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TELEPORTATION IMPROVEMENT BY NON-DETERMINISTIC NOISELESS LINEAR AMPLIFICATION

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We address de-Gaussification of continuous variables Gaussian states by optimal non-deterministic noiseless linear amplifier (NLA) and analyze in details the properties of the amplified states. In particular, we investigate the entanglement content and the non-Gaussian character for the class of non-Gaussian entangled state obtained by using NL-amplification of two-mode squeezed vacua (twin-beam, TWB). We show that entanglement always increases, whereas improved EPR correlations are observed only when the input TWB has low energy. We then examine a Braunstein-Kimble-like protocol for the teleportation of coherent states, and compare the performances of TWB-based teleprotation with those obtained using NL-amplified resources. We show that teleportation fidelity and security may be improved for a large range of NLA parameters (gain and threshold).

Keywords: Quantum teleportation, probabilistic noiseless linear amplification, non-Gaussianity, entanglement, EPR correlations
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1 Introduction

Quantum teleportation has proven to be one of the most relevant manifestations of entanglement [1, 2]. In this protocol, an unknown quantum state is teleported from a sending station to a remote receiving terminal by exploiting a quantum channel made by a two mode entangled state. In the framework of continuous-variable (CV) systems several teleportation protocols exploiting Gaussian entangled optical resources have been suggested and experimentally realized [3, 4, 5]. The Gaussian entangled resource commonly employed in CV quantum information protocols is the two-mode squeezed vacuum state (also termed twin-beam or EPR state) that may be produced by the process of parametric downconversion. Nonetheless, it has been shown that Gaussian entangled resources as twin-beams and any bipartite Gaussian states generated by Gaussian operations are subjected to restrictions. For instance, local Gaussian operations applied to bipartite Gaussian states cannot lead to entanglement distillation [6, 7]. Likewise, in the framework of quantum communication protocols, it has been shown that the security of Gaussian states against Gaussian errors cannot be improved by Gaussian operations [8]. Besides those restrictions, engineering high squeezed twin-beams is an arduous task even when the down conversion process is implemented in a resonant cavity. In fact, the highest amount of squeezing for the twin-beam attainable with current technology is around 10 dB [9] whereas the state-of-the-art of single mode squeezing boarder on 15 dB [10].

Thereby, a particular attention was drawn to non-Gaussian entangled resource that are more suited for quantum communication tasks. For this purpose, divers de-Gaussification protocols have been considered previously so as to improve the efficiency of quantum information processing. In particular, photons subtraction, photons addition and their coherent superposition performed on twin-beams has proven to enhance the degree of entanglement along with the teleportation fidelity [11, 12, 13, 14, 15, 16]. Recently, our investigations [17] on the action of an optimal non-deterministic noiseless linear amplifier (NLA) [18, 19, 20] on twin-beams revealed its robustness for the generation of non-Gaussian bipartite states and the enhancement of the degree of entanglement captured by the excess Von Neumann entropy [21, 22]. Previous works have demonstrated that some particular properties of non-Gaussian entangled resources are crucial to enhance the quality of quantum teleportation. Particularly, Dell'Anno *et. al*. [13] showed that optimization of the teleportation with a B-K-like protocol using certain non-Gaussian resources is made possible by adjusting their amount of entanglement, non-Gaussianity (NG) and squeezed vacuum affinity (SVA). Moreover, in [3], Einstein-Podolsky-Rosen (EPR) correlation of the entangled resources is pointed out to be an indicator for quantum teleportation.

In this work, the characteristics of the successfully amplified twin-beams generated from the action of the optimal NLA along with their performance on teleportation of coherent states are investigated. The degree of entanglement quantified by the excess Von Neumann entropy, the non-Gaussianity and the EPR correlations of the non-Gaussian twin-beams are discussed. Their performances in (CV) quantum teleportation of coherent states in a B-K-like protocol are then assessed. Finally, we identify the characteristics of the non-Gaussian amplified twin-beams that lead to an improvement of the teleportation success (quantified by the average fidelity of the output state with respect to the input unknown one), compared to the standard twin-beams. We will show that substantial improvements of the teleportation success are achieved in the regime where the EPR correlation of the entangled resource are lowered while a non-trivial dependence on the degree of entanglement and the non-Gaussianity quantified by the quantum relative entropy [23], is reported. It's worth noting that noiseless amplification was previously proposed to be employed in a teleportation set-up to achieve continuous variable error correction on Gaussian states that experienced Gausssian noise occasioned by loss [24].

The paper is structured as follows: in Sec 2, we analyse relevant properties of two-mode non-Gaussian entangled states engineered via an optimal NLA and confront them with that of the standard twin-beam. Entanglement content captured by the excess Von Neumann entropy and EPR correlation along with the entropic non-Gaussianity are discussed. Sec 3, is devoted to continuous-variables quantum teleportation with a class of non-deterministically amplified twin-beams. The paradigmatic instance of coherent states as inputs is discussed. Finally, in Section 4 we summarize our results and set forth our conclusions.

2 Characteristics of the amplified twin-beams

Aiming to examine the properties of the non-Gaussian amplified twin-beam, we first review some details regarding its generation and recall certain characteristics of the Gaussian standard twin-beam.

