Optimal strategies to infer the width of an infinite square well by performing measurements on the particle(s) contained in the well

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Abstract
We seek the optimal strategy to infer the width $a$ of an infinite potential well by performing measurements on the particle(s) contained in the well. In particular, we address quantum estimation theory as the proper framework to formulate the problem and to determine the optimal quantum measurement, as well as to evaluate the ultimate bounds to precision. Our results show that in a static framework the best strategy is to measure position on a delocalized particle, corresponding to a width-independent quantum signal-to-noise ratio (QSNR), which increases with delocalisation. Upon considering time-evolution inside the well, we find that QSNR increases with time as $t^2$ (at least for small $t$). On the other hand, it decreases with $a$ and thus time-evolution is a metrological resource only when the width is not too large compared to the available time evolution. Finally, we consider entangled particles in the well and observe super-additivity of the QSNR: it is the sum of the single-particle QSNRs, plus a positive definite term, which depends on their preparation and may increase with the number of entangled particles. Overall, entanglement represents a resource for the precise characterization of potential wells.

Keywords: quantum estimation, quantum wells, quantum metrology

(Some figures may appear in colour only in the online journal)