The illusionist game and hidden correlations

G Brida[1]**, I P Degiovanni**[1]**, M Genovese**[1]**, A Meda**[1]**, S Olivares**[2,3,4]
and M Paris[2,4]

[1] INRIM, Strada delle Cacce 91, I-10135 Torino, Italy

[2] Dipartimento di Fisica, Università degli Studi di Milano, I-20133 Milano, Italy

[3] Dipartimento di Fisica, Università degli Studi di Trieste, I-34151 Trieste, Italy

[4] CNISM UdR Milano Statale, I-20133 Milano, Italy

E-mail: m.genovese@inrim.it

Abstract

We suggest and demonstrate a new protocol based on correlated beams of light: the 'optical illusionist game'. An 'illusionist' at first shows that if two uncorrelated light beams excited in the same Gaussian state are mixed in a beam splitter, then no correlations arise between them, as it was not present. On the other hand, when correlations with an ancillary state are exploited, the presence of the beam splitter can be unveiled.

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(Some figures may appear in color only in the online journal)

1. Introduction

Gaussian states, i.e. states with Gaussian Wigner functions (e.g. coherent, thermal and squeezed states), play a crucial role in developing quantum technologies [1–21] and in quantum optics.

They possess the interesting property of being generated through linear and bilinear interactions feasible in quantum optics laboratories and, remarkably, led to the realization of the first quantum protocols, such as continuous variable quantum teleportation [22]. Furthermore, in order to optimize the generation of entanglement and to preserve it during propagation through realistic channels, very recently interest has emerged in analyzing the effects of linear optical elements on the correlations of Gaussian states [22–29].

In this paper, we address the general question of the invariance of two uncorrelated Gaussian states, irrespective of whether classical (e.g. thermal) or quantum (e.g. squeezed vacuum), mixed at a beam splitter (BS); in particular, we demonstrate that, even when they are left unchanged, as when the BS is not present, one can distinguish between the two situations when a third ancillary state is included. In the following, we present this physical phenomenon as a fancy game.

2. The illusionist game: theory

An illusionist displays on the stage an amazing game involving two light beams, a beam splitter (BS_I) and two photodetectors able to measure the correlations between the beams. He specifies that he will use only Gaussian states of light with zero first moments. The illusionist starts his performance by crossing two uncorrelated beams of light excited in the same Gaussian state and showing, as one may expect, that no correlations arise between them (figure 1(a)). Then, he inserts BS_I and shows that the counters continue to reveal zero correlations (figure 1(b)). This effect is a consequence of the linear interaction of modes performed by the BS_I and the Gaussian nature of the states [18, 19]. In fact, the most general single-mode Gaussian state can be written as $\varrho = D(\alpha)S(\xi)\nu_{\text{th}}(N)S^\dagger(\xi)D^\dagger(\alpha)$, where $\nu_{\text{th}}(N) = \sum_{n=0}^{\infty}(N)^n/(1+N)^{n+1}|n\rangle\langle n|$ is a thermal equilibrium state with N average number of quanta, $S(r) = \exp\{\frac{1}{2}[\xi(\hat{a}^\dagger)^2 - \xi^*\hat{a}^2]\}$ and $D(\alpha) = \exp[\alpha\hat{a}^\dagger - \alpha^*\hat{a}]$ are the squeezing and the displacement operators, respectively, a being the annihilation operator. The state ϱ is fully characterized by its 2×2 covariance matrix (CM) $\boldsymbol{\sigma}$, $[\boldsymbol{\sigma}]_{hk} = \frac{1}{2}\langle R_h R_k + R_k R_h\rangle - \langle R_h\rangle\langle R_k\rangle$, $k = 1, 2$, and first moment vector $\overline{\boldsymbol{X}}^{\text{T}} = \langle \boldsymbol{R}^{\text{T}}\rangle = \sqrt{2}(\Re e[\alpha], \Im m[\alpha])$, with $\langle A\rangle = \text{Tr}[A\varrho]$,
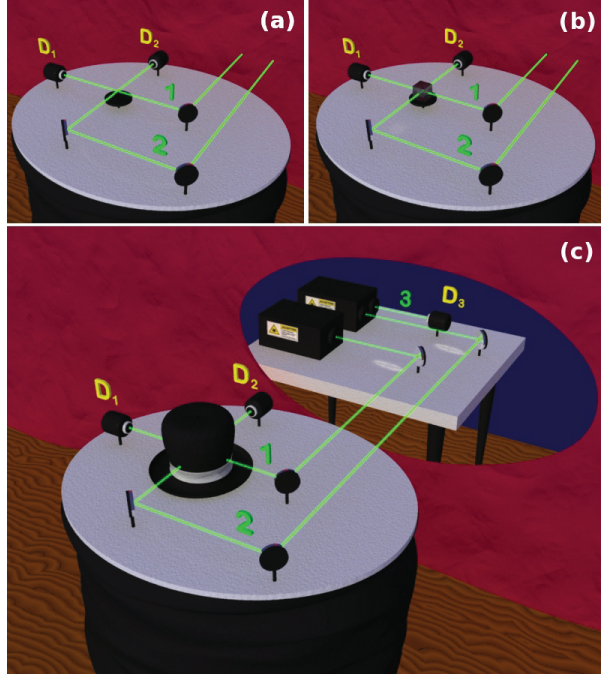
Figure 1. The illusionist game. (a) Two uncorrelated light beams excited in the same Gaussian state are crossed: no correlations arise. (b) A beam splitter (BS_I) lets the two beams interfere: still no correlations are detected. (c) The illusionist can always say whether the BS_I is hidden under the hat or not: the trick is behind the curtain (where the illusionist has access to a further beam). D_k, $k = 1, 2, 3$, are detectors to measure the correlations.