2.1 The standard twin-beam bipartite state

The two-mode squeezed vacuum is a broadly used entangled resource in divers quantum information protocols. Its generation involves the process of parametric down-conversion where a $\chi^{(2)}$ non-linear crystal serving as an amplifying medium is pumped with light at frequency ω_p. A fraction of it gives then arise to a pair of photons with frequencies ω_a and ω_b, obeying to the constraint $\omega_p = \omega_a + \omega_b$. Formally, its expression is obtained by applying the two mode squeezing operator $S_{ab}(r) =$

$\exp -r(\hat{a}^\dagger\hat{b}^\dagger - \hat{a}\hat{b})$ [25] to the vacuum:

$$|\chi\rangle = \exp\left(r(\hat{a}^\dagger\hat{b}^\dagger - \hat{a}\hat{b})\right)|00\rangle = \sqrt{1-\chi^2}\sum_{n=0}^{\infty}\chi^n\,|nn\rangle\,, \tag{1}$$

where r is assumed real without loss of generality, $\hat{a}^\dagger(\hat{a})$ and $\hat{b}^\dagger(\hat{b})$ are the creation(annihilation) operators of the two modes and $0 < \chi = \tanh r < 1$ depends on the non-linear susceptibility and an effective interaction length. The standard twin-beam has a Gaussian Wigner function. Since it is a pure (CV) state, a good measure of its entanglement content is the excess Von Neumann entropy defined as the Von Neumann entropy of the reduced density operator $E\left[\varrho_{ab}\right] = \hat{S}\left[\rho_a\right] = -Tr\left[\rho_a \ln \rho_a\right]$ [26], where the reduced density operator ρ_a is obtained by tracing over the mode b of the twin-beam. Its Von Neumann entropy is found to be

$$S\left[\rho_a\right] = -\ln\left(1-\chi^2\right) - \frac{\chi^2\ln\left(\chi^2\right)}{\left(1-\chi^2\right)}. \tag{2}$$

Besides the degree of entanglement characterizing an entangled resource, Einstein-Podolsky-Rosen correlation of a bipartite state has proven to be a prominent element to carry out quantum teleportation. Instead of the position and momentum operators of a massive particle discussed in the celebrated paper on the completeness of quantum mechanics [27], the quantities of interest for an optical system are the quadrature operators defined as

$$\begin{aligned} \hat{x}_a &= \frac{1}{\sqrt{2}}(\hat{a} + \hat{a}^\dagger), \qquad \hat{p}_a = \frac{-i}{\sqrt{2}}(\hat{a} - \hat{a}^\dagger), \\ \hat{x}_b &= \frac{1}{\sqrt{2}}(\hat{b} + \hat{b}^\dagger), \qquad \hat{p}_b = \frac{-i}{\sqrt{2}}(\hat{b} - \hat{b}^\dagger), \end{aligned} \tag{3}$$

Regarding a bipartite optical state, the EPR correlation reads [28]

$$\Delta z^2 = \Delta(\hat{x}_a - \hat{x}_b)^2 + \Delta(\hat{p}_a + \hat{p}_b)^2, \tag{4}$$

which, after expressing the quadrature operators in term of the annihilation and creation operators becomes

$$\Delta z^2 = 2\left[1 + \langle\hat{a}^\dagger\hat{a}\rangle + \left\langle\hat{b}^\dagger\hat{b}\right\rangle - \left\langle\hat{a}^\dagger\hat{b}^\dagger\right\rangle - \left\langle\hat{a}\hat{b}\right\rangle - (\langle\hat{a}\rangle - \left\langle\hat{b}^\dagger\right\rangle)(\langle\hat{a}^\dagger\rangle - \left\langle\hat{b}\right\rangle)\right]. \tag{5}$$

A zero EPR correlation reveals perfect correlation between the two modes as for the ideal EPR state introduced in [27]. Indeed, both variances of the position-like operators difference and momentum-like operators sum are zero, thus indicating that knowing the quadratures of say mode 'a' enables to exactly determine the quadratures of the mode 'b'. Contrariwise, a value exceeding 2 indicates classical separable two-mode states. For a two-mode squeezed vacuum, the EPR correlation is found to be

$$\Delta z^2 = \frac{2(1-\chi)}{(1+\chi)}. \tag{6}$$

As long as the squeezing parameter χ belongs to $]0, 1[$, the EPR correlation of the standard twin-beam remains lower than 2, thus indicating the presence of quantum correlations. Moreover, when χ broads on 1 (in the limit of infinite squeezing), the EPR correlation becomes null, testifying for maximum entanglement while in the case of weak squeezing, it approaches 2. Thereby, for a two mode squeezed state, highest squeezing strength lead to strongest correlations. Finally, we recall that at fixed energy, the twin-beams are known to be the maximally entangled (CV) bipartite states [29].

2.2 The amplified twin-beam

Before going deeper into our survey, we propose to expose in short, some of the most emblematic experimental realisations of the non-Gaussian operation discussed here, namely, the noiseless linear amplification. Since the pioneer work of Ralph and Lund [30] where the idea of probabilistic noise-free amplifiers was coined, several implementations of the NLA has been achieved by different groups. The first realisations were physical: a suitably engineered device involving optical components was required. The operating principles of the proposed schemes range from generalized quantum scissors involving auxiliary single-photon sources and multiphoton interferometric set-ups [31, 32, 33] to coherent superposition of single photons addition and subtraction [35, 36], and include concentration of phase information via noise addition [34]. Thereafter, given the demanding resources involved in the physical designs, virtual noiseless amplifiers were theoretically examined [37, 38] then experimentally achieved [39, 40]. Surprisingly, these alternative schemes, commonly termed "Measurement-based NLA" only require a postselection of heterodyne detection outcomes that emulate the quantum filter induced by the physical non-deterministic amplification. In the end, each of the designs mentioned so far along with the optimal architecture considered in our work [20] naturally afford different performances in terms of success probability and fidelity to the ideally amplified input, and are found to be approximate realizations of the ideal noise-free phase-insensitive amplifier achieving the operation $\hat{g}^{a^\dagger a}$.

