Eq. (22) $|x\rangle_\phi$ represents the eigenvector of the quadrature a_ϕ, namely

$$|x\rangle_\phi = (2/\pi)^{1/4}e^{-x^2}\sum_{n=0}^{\infty}\frac{H_n(\sqrt{2}x)}{\sqrt{2^n n!}}\,e^{-in\phi}\,|n\rangle. \tag{23}$$

The probability distribution of the detector is now obtained by tracing the POM with the two identical density matrices of the input fields

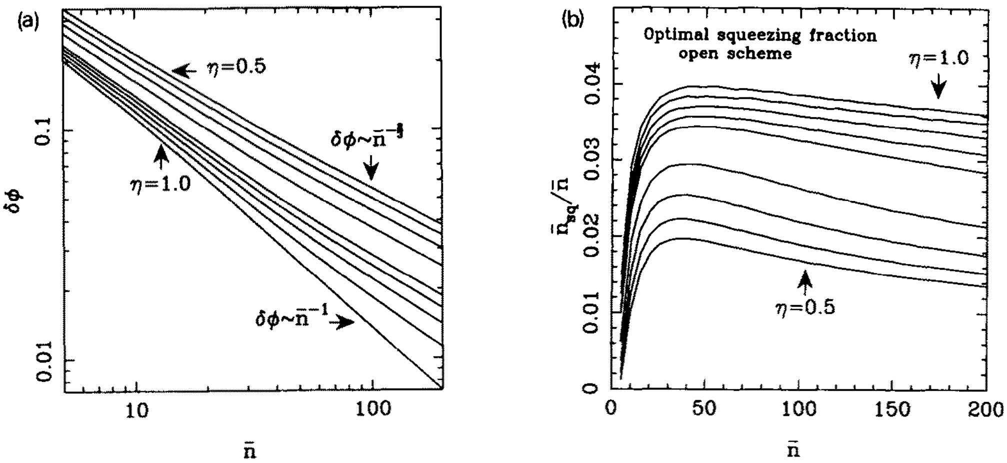

Fig. 5. Optimal phase sensitivity and corresponding fraction of squeezing photons (open double homodyne detector) versus total average number of photons $\bar{n}$ for different values of η. (The same comments of Fig. 4 hold. Here, notice also that the weak curvature of the log-log plot $\delta\phi^{\rm O}_\eta$ versus $\bar{n}$ is negative only for $\eta = 1$.)

$$\mathrm{d}P^{\rm O}_\eta(\alpha,\bar{\alpha}) = \mathrm{Tr}[\rho\otimes\rho\ \mathrm{d}\mu^A_\eta(\alpha,\bar{\alpha})]\ . \tag{24}$$

For input states which are coherent or squeezed, the probability given by Eq. (24) coincides with the Wigner function for $\eta = 1$, whereas it is a Gaussian convolution of the Wigner function for $\eta < 1$ [8]. The states should be squeezed in the direction of one of the two quadratures, corresponding to a real squeezing parameter. In this case, in fact, the Wigner function is factorized into the product of the two independent probabilities of the quadratures. More generally, for $\eta \leqslant 1$, the Gaussian convolution is a generalized Wigner function with ordering parameter $s = 1-\eta^{-1}$. Choosing for simplicity states with zero mean phase (namely with real positive amplitude β) one has

$$W_s(\alpha,\bar{\alpha}) = \frac{1}{2\pi\sigma_{1s}\sigma_{2s}}\exp\left(-\frac{(\mathrm{Re}\,\alpha-\beta)^2}{2\sigma^2_{1s}}-\frac{(\mathrm{Im}\,\alpha)^2}{2\sigma^2_{2s}}\right),\qquad s = 1-1/\eta\ , \tag{25}$$

where the variances σ_{is} are

$$\sigma^2_{1s} = \tfrac{1}{4}(\mathrm{e}^{2r}-s)\ ,\qquad \sigma^2_{2s} = \tfrac{1}{4}(\mathrm{e}^{-2r}-s)\ . \tag{26}$$

The marginal phase probability is given by

$$P^{\rm O}_\eta(\phi) = \frac{1}{4\pi\sigma_{1s}\sigma_{2s}\kappa(\phi)}\exp\left(-\frac{\bar{n}-\sinh^2 r}{2\sigma^2_{1s}}\right)\left[1+\mathrm{e}^{\lambda(\phi)}\sqrt{\pi\lambda(\phi)}\left(1+\frac{2}{\sqrt{\pi}}\int\limits_0^{\sqrt{\lambda(\phi)}}\mathrm{d}t\ \mathrm{e}^{-t^2}\right)\right], \tag{27}$$

where

$$\lambda(\phi) = \frac{(\bar{n}-\sinh^2 r)\sigma^2_{2s}}{2\sigma^2_{1s}(\sigma^2_{2s}+\sigma^2_{1s}\tan^2\phi)}\ , \tag{28}$$

$$\kappa(\phi) = \frac{1}{2}\left(\frac{\cos^2\phi}{\sigma^2_{1s}}+\frac{\sin^2\phi}{\sigma^2_{2s}}\right)\ . \tag{29}$$

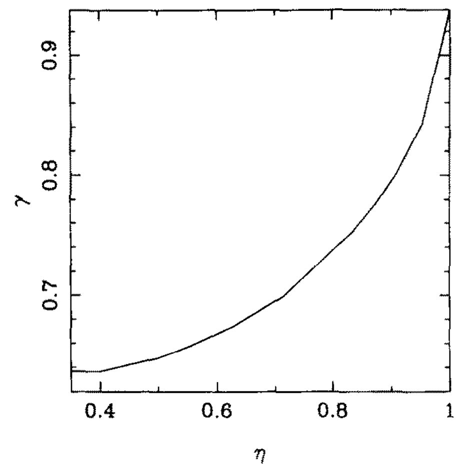

Fig. 6. Exponent γ of phase sensitivity $\delta\phi_\eta \sim \bar{n}^{-\gamma}$ versus quantum efficiency of photodetectors.