where $\boldsymbol{R}^{\mathrm{T}} = (\hat{R}_1, \hat{R}_2) \equiv (\hat{q}, \hat{p})$ and $\hat{q} = (\hat{a} + \hat{a}^{\dagger})/\sqrt{2}$ and $\hat{p} = (\hat{a} - \hat{a}^{\dagger})/(\mathrm{i}\sqrt{2})$ are the position- and momentum-like operators. In the following, according to the requirements of the performer, we set $\alpha = 0$, i.e. $\overline{\boldsymbol{X}}^{\mathrm{T}} = 0$. Now, when two uncorrelated, single-mode Gaussian states ϱ_k with CMs $\boldsymbol{\sigma}_k$, $k = 1, 2$, interact through a BS_I with transmissivity τ, the initial 4×4 CM $\boldsymbol{\Sigma}_0 = \boldsymbol{\sigma}_1 \oplus \boldsymbol{\sigma}_2$ of the two-mode state $\varrho_1 \otimes \varrho_2$ transforms as [30–33]

$$\boldsymbol{\Sigma}_0 \to \boldsymbol{\Sigma}^{(\mathrm{out})} = \begin{pmatrix} \boldsymbol{\Sigma}_1 & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{12} & \boldsymbol{\Sigma}_2 \end{pmatrix}, \qquad (1)$$

where

$$\boldsymbol{\Sigma}_1 = \tau \boldsymbol{\sigma}_1 + (1 - \tau) \boldsymbol{\sigma}_2, \quad \boldsymbol{\Sigma}_2 = \tau \boldsymbol{\sigma}_2 + (1 - \tau) \boldsymbol{\sigma}_1$$

and

$$\boldsymbol{\Sigma}_{12} = \tau (1 - \tau)(\boldsymbol{\sigma}_2 - \boldsymbol{\sigma}_1).$$

Note that $\boldsymbol{\Sigma}_{12} \neq \boldsymbol{0}$ denotes the presence of correlation between the outgoing modes. It is now clear that if the input states are in the same initial state, namely $\boldsymbol{\sigma}_k = \boldsymbol{\sigma}$ and $\boldsymbol{\Sigma}_0 \equiv \boldsymbol{\Sigma}^{(\mathrm{out})}$, then the output beams are left uncorrelated ($\boldsymbol{\Sigma}_{12} = \boldsymbol{0}$).

In the second part of the game, the public is now asked to insert at will the BS_I hiding it under a hat (figure 1(c)). The illusionist argues that he is able to say whether it is present or not and he succeeds, despite the fact that the interaction through the BS_I does not seems to affect the output states.