Generation of non-Gaussian entangled bipartite state via photon subtraction(addition) or their coherent superposition on two-mode squeezed vacuum states was revealed useful in quantum information processing. Likewise, non-Gaussian states produced by mixing photon subtracted squeezed vacuum states with vacuum in a beam splitter showed interesting improvement in the quality of quantum teleportation of coherent states [41]. Here we discuss a process of de-Gaussification based on the action of an optimal NLA on a twin-beam. One mode of the twin-beam is coupled to a measurement device (MD) consisting of a two-level system (as it happens, $|S\rangle$ and $|F\rangle$, where "S" refers to success whereas "F" stands for failure) through a unitary transformation. The (MD) is then projected into one of its initial states $|S\rangle$ or $|F\rangle$, that heralds the success or the failure of the amplification. We consider only the successfully amplified states that we refer to as the amplified twin-beam. The process is thus inherently conditional and was presented in details in [17]. Here we just recall the expression of the amplified twin-beam derived from the action of the Krauss operator accounting for a successful run of the optimal device [20, 19]

$$|\chi_s\rangle = \frac{\hat{E}_s^p \otimes \mathbb{I}_b \, |\chi\rangle}{\sqrt{P_{s,\chi}}} \tag{7}$$

$$= \sqrt{\frac{1-\chi^2}{P_{s,\chi}}} \left(g^{-p} \sum_{n=0}^{p} (g\chi)^n \, |nn\rangle + \sum_{n=p+1}^{\infty} \chi^n \, |nn\rangle \right), \tag{8}$$

where g and p are the intrinsic parameters of the optimal NLA and denote respectively the gain and the integer setting the truncation order in the Fock basis that we dub threshold. $P_{s,\chi}$ represents the probability to successfully implement the desired amplification on an initial twin-beam and reads

$$P_{s,\chi} = (1-\chi^2)\left[g^{-2p}\sum_{n=0}^{p}(g\chi)^{2n} + \sum_{n=p+1}^{\infty}\chi^{2n}\right]. \tag{9}$$

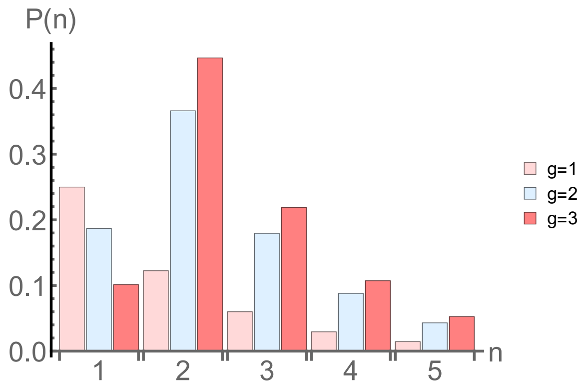

Fig. 1. (Color online) Bar chart representation of the photon number distribution for various bipartite states: the standard twin-beam $g = 1$ (light-red bar) and amplified twin-beams generated through different configurations: $g = 2$ (light-blue bar), $g = 3$ (pink bar). The threshold is kept fix $p = 2$ and the squeezing parameter is setted equal to 0.6.

As we clearly see, the NLA tends to privilege the occurrence of higher photon numbers, thus inducing a modification in the photon number distribution. Fig. 1 shows the bar chart representation of the photon number distribution assigned to the amplified twin-beams and their standard version for a fixed squeezing parameter and different configurations of the NLA. We notice that indeed, the coincidence of measuring higher photons number in the two modes of the amplified twin-beam has a greater weighting then its standard version while the inverse is observed for single photons. Moreover, we remark that stronger is the amplification (great values of the gain), more pronounced is the variation of the photon number distribution induced by the NLA. This observation remains valid for any values of the squeezing parameter χ and the threshold p.

We remind the reader that earlier works [30, 19] have considered in some details the effect of a noiseless linear amplifier on EPR states. In fact, in the pioneer survey by Ralph and Lund [30], the authors examined the action of a particular NLA consisting of multiple blocks of quantum scissors on EPR states both for ideal and lossy channels. On the other hand, a different theoretical description of the amplifier has been examined in[19], where performances of an optimal device on a lossy EPR state are drawn up.

2.3 Non-Gaussianity

As emphasized previously, non-Gaussianity (nG) has proven to be a resource for various tasks in quantum information processing. In order to examine the effect of non-Gaussianity on the performances of the amplified twin-beam, we make use of the entropic measure introduced in [23]. For a generic quantum state , the entropic nG measure is defined as the quantum relative entropy between the state under study and its reference Gaussian state $\hat{\varrho}_G$ (that is completely defined through its vector of mean values and covariance matrix)

$$\delta_{\mathrm{nG}}[\hat{\varrho}] = S(\hat{\varrho}_G) - S(\hat{\varrho})\,, \tag{10}$$

where $S(.)$ remains for the Von Neumann entropy. The amplified twin-beam being pure, its Von Neumann entropy vanishes and the evaluation of its non-Gaussian character reduces to the Von Neumann entropy of $\hat{\varrho_G}$. For a generic bipartite Gaussian state, the Von Neumann entropy is given by

$$S(\hat{\varrho}) = h(d_+) + h(d_-)\,, \tag{11}$$

where d_+ and d_- are the symplectic eigenvalues of the covariance matrix assigned to the amplified twin-beam and $h(x)$ a function defined as

$$h(x) = \left(x + \frac{1}{2}\right)\ln\left(x + \frac{1}{2}\right) - \left(x - \frac{1}{2}\right)\ln\left(x - \frac{1}{2}\right). \tag{12}$$

The entropic nG measure is then found to be

$$\delta_{\mathrm{nG}}\Big[|\chi_s\rangle\langle\chi_s|\Big] = 2h(d_+) = 2h(\sqrt{I_1 + I_3}) \tag{13}$$

where $I_1 = \frac{1}{2} + \bar{N}_\chi$ and $I_3 = \mathrm{Tr}[|\chi\rangle\langle\chi|(\hat{a}\hat{b})]$. $\bar{N}_\chi$ denotes the average photons number in one mode of the amplified twin-beam. For detailed calculations, see [17]. In Fig. 2 are reported the plots of

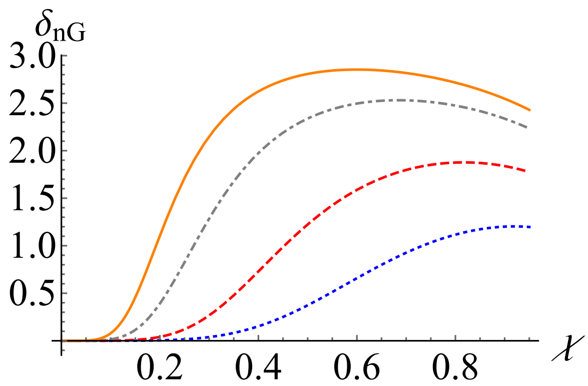

Fig. 2. (Color online) Entropic Non-Gaussianity measure δ_{nG} as a function of the squeezing parameter for the bipartite states generated by an optimal NLA with different calibrations. The threshold p is fixed to 2 while varying the gain: the blue dotted line represents $g = 1.5$, the red dashed line $g = 2$, the grey dotted-dashed line denotes $g = 3$ and finally, the orange full line $g = 4$.