The phase sensitivity is optimized numerically by varying the fraction of squeezing photons $\bar{n}_{sq}$ at fixed total average photon number $\bar{n}$. The optimal $\delta\phi$ versus $\bar{n}$ is plotted in Fig. 5 for different values of η, giving also the corresponding optimal fraction $\bar{n}_{sq}/\bar{n}$ of squeezing photons. The phase sensitivity obeys the power law

$$\delta\phi_\eta^{\mathrm{O}} \sim \bar{n}^{-\gamma(\eta)} , \tag{30}$$

where the exponent γ versus η is plotted in Fig. 6. For decreasing η the sensitivity is degraded: one has approximately $\gamma \simeq 1 - \eta\sqrt{1-\eta}$ for $\eta > 0.5$, whereas for $\eta < 0.5$ one has $\gamma \simeq \frac{2}{3}$ (with small variations) similar to the exponent pertaining the closed scheme. Only a few percent of squeezing photons are needed for optimal sensitivity, and less squeezing photons for less efficient detectors. The best sensitivity is obviously attained for unit quantum efficiency. In this case the power law is given by

$$\delta\phi_1^{\mathrm{O}} \simeq \frac{2.72}{\bar{n}} . \tag{31}$$

Eq. (31) differs from the ultimate sensitivity (3) by a factor two.

Before ending this section some remarks on the probability of the currents of the open scheme are in order. We have seen that for $\eta = 1$ the probability is the Wigner function of the field when input states are squeezed in the direction of one of the two quadratures. This is due to the special analytic form of the Wigner function for these states, which is factorized as the product of the two probabilities for each quadrature (for general input states this is no longer true, and the Wigner function is generally nonpositive definite). The marginal phase probability of the Wigner function can be sharper than the ideal one, but generally it is itself not positive definite. For squeezed states it is positive, and has a central peak which is sharper and narrower than the corresponding ideal one. The r.m.s. phase sensitivity, however, is larger than the ideal value, due to the enhanced tails of the distribution near the edges of the phase window: this mechanism is illustrated in Fig. 7.

4. Conclusions

We have analyzed the detection of the phase of a single mode of the field that corresponds to the polar angle between two real photocurrents. For double homodyne detectors the two photocurrents are proportional to a couple of conjugated quadratures of the field. We considered both the cases of joint measurements (closed scheme) and independent measurements (open scheme). For $\eta = 1$ the former has a probability distribution

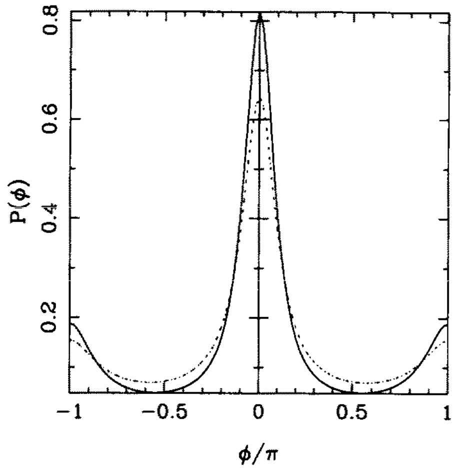

Fig. 7. Comparison between the marginal phase probability of the Wigner function and the ideal phase distribution for a squeezed state with amplitude $\alpha = 0.25$ and squeezing factor $r = 0.5$. The solid line is the marginal Wigner distribution and the dotted one represents the ideal distribution. The central peak is sharper and narrower for the marginal Wigner distribution which, however, exhibits higher tails than the ideal distribution at the edges of the $[-\pi, \pi]$ phase window.

equal to the Q-function, whereas the latter provides the Wigner function, but only for squeezed states with squeezing in the direction of one of the detected quadratures. For $\eta < 1$ the probabilities are Gaussian convoluted, corresponding to generalized Wigner functions, and resulting in an additional detection noise. The phase probability is the marginal distribution of the Wigner function. We optimized the phase sensitivity $\delta\phi$ for these probabilities for fixed average photon number $\bar{n}$. We found that the open scheme (independent measurements of the conjugated quadratures) achieves the best sensitivity, with the same power law $\delta\phi \sim \bar{n}^{-1}$ of ideal phase detection. This scheme of detection, however, works only for squezed states. On the other hand, the closed scheme (joint measurement) is a genuine phase detector - i.e. valid for any arbitrary input states, but has a sensitivity which is much degraded. The closed scheme does not reach the ideal power law even for $\eta = 1$, because for any η it has the corresponding sensitivity of the open scheme at $\eta/2$: this "effective quantum efficiency" of the closed scheme is due to the added noise related to the joint measurement of both quadratures [9].

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