Now we explain how the use of a third ancillary mode correlated with one of the two input modes allows us to discri-

minate between the two cases. Although the public sees only modes 1 and 2 on the stage excited in the state $\varrho_1 = \varrho_2 = \varrho$, however, mode 2 is actually correlated with mode 3 (kept by the illusionist behind the curtain, figure 1(c)), i.e. $\varrho_2 = \mathrm{Tr}[\varrho_{23}]$ with $\varrho_{23} \neq \varrho_2 \otimes \varrho_3$. On the one hand, modes 1 and 2 are left unchanged and uncorrelated after the interference; because of the interaction, part of the correlations shared between modes 2 and 3 are now shared between modes 1 and 3. This can be seen by looking at the evolved CM of the whole state of the three modes in the presence of the BS_I. The 6×6 CM of the initial state $\varrho_{123} = \varrho_1 \otimes \varrho_{23}$ reads

$$\boldsymbol{\Sigma}_{123} = \begin{pmatrix} \boldsymbol{\sigma}_1 & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\sigma}_2 & \boldsymbol{\delta}_{23} \\ \boldsymbol{0} & \boldsymbol{\delta}_{23}^{\mathrm{T}} & \boldsymbol{\sigma}_3 \end{pmatrix}, \qquad (2)$$

where $\boldsymbol{\sigma}_k$ is the 2×2 single-mode CM of mode $k = 1, 2, 3$, $\boldsymbol{\sigma}_1 = \boldsymbol{\sigma}_2 = \boldsymbol{\sigma}$ and the matrix $\boldsymbol{\delta}_{23} \neq \boldsymbol{0}$ contains the correlations between modes 2 and 3. Since the BS_I acts only on modes 1 and 2, we have

$$\boldsymbol{\Sigma}_{123} \to \boldsymbol{\Sigma}_{123}^{(\mathrm{out})} = \begin{pmatrix} \boldsymbol{\sigma} & \boldsymbol{0} & \sqrt{1-\tau}\,\boldsymbol{\delta}_{23} \\ \boldsymbol{0} & \boldsymbol{\sigma} & \sqrt{\tau}\,\boldsymbol{\delta}_{23} \\ \sqrt{1-\tau}\,\boldsymbol{\delta}_{23}^{\mathrm{T}} & \sqrt{\tau}\,\boldsymbol{\delta}_{23}^{\mathrm{T}} & \boldsymbol{\sigma}_3 \end{pmatrix}. \qquad (3)$$

The comparison between equations (2) and (3) shows that while the states of modes 1 and 2 are (locally) left unchanged and, in turn, uncorrelated, both modes 1 and 2 are now correlated with mode 3 (again, the presence of non-zero off-diagonal blocks denotes the presence of correlations between the corresponding modes). Furthermore, the degree of correlations between modes 2 and 3 is decreased ($\boldsymbol{\delta}_{23} \to \sqrt{\tau}\,\boldsymbol{\delta}_{23}$) for the benefit of the birth of correlations between the previously uncorrelated modes 1 and 3 ($\boldsymbol{0} \to \sqrt{1-\tau}\,\boldsymbol{\delta}_{23}$). We conclude that BS_I is present, since mode 1 becomes correlated with mode 3 at the expense of the initial correlations between modes 1 and 2. Nevertheless, this happens in such a way that no overall correlations arise between the interacting modes. The illusionist, who has access also to the third mode can discover the presence of the BS_I by exploiting the 'hidden correlations'. Remarkably, the correlations are not hidden 'behind the curtain': in fact, the two beams are actually mixed (part of the photons from each beam are reflected and transmitted according to the interference rules of quantum physics) and, shot by shot, correlation may arise due to the quantized nature of the light, but, overall, this effect is 'washed out' by the very statistical nature of the Gaussian states and the linear interaction itself. Thus, the use of the ancillary mode allows us to retrieve this particular information.

For the sake of clarity, we described the illusionist game addressing only single-mode beams; however, the same results hold also in the presence of multimode Gaussian beams, provided that each mode interferes with the corresponding one. This is essentially due to the tensor product nature of the multimode state and the pairwise interaction.
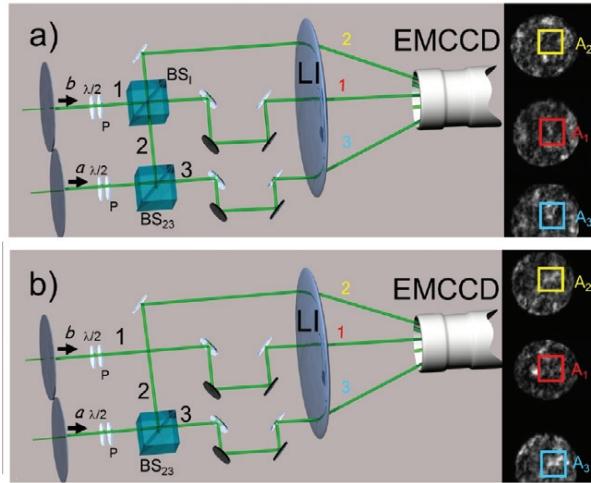
Figure 2. Sketch of the experimental setup. Implementation of the illusionist game with the BS_I (a) and without it (b). In the right panels of figures (a) and (b) the thermal beams detection area A_k are reported. The numbers of photoelectrons $I_k^{(j)}$ per frame j collected in the area A_k are used to evaluate the correlation coefficients $c_{1,2}$, $c_{1,3}$ and $c_{2,3}$ according to equation (4).