the entropic non-Gaussianity of the amplified twin-beam as a function of the squeezing parameter. Different configurations of the NLA are considered. We observe a non-monotonic behaviour for the entropic nG with respect to the squeezing parameter χ. Two regimes are clearly noticeable: first, the nG increases with the squeezing parameter to reach its peak then tends to diminish softly. The rate at which the entropic nG increases, the value at which the peak is achieved and its magnitude depend on the configuration of the NLA. We observe that the more intense is the configuration, the swiftest is the increment of the nG and the higher is its peak. Moreover, for intense configurations, the peak is reached at lower squeezing parameters, thus indicating that starting from a weakly squeezed twin-beam, one could engineer highly non-Gaussian bipartite state via the NLA. Finally, we notice that for relatively weak amplifications and squeezing parameters, the emerging bipartite state remains Gaussian.

2.4 Degree of entanglement

Here we intend to quantify the effect of the optimal NLA on the entanglement content of the twin-beam. From the moment that the resulting amplified twin-beam remains pure (the quantum operation describing a successful amplification is represented by a single Krauss operator [20]) we turn again towards the Von Neumann excess entropy to quantify its degree of entanglement. In Fig. 3 are reported the plots of the excess Von Neumann entropy as a function of the squeezing parameter for different configurations of the NLA. The threshold is kept fixed while varying the gain. For the sake of comparison, we also report the entanglement content of the standard twin-beam. As expected, all the considered quantities are increasing functions of the squeezing parameter. We remark that the degree of entanglement of the amplified twin-beams is greater than that of the standard bipartite squeezed vacuum, and that for all the considered configurations and at any value of the energy, particularly in the interval $0 < \chi < 0,75$ experimentally available. Moreover, strong amplifications tend to enhance the degree of entanglement of twin-beams characterized by weak energies while the opposite is observed for weak amplification. Finally, as observed for the entropic non-Gaussianity, the more intense are the NLA configurations, lowest are values of the squeezing parameter at which the peaks of the excess Von Neumann entropy are reached.

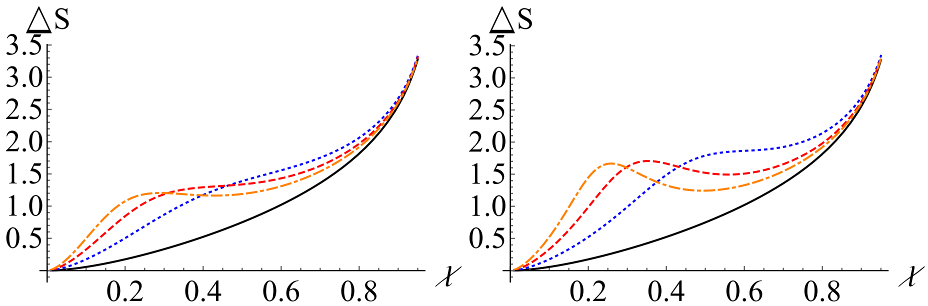

Fig. 3. (Color online) Excess Von Neumann entropy of the amplified and standard twin-beams as a function of the squeezing parameter. Different NLA configurations are considered: the blue dotted line denotes $g = 2$, the red dashed line $g = 3$ and the orange dotted-dashed line represents $g = 4$. The black full line remains for the standard twin-beam. In the left panel, $p = 2$ whereas in the right panel $p = 4$.

2.5 EPR correlation

Previously, we addressed EPR correlation between the optical field quadratures as an alternative approach to quantify entanglement content of a bipartite state. In This subsection, we discuss the effect of an optimal non-deterministic noiseless amplification on the EPR correlation of a two-mode squeezed vacuum. We underline that for the bipartite states of interest, the variance of the position difference and the momentum sum are equal, thus the EPR correlation resumes to $\Delta z^2 = 2\Delta\left(\hat{x}_a - \hat{x}_b\right)^2$. Fig. 4 shows the variations of the EPR correlation with respect to the squeezing parameter. Different configurations where the threshold is fixed and the gain varies are considered and compared to that of the standard twin-beam. We remark that apart from the standard twin-beam whose position difference variance is a monotonically decreasing function of the squeezing parameter, the quantity of interest shows a non-monotonic behaviour for all the generated bipartite states from different configurations. We also notice that if χ is smaller than a threshold that depends on the intrinsic parameters of the NLA, the EPR correlation is lower than that of the standard twin-beam. Thereby witnessing for the robustness of an optimal non-deterministic noiseless amplifier acting in a certain regime to increase the entanglement content of a twin-beam of a weak energy. In the contrary, beyond that threshold, the variance of the amplified twin-beams is larger than that of the standard two-mode squeezed vacuum. Especially, regarding strong amplifications, the EPR correlation attains values exceeding 2 that separate quantum entangled bipartite states from separable states, thus testifying for the inefficiency of the NLA in that range of energies.

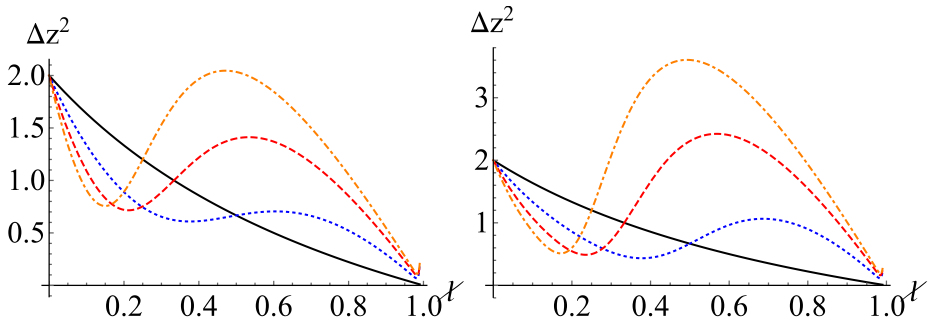

Fig. 4. (Color online) The EPR correlation Δz^2 as a function of χ for a two mode squeezed vacuum (black full line) and amplified twin-beams generated from different configurations of the NLA: $g = 2$ (blue dotted line), $g = 3$ (red dashed line), $g = 4$ (orange dotted-dashed line). In the left panel, p is setted to 2 while in the right panel $p = 4$.

According to Fig.3 and Fig.4, the NLA enhances the entanglement content captured by the excess Von Neumann entropy of a twin-beam regardless of its initial squeezing parameter while a different effect is observed when entanglement is quantified by the EPR correlation, where enhancement is observed only in the interval of weak energies. We conclude that an increase of the excess Von Neumann entropy doesnt systematically imply strongest EPR correlation. In the light of our upshot, one may expect the non-Gaussian amplified twin-beams to enhance the fidelity of quantum teleportation in the range of weak squeezing parameter. This stems from the fact that in one hand, the Braunstein-Kimble protocol is based on two-mode squeezed vacua as entangled resources that are subjected to drastic restrictions [6], in the other hand, we showed that the entanglement content of the non-deterministically amplified twin-beams captured by both the excess Von Neumann entropy and the EPR correlation is improved by the action of the NLA in the range of weak energies. We recall that the interval extent depends on the NLA intrinsic parameters.

3 Teleportation improvement assisted by NLA

Quantum teleportation was initially proposed by Bennet *et. al* . in the discrete variable regime where an unknown quantum state of a half-spin particle is transferred from a sending station to a distant receiver via a dual combination of a quantum channel consisting of EPR states and classical communication. Later Vaidman proposed a similar scheme in the domain of continuous-variable (CV) systems where the information is encoded in the position and momentum of the system [42] . Thereafter, an example of (CV) quantum teleportation in the optical domain was presented by Braunstein and Kimble where information is encoded in the quadratures of the optical system and the two-mode squeezed vacuum used as an entangled resource [3]. Subsequently, Furusawa and collaborators implemented the protocol experimentally [4]. Here we present our theoretical results on (CV) quantum teleportation assisted by NLA where the entangled resource is a non-deterministically amplified two-mode squeezed vacuum. We further draw up a comparative study with respect to the conventional B-K protocol with the standard twin-beam. We are particularly interested in the teleportation of coherent Gaussian states. Foremost, let us swiftly recall the main steps of the standard BK protocol. Two protagonists Alice and Bob, share beforehand an entangled bipartite resource of modes a and b. Alice possess a single-mode input unknown state $|\psi_{in}\rangle$ that she intends to transfer to Bob and thus corresponds to the state to be teleported. Her task is achieved as follows: she mixes the state in mode in on her possession with the mode a of the shared bipartite resource state at a balanced beam splitter, resulting in the output modes c and d. An homodyne detection is then performed by Alice on the output modes and the result $\beta = x_- + ip_+$ transmitted to Bob through a classical channel. The beam splitter being balanced, the real and imaginary parts of β are respectively the eigenvalues of the position-like difference and momentum-like sum

$$\frac{1}{\sqrt{2}}(\hat{x}_{in} - \hat{x}_a), \qquad \frac{-i}{\sqrt{2}}(\hat{p}_{in} + \hat{p}_a), \tag{14}$$

where $\hat{x}_{in}$, $\hat{p}_{in}$ are the quadrature operators of the mode in and $\hat{x}_a$, $\hat{p}_a$ those of the mode a. The quadrature operators appearing in Eq. (14) are the observables measured through the homodyne detection and are related to the input modes operators via the unitary transformation induced by the $50/50$ beam-splitter. Finally, Bob applies a displacement $\hat{D}(\beta) = \exp\left(\beta\hat{b}^\dagger - \beta^*\hat{b}\right)$, conditioned by the result β, on the remote mode b of the entangled resource yielding the teleported state. We precise that throughout the paper, the gain of the teleporter is set to unity.

Formally, various descriptions of the above protocol have been presented in the literature, ranging from the Wigner function formalism originally introduced by Braunstein and Kimble [3], the Fock state expansion approach [43], the transfer operator description [44] to the characteristic function based formalism [45]. Although this latter is extensively used when non-Gaussian entangled resources are involved in quantum teleportation, we adopt the transfer operator description that is more suited for our work. So as to assess the performances of quantum teleportation with the non-Gaussian amplified twin-beam, we establish the expression of the average fidelity. In the formalism of the transfer operator, the expressions of the teleportation fidelity, the probability for the homodyne detection to display the outcome β and the average fidelity are evaluated through the action of a certain transfer operator on the quantum states involved in the teleportation process. The quantum state in Bob's possession arising from the whole teleportation process (transfer of information through the quantum and classical channels) and that we dub output state is obtained through the action of the transfer operator $\hat{T}(\beta)$ on the input state

$$|\psi_{out}(\beta)\rangle = \hat{T}(\beta)\,|\psi_{in}\rangle\,, \tag{15}$$

we note that the output state under consideration is conditioned by the result of the homodyne measurement and is thus not normalized. The probability of a given outcome to occur reads

$$p(\beta) = \langle\psi_{out}(\beta)|\psi_{out}(\beta)\rangle\,. \tag{16}$$

Given those expressions, the fidelity of teleportation $\mathcal{F}(\beta)$ defined as the overlap between the input $|\psi_{in}\rangle$ and the output $|\psi_{out}(\beta)\rangle$ states is found to be

$$\mathcal{F}(\beta) = \frac{1}{p(\beta)} \left| \langle\psi_{in}| \, \hat{T}(\beta) \, |\psi_{in}\rangle \right|^2, \tag{17}$$

whereas the average fidelity, being the fidelity of teleportation averaged over the possible outcomes β assume the following expression

$$\bar{\mathcal{F}} = \int d\beta^2 p(\beta)\mathcal{F}(\beta) = \int d\beta^2 \left| \langle\psi_{in}| \, \hat{T}(\beta) \, |\psi_{in}\rangle \right|^2. \tag{18}$$