3. The illusionist game: experiment

We have experimentally realized the protocol by addressing the most simple Gaussian states, i.e. thermal beams (figure 2). Two speckled, i.e. spatial multimode and single temporal mode (pseudo)-thermal beams, a and b, were generated by scattering two 1 ns laser pulses ($\lambda = 532$ nm and 12.4 Hz repetition rate) on two independent rotating ground glasses (R1 and R2) [34]. While beam b is directly sent to BS_I as beam 1 (figure 2(a)), beam a is divided into the two correlated beams 2 and 3, needed for the trick, by means of the balanced beam splitter BS_{23}. A system of half waveplates ($\lambda/2$) and polarizers (P) compensated unbalancing in intensity of the modes and selected only modes with the same polarization, in order to realize the theoretical condition $\varrho_1 = \varrho_2$. The lens LI (focal length 20 cm) imaged the beams on an electron multiplying charge coupled device (EMCCD) array (Andor Luca, sensitive area 658×498 pixels, squared pixel dimension 10μm) with 0.16 magnification of the beams. Each mode of the beams corresponded to a point on the EMCCD array. For each beam $k = 1, 2, 3$ we selected an area A_k with 40×40 pixels collecting $M \approx 41$ modes. Each mode collected in the area A_k interferes with its correlated one in the corresponding other areas. Optimal selection of the correlated areas A_k is performed maximizing the (pixel-by-pixel) spatial correlation [35].

The pseudo-thermal beams show the typical speckle structure, and we put the BS_I in their far field zone in order to associate each mode of the multimode thermal beam with a point on the BS_I, i.e. on the detection plane. In this way, our setup allows us to identify properly each pair of single modes and make them interacting at the BS_I.

To evaluate the intensity correlations between the modes, which are due to the presence of non-zero off-diagonal blocks of the CM, each beam $k = 1, 2, 3$ was registered by an area A_k of an EMCCD array (right panel of figure 2), corresponding to the detector D_k of figure 1. We actually measure the number

of photoelectrons $I_k^{(j)}$ corresponding to the light intensity impinging on the whole area A_k. By averaging $I_k^{(j)}$ over the number of frames, due to the presence of many spatial modes per area, one has a reasonable estimate of the operator $\hat{\mathbb{I}}_k = \eta \sum_{m=1}^{M} \langle \hat{a}_{m,k}^{\dagger} \hat{a}_{m,k} \rangle$, where η is the quantum efficiency of the EMCCD and $\hat{a}_{m,k}$ is the field operator of the mth mode impinging on A_k and M is the total number of collected modes. The correlation between the beams h and k ($h, k = 1, 2, 3$) is estimated by using the second-order correlation coefficient $c_{h,k}$, namely

$$c_{h,k} = \frac{\langle I_k I_h \rangle_{\text{fr}} - \langle I_h \rangle_{\text{fr}} \langle I_k \rangle_{\text{fr}}}{\Delta_{\text{fr}}(I_h) \Delta_{\text{fr}}(I_k)}, \tag{4}$$

where $\langle F \rangle_{\text{fr}} = (N_{\text{frame}})^{-1} \sum_{j=1}^{N_{\text{frame}}} F^{(j)}$ is the average over N_{frame} frames and $\Delta_{\text{fr}}(I_k)^2 = \langle I_k^2 \rangle_{\text{fr}} - \langle I_k \rangle_{\text{fr}}^2$. If $c_{h,k} = 1$, one has maximum correlation; if $c_{h,k} = 0$, the beams are uncorrelated. It is remarkable to note that if all the spatial modes belonging to a beam have the same intensity, the correlation coefficient $c_{h,k}$ is independent of the number of modes M, i.e. $c_{h,k}$ is equal to the one calculated for the single mode.