In order to evaluate the quantities of concern, one has to establish the expression of the transfer operator that depends on the bipartite entangled resource. In [44], Hofmann *et. al.* draw up the expression that the transfer operator assumes when the entangled resource consists of the standard twin-beam

$$\hat{T}(\beta) = \sqrt{\frac{1-\chi^2}{\pi}} \sum_{n=0}^{\infty} \chi^n \hat{D}(\beta)|n\rangle\langle n|\hat{D}(-\beta), \tag{19}$$

where $\hat{D}(\beta)$ denotes the usual displacement operator acting on the input state. Moreover, it has been noticed in [12] that if the entangled bipartite state assumes the following form

$$|\phi\rangle = \mathcal{N} \sum_{n=0}^{\infty} k_n |n, n\rangle, \tag{20}$$

where $\mathcal{N}$ is a normalisation constant and k_n the coefficients of the entangled resource when expanded in a Fock basis, the transfer operator generalizes to

$$\hat{T}(\beta) = \frac{\mathcal{N}}{\sqrt{\pi}} \sum_{n=0}^{\infty} k_n \hat{D}(\beta)|n\rangle\langle n|\hat{D}(-\beta), \tag{21}$$

As we can clearly see, our entangled state arising from the action of an optimal NLA on a standard EPR-state is of the form Eq. (20) where the normalisation constant and coefficients respectively read as follows

$$\mathcal{N} = \sqrt{\frac{1-\chi^2}{P_{s,\chi}}}, \tag{22}$$

$$k_n = \begin{cases} g^{n-p}\chi^n, & \text{if } n \leq p \\ \chi^n, & \text{otherwise} \end{cases} \tag{23}$$

3.1 Teleportation of coherent states

In order to assess the performances of the entangled resource considered in our work, we focus on the paradigmatic case of coherent states teleportation. For the sake of comparison, we first recall some results regarding the standard protocol with the two-mode squeezed vacuum as entangled state. The output state arising from the teleportation process performed on an input coherent state of amplitude α is found to be

$$\hat{T}(\beta)|\alpha\rangle = \sqrt{\frac{1-\chi^2}{\pi}} \exp\left[-(1-\chi^2)\frac{|\alpha-\beta|^2}{2}\right] \exp\left[(1-\chi)\frac{\alpha\beta^* - \alpha^*\beta}{2}\right] |\chi(\alpha-\beta)\rangle. \tag{24}$$

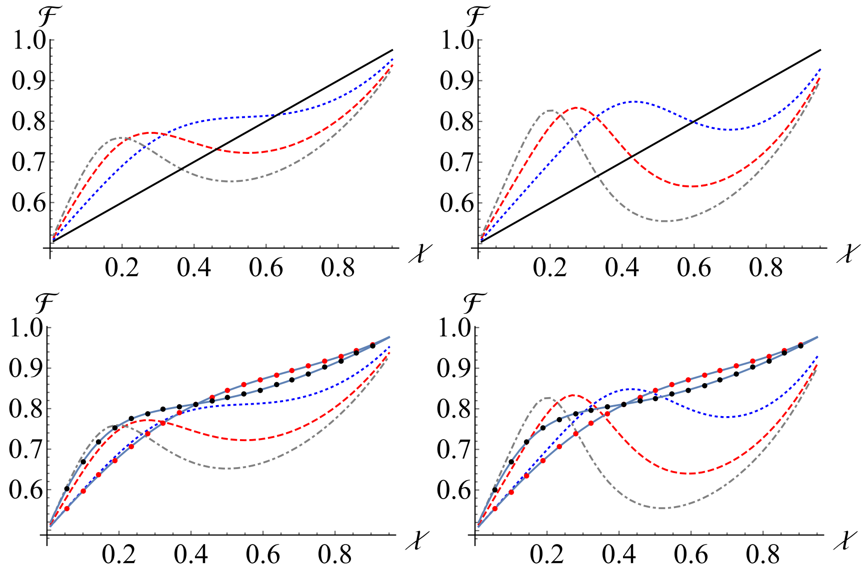

Fig. 5. (Color online) The average fidelity $\bar{\mathcal{F}}$ as a function of χ for a two mode squeezed vacuum (black full line), photon-subtracted twin-beam (red circles), photon-added-then-subtracted twin-beam (black circles) and amplified twin-beams generated from different configurations of the NLA: $g = 2$ (blue dotted line), $g = 3$ (red dashed line), $g = 4$ (grey dotted dashed line). In the left panels, p is setted to 2 while in the right panels $p = 4$.

We note that the output is also a coherent state whose amplitude coincides with the displaced then attenuated input amplitude respectively by the factors β and χ. The teleportation fidelity averaged over all the homodyne measurement outcomes β is independent of the input amplitude and reads

$$\bar{\mathcal{F}} = \frac{1}{2}(1 - \chi). \tag{25}$$

Regarding our B-K like protocols where the standard EPR state is substituted by the conditional amplified twin-beam, the fidelity of teleportation and the probability of occurrence for a given outcome β are calculated by means of the following formula

$$\langle n|\hat{D}(\beta)|\alpha\rangle = (n!)^{-1/2}(\alpha+\beta)^n \exp\frac{1}{2}|\alpha+\beta|^2 \exp\frac{1}{2}(\alpha^*\beta - \alpha\beta^*). \tag{26}$$

The average fidelity is then carried out numerically by truncating the Fock basis to a relevant order. In Fig. 5 are reported the plots of the average fidelity $\bar{\mathcal{F}}$ for input coherent state $\alpha = 2$ as a function of the squeezing parameter χ. A comparative survey between the non-deterministically amplified twin-beams and their standard version along with different instances of commonly used de-Gaussified twin-beams is drawn up. As it is apparent from the two sub-figures on top, there is a range of the squeezing parameter where the average fidelity for quantum teleportation with our conditional entangled resource is enhanced when compared with the standard B-K protocol. We remark that although the amplified twin-beams present a higher excess Von Neumann entropy regardless of the squeezing parameter and the NLA parameters, the average fidelity is enhanced only in a certain region of the input energy. In addition, we notice that the critical value delimiting that region depends on the NLA gain g for a fixed