To ensure proper mode matching at BS_I and perfect superposition of the correlated areas A_1 and A_2, we alternatively stopped beams a or b and measured and maximized the correlation $c_{1,2}^{(a)}$ and $c_{1,2}^{(b)}$, respectively, between beams 1 and 2 outgoing the BS_I. We then investigated the two configurations proposed by the illusionist: with the BS_I and without it. Our results fully demonstrate the theoretical protocol. Figure 3 shows the correlation coefficient $c_{1,2}$ for different configurations and as a function of the number of frames. With the BS_I inserted, the correlation falls from almost 1 in the presence of only one of the two beams a and b ($c_{1,2}^{(a)}$ and $c_{1,2}^{(b)}$), to almost zero when both beams are present, thus efficiently hiding the correlation ($c_{1,2}^{(BS)}$). A very small residual correlation persists due to experimental imperfections, such as the non-perfect superpositions of the optical modes and the unbalancing of the BS_I. The question that naturally arises is whether this residual correlation is enough to discriminate the presence from the absence of the BS_I. The answer is 'no' where a relatively small number of frames is used to estimate $c_{1,2}$. As shown in the insets of figure 3, the large overlap between the probability density functions (pdf) [36] of $c_{1,2}^{(BS)}$ and of $c_{1,2}^{(no)}$ (obtained for $N_{\text{frame}} = 10, 30, 50$) does not allow us to discriminate between the two cases with sufficiently small error probability and the 'hidden correlations' are needed.

In figure 4(a) we report $c_{1,3}$ with and without BS_I: the correlation between beams 1 and 3, which are initially uncorrelated, is evident in the presence of BS_I. In the absence of BS_I, beams 1 and 3 are uncorrelated, hence $c_{1,3}^{(no)} \approx 0$; inserting BS_I, correlations appear and $c_{1,3}^{(BS)} \approx 0.5$. Similar results concern $c_{2,3}$ (figure 4(b)): one has $c_{2,3}^{(no)} \approx 1$ without BS_I and $c_{2,3}^{(BS)} \approx 0.5$ with BS_I. However, to perform the trick, the illusionist should be able to discriminate between the two cases with low error probability and, thus, he should choose a suitable N_{frame}. In figure 4(c) we plotted the probability of error $P_{\text{err}}^{(1,2)}$ as expected by the public, i.e. addressing $c_{1,2}$, and that of the illusionist, $P_{\text{err}}^{(1,3)}$ or $P_{\text{err}}^{(2,3)}$, i.e. addressing $c_{1,3}$ or $c_{2,3}$, respectively. Also in the presence
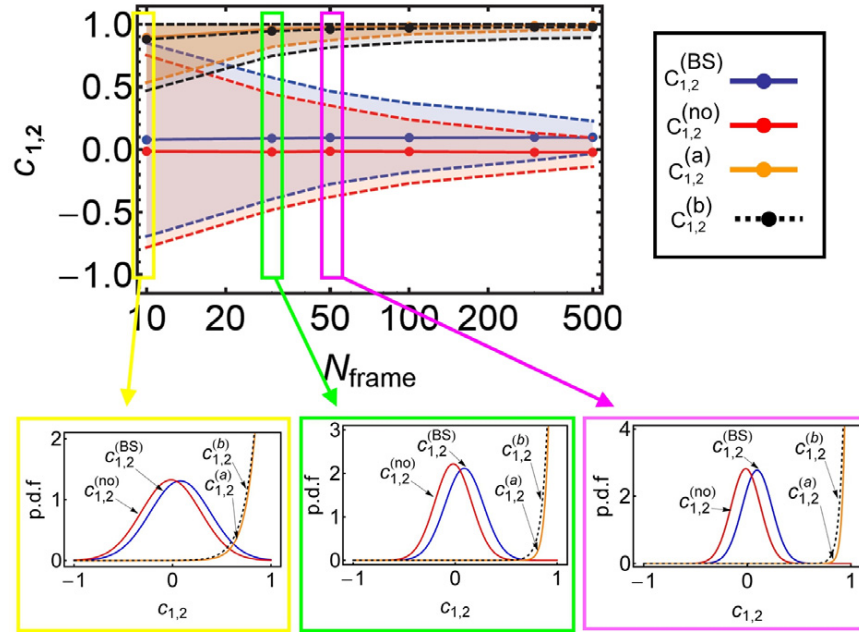
Figure 3. Plot of the correlation $c_{1,2}$ for different experimental configurations (dots), together with the confidence intervals at 99% (shaded regions), as a function of the number of frames used to evaluate it. $c_{1,2}^{(a)}$ (black) and $c_{1,2}^{(b)}$ (orange) correspond to the correlation of beams 1 and 2 outgoing BS$_I$ when only mode *a* or *b*, respectively, is switched on. $c_{1,2}^{(BS)}$ (blue) and $c_{1,2}^{(no)}$ (red) refer to the correlation of modes 1 and 2 with and without the BS$_I$, respectively, when both *a* and *b* are switched on. As expected, the width of the confidence intervals decreases as N_{frames} increases. The insets of the figure show the probability density function of the correlation coefficients for $N_{frame} = 10, 30$ and 50.
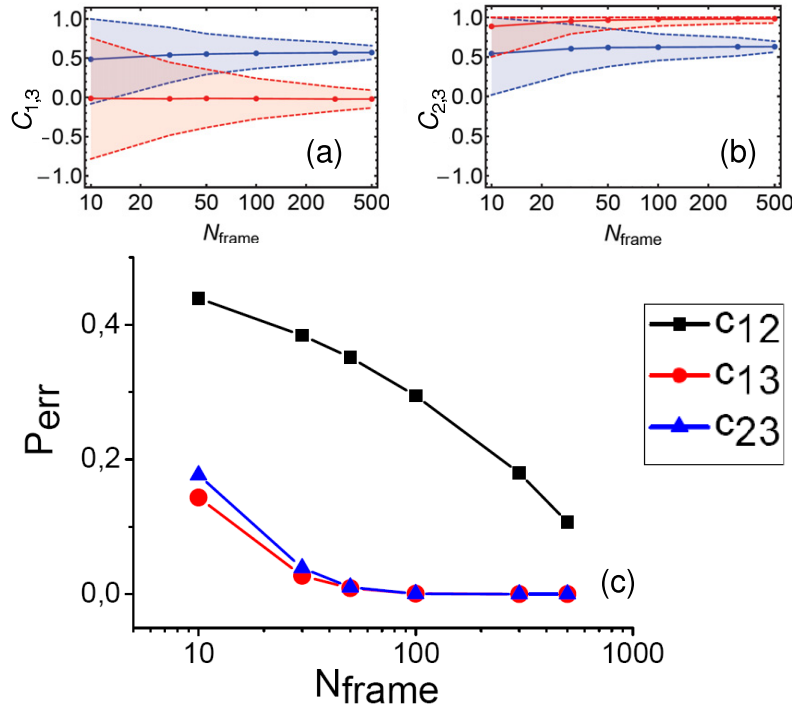

Figure 4. (a) Plot of $c_{1,3}$ (dots), together with their confidence intervals at 99% (dashed lines and shaded regions), as functions of N_{frame}. $c_{1,3}^{(BS)}$ (blue) and $c_{1,3}^{(no)}$ (red) refer to correlations between beams 1 and 3 with and without BS$_I$, respectively. (b) Plot of $c_{2,3}^{(BS)}$ (blue) and $c_{2,3}^{(no)}$ (red), referred to correlations between beams 2 and 3 with and without BS$_I$, respectively. (c) Log-linear plot of the error probability $P_{err}^{(h,k)}$ of the discrimination presence/absence of BS$_I$ as a function of N_{frame} obtained from $c_{h,k}$.

of experimental imperfections, for $N_{frame} \approx 50$, he is indeed able to discriminate between the two configurations, since $P_{err}^{(1,3)} \approx 1\%$ ($P_{err}^{(2,3)} \approx 1\%$) and $P_{err}^{(1,2)} \approx 35\%$: we conclude that the illusionist succeeded in his game!

4. Conclusions

In summary, we have suggested and demonstrated a novel scheme to illustrate and exploit the correlations of quantized

light. In our illusionist game, two identical Gaussian beams of light are addressed to the BS_I and do not acquire any correlation revealing the action of the BS itself. On the other hand, the presence of the BS can be identified by using the pre-existing correlations of one of the beams with a third one. Our protocol represents a paradigmatic example of hidden correlations of quantized optical beams, and paves the way for future applications, e.g. innovative interferometric schemes and generation scheme for highly excited photon number entangled states [37].

Acknowledgments

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