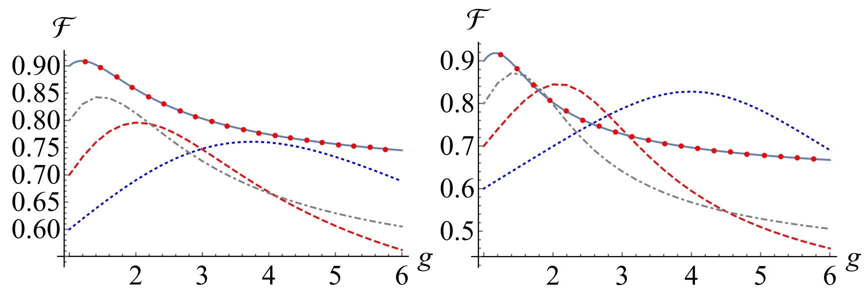

Fig. 6. (Color online) The average fidelity $\bar{\mathcal{F}}$ as a function of g for entangled resources with different input energies: $\chi = 0.22$ (blue dotted line), $\chi = 0.4$ (red dashed line), $\chi = 0.6$ (grey dotted dashed line) and $\chi = 0.8$ (red circles). In the left panel, p is fixed at 2 whereas in the right panel $p = 4$.

value of the truncation order p in such a way that the strongest is the amplification the smallest is that critical value. Through sub-figures on bottom of Fig. 5, we compare the performances of coherent states teleportation protocol exploiting the de-Gaussification process based on an optimal NLA and two well-known de-Gaussification schemes that respectively involve photons subtraction and photons addition then subtraction. Expressions of their average fidelities are independent of the input coherent amplitude and are reported in [14], where, it was also shown that the photon-added-then-subtracted twin-beam beats the performance of the photon-subtracted twin-beam in the range of weak energies. It appears that, for strong calibrations of the NLA (high values of g and $p = 4$), the amplified twin-beams outperform the photon-added-then-subtracted two-mode squeezed state when the input energy belongs to a certain interval, whereas being detrimental to the quality of teleportation in the remaining region. A different behaviour is observed when the threshold is fixed at $p = 2$. Indeed, when the gain is below $g = 3$, enhancement is achieved compared with the photon-subtracted resource whereas the quality of teleportation remains below that of the photon-added-then-subtracted twin-beam. Finally, We notice that numerical results of the average fidelity for a broad set of input amplitudes ranging from -20 to 20 that includes also complex values show an identical behaviour, we thus state that the teleportation quality captured by the fidelity is state independent. Hence, the plots produced for the specific value ($\alpha = 2$) of the input amplitude considered throughout the paper generalize to arbitrary coherent input state.

As it appears from the upshots displayed in Fig. 5, noiseless amplifications operating in the strong working regime (high values of the gain) tend to better enhance the average fidelity when entangled resources with low energies are considered whereas being detrimental for the teleportation quality for EPR states with high squeezing parameter. Identification of the values of the gain leading to the most substantial improvement afforded by the NLA is of great interest. In Fig. 6, the average fidelity of teleporting coherent states with entangled resources of different squeezing parameters is plotted as a function of the NLA's gain. The intersections of the plots with the "y" axis characterize the average fidelity of the standard B-K protocol. As expected, and in accordance with our prior observations, the improvements induced by the NLA on the teleportation quality are optimized by high values of the gain for twin-beams with low input energies while weak amplifications lead to more substantial fidelities when performed on high squeezed entangled resources. Due to the tunable nature of the NLA, its employment maybe optimized with respect to its intrinsic parameters for each considered

EPR states and hence take full advantage of the de-Gaussification process. We notice that beyond a certain value of the input squeezing of the twin-beam, the NLA is no more useful irrespectively of its calibration.

An interesting feature of non-deterministically amplified EPR states is thus to lead up high fidelities starting from a standard EPR state of weak energy whose experimental generation is well held down. Hence, so as to achieve high average fidelities for the continuous variable quantum teleportation, one may exploit the advantages of the optimal NLA that acts as an entanglement distiller instead of seeking to engineer intense squeezers. Finally, we point out that even when moderated amplifications are implemented, the amplified entangled resources show, in a certain region of weak energies, a more substantial improvement than that of the photon-subtracted EPR state extensively studied in previous works [13, 14, 46]. A similar observation stands for its inconclusive version [15].

Information transmission in a teleportation process is said truly nonlocal if its average fidelity $\bar{\mathcal{F}}$ exceeds $1/2$ [47]. As shown in Fig. 5, both the standard and amplified twin-beams surpass this limit independently of the NLA configuration. A more significant boundary ($\bar{\mathcal{F}} > 2/3$), witnessing that the state in Bob's possession is the best existing copy was derived in [48]. The meaning of this boundary is that when a secure transmission is required, even if Bob is not able to avert an eavesdropping action that alters or clones the transferred state, he can check if he's the only owner of this latter [49, 50]. Indeed, this is made possible by considering the average fidelity of the teleportation process. When the fidelity beats the boundary $2/3$, Bob knows unambiguously that his state was not duplicated and the teleportation is said secure. In Fig. 7 are plotted the average fidelities of the standard twin-beam and its non-deterministically amplified homologous with the configuration ($g = 2$ and $p = 4$) as functions of the squeezing parameter χ, and where a coherent state of amplitude $\alpha = 2$ is teleported. Fig. 7 highlights a region (the light-blue shaded area) where the average fidelity of the teleportation with our non-Gaussian resource exceeds $2/3$ whereas the quality of the standard protocol with the two-mode squeezed vacuum remains below that boundary. The vertical and horizontal lines serve to mark out the shaded region. According to those results, our proposed conditional scheme based on non-deterministic noiseless amplification enables to improve the quality of coherent state teleportation and makes the process secure in a region where the standard EPR resource does not.

3.2 Identification of the source of improvement: entanglement content and non-Gaussianity

In this section, we will discuss the properties of the amplified twin-beam that led to enhance the quality of quantum teleportation. We confront our results on the degree of entanglement and the non-Gaussianity of our conditional entangled resource presented in subsesction (2.2) to the content of Fig. 5 regarding the average fidelity of coherent state teleportation. First, we focus on the entanglement content quantified by the excess Von Neumann entropy for different configurations of the NLA. We observe from Fig. 3 and Fig. 5 that the degree of entanglement for all the considered configurations of the NLA is higher than that of the standard twin-beam, at the opposite of the average fidelity, which is degraded whenever the squeezing parameter exceeds a certain threshold. Thereby, the excess Von Neumann entropy does not provide full explanation about the behaviour of the quality of teleportation. Contrariwise, regarding the non-Gaussian states arising from different configurations, the more substantial is the excess Von Neumann entropy, the higher is the average fidelity of the entangled resource. Hence, we identify the entanglement content captured by the excess Von Neumann entropy as a relevant property to compare the quality of teleportation attainable with the different non-Gaussian

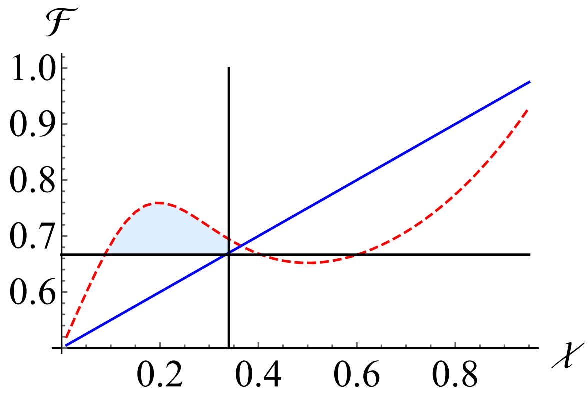

Fig. 7. (Color online) Dependence of the quality of teleporting a coherent state of amplitude $\alpha = 2$ with respect to the squeezing parameter χ by means of the standard twin-beam (blue solid line) and the amplified twin-beam under the calibration ($g = 2$ and $p = 4$) (red dotted line). The light-blue shaded area represents where the amplified twin-beam achieves an average fidelity exceeding the boundary $2/3$ while the standard one does not. The horizontal and vertical lines delimit the region of concern.

resources, though not sufficient to elucidate its whole behaviour.

Naturally, the next step is to examine how the amount of non-Gaussianity of the entangled bipartite states generated from the NLA action acts on the quality of the teleportation. For that purpose, we jointly analyse the results of Figs. 2 and Fig. 5. It is clear that there is no trivial dependence between the entropic non-Gaussianity and the fidelity of teleportation for all the considered configurations. Moreover, in the range of high values of the squeezing parameter, the non-Gaussianity seems to be detrimental to the quality of teleportation. Entropic non-Gaussianity doesn't offer any clarification on the origin of the improvement observed for the teleportation quality.

At last, we consider the dependence of the average fidelity of the quantum teleportation with respect to the EPR correlation that characterizes the entanglement content of the bipartite states. Based on the results reported in Fig. 4 and Fig. 5, we establish a comparison between the two quantities of interest. First of all, we notice two distinct regions for both the EPR correlation and the quality of the teleportation for a given configuration of the amplified twin-beam. For $0 < \chi < \chi_{c1}$, where χ_{c1} is a critical value that depends on the NLA's calibration, the entanglement content captured by the EPR correlation is lowered when compared with that of the standard EPR state, whereas in the range of squeezing parameters surpassing χ_{c1} , the EPR correlation is greater than that of the standard twin-beam. A similar behaviour is observed for the average fidelity where the two regions are delimited by χ_{c2} instead of χ_{c1}. Furthermore, in the range of weak energies, it appears that the more intense is the amplification, the more correlated are the two-modes and better is the quality of teleportation. Our results suggest that EPR correlation is a good witness for an improved average fidelity of the teleportation and may explain its origin. Finally, we remark that in the regime of intense amplifications ($g = 3, 4$ and $p = 4$) the presence of an area where Δz^2 indicates non-correlated resource and still leads to an average fidelity $\bar{\mathcal{F}} > 1/2$. This observation meets the result highlighted in [51, 52] according to which EPR correlation is not a necessary condition for quantum teleportation.

4 Conclusion

In summary, we have examined the properties of the non-Gaussian bipartite states resulting from the non-deterministic noiseless linear amplification of two-mode squeezed vacua. Various characteristic attributes related to the entanglement content and the non-Gaussian character of the amplified non-Gaussian resources have been considered and then compared with respect to the different configurations of the NLA and those of the standard twin-beam. We show that the excess Von Neumann entropy of the generated states is greater than that of the standard twin-beam independently of the configuration of the NLA. In particular, weak amplifications promote the increase of the degree of entanglement in the range of strong squeezing whereas intense calibrations of the NLA tend to optimise it in the interval of weak energies. Regarding the EPR correlation, a different behaviour has been observed: the amplification appears to enhance the entanglement content in the region of low energies whereas being detrimental when the squeezing of the input standard twin-beam exceeds a certain critical value.

In the light of these results, we have investigated a B-K like teleportation protocol where the standard entangled resource is substituted by the non-deterministically amplified twin-beam. We then drew up an elaborated comparison of the performance of the amplified non-Gaussian entangled resource and its Gaussian homologous in the quantum teleportation of coherent input light. Our upshot identify two range of the input energy delimited by a critical value that depends on the NLA calibration: for an input energy lower than the critical value, substantial improvement of the quality of teleportation can be reached whereas in the remaining region, amplification appears to be detrimental to the average fidelity. Thereby, the NLA is identified as a robust resource for quantum teleportation when performed on initial EPR-states with low energies. Furthermore, we emphasize the existence of a region where the non-Gaussian resources achieve secure quantum teleportation while the standard Gaussian bipartite state does not. Finally, we show that the enhancement of the quality of teleportation and the EPR correlation pursue approximately the same behaviour in opposition with the excess Von Neumann entropy and the non-Gaussianity that show a non-trivial dependance. Thus, we identify EPR correlation as a good indicator to gauge the quality of teleportation that may explain the origin of the noticeable improvement.

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