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Measuring high-order photon-number correlations in experiments with multimode pulsed quantum states

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We implement a direct detection scheme based on hybrid photodetectors to experimentally investigate high-order correlations for detected photons by means of experimentally accessible quantities. The scheme is self-consistent, allowing the estimation of all the involved parameters (quantum efficiency, number of modes, and average energy). In particular, we show how high-order correlation functions can be exploited to fully characterize bipartite multimode states in regimes realistic for quantum technology, that is, in the mesoscopic photon-number domain and with limited quantum efficiency. Furthermore, we introduce a nonclassicality criterion based on a simple linear combination of high-order correlation functions.

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I. INTRODUCTION

Correlations play a fundamental role in the investigation of optical coherence [1]. Since their introduction [2], correlation functions have been extensively investigated in connection with quantum-state characterization, to define nonclassicality criteria [3], and for the enhancement of ghost-imaging protocols [4,5]. From the experimental point of view, the study of this topic dates back to the pioneering work in which Hanbury Brown and Twiss discovered photon bunching in light emitted by a chaotic source [6]. In the last decade, many experiments have been developed in which the photon-number correlations have been used to characterize the entangled states generated by parametric down conversion [7–10]. In all the mentioned cases, the detection was performed by means of single or arrays of avalanche photodiodes so that the possibility to recover the correlation of the number of photons was quite straightforward [11,12], although the intensity range actually investigated by these systems was limited to much less than one mean photon [13]. However, the investigation of more intense light beams is of extreme interest, especially in view of possible applications to quantum technology: pulsed optical states endowed with sizable numbers of photons represent a useful resource as they are robust with respect to losses and their reliable experimental detection and characterization is relevant, especially for establishing nonclassicality, which plays a major role in quantum information. Furthermore, the very multimode nature of the bipartite states used in many quantum protocols requires both theoretical and experimental efforts to find suitable detection schemes to retrieve as much information as possible from the state itself up to the applicative level. In this case a fundamental requirement is represented by the possibility to have a scheme that is reliable for different intensity regimes as well as for limited quantum efficiency and that could be embedded in the experimental setup used to implement a particular protocol.

In this paper we report on a direct detection scheme aimed at measuring high-order correlations by means of a pair of hybrid photodetectors [14] in the presence of low quantum efficiency and bipartite multimode input states. In order to investigate the reliability and performance of the method, we will consider correlated bipartite optical states, namely, a multimode twin-beam state (TWB) and a bipartite pseudothermal state generated in the mesoscopic photon-number domain [15]. We define and derive the analytical expression of correlation functions at any order by only using quantities that can be experimentally accessed by direct detection, taking into account the nonunit quantum efficiency of the detection scheme. We then show that high-order correlations represent a useful discriminating tool of the nature of the state and demonstrate that, at increasing correlation order, the differences between classical and quantum states become more and more evident.

II. THEORY

The correlation functions $g_{\hat{n}}^{jk}$ are usually defined in terms of the normally ordered creation and annihilation operators [16]:

$$g_{\hat{n}}^{jk} = \frac{\langle : \hat{n}_1^j \hat{n}_2^k : \rangle}{\langle : \hat{n}_1 : \rangle^j \langle : \hat{n}_2 : \rangle^k} = \frac{\langle \hat{a}_1^{\dagger j} \hat{a}_1^j \hat{a}_2^{\dagger k} \hat{a}_2^k \rangle}{\langle \hat{a}_1^\dagger \hat{a}_1 \rangle^j \langle \hat{a}_2^\dagger \hat{a}_2 \rangle^k}, \qquad (1)$$

where $\hat{a}_k$ is the field operator of the kth mode and $\hat{n}_k = \hat{a}_k^\dagger \hat{a}_k$. $g_{\hat{n}}^{jk}$ have a well-recognized meaning in connection with coherence properties of light and the n-photon absorption process [17]. However, in a realistic direct detection scheme, we only have access to the shot-by-shot detected photons,

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and this requires a suitable description of the actual operation performed by the detector. If we assimilate the real detection to a Bernoullian process having efficiency η, we can express all the operatorial moments of the detected-photon distribution as a function of those of the photon distribution, i.e., $\hat{m}_k^p = \sum_{h=1}^{p} c_h(\eta)\hat{n}_k^h$, where the coefficients $c_h(\eta)$ are given in Ref. [18]. In this way we can build correlation functions in analogy with normally ordered ones:

$$g_{\hat{m}}^{jk} = \langle \hat{m}_1^j \hat{m}_2^k \rangle (\langle \hat{m}_1 \rangle^j \langle \hat{m}_2 \rangle^k)^{-1}\ , \tag{2}$$

where $\hat{m}_k$ is the operator describing the actual number of detected photons in the kth arm of the bipartite state. Indeed, we can express detected-photon correlations in terms of normally ordered ones, i.e., $g_{\hat{m}}^{jk} = \sum_{s,t=0}^{j,k} \varepsilon_{s,t}\, g_{\hat{n}}^{st}$, where $g_{\hat{n}}^{00} \equiv 1$ and $\varepsilon_{s,t}$ depend on the physical parameters of the system under investigation.

The expected results for correlations should be evaluated by taking into account all the realistic experimental conditions, that is, not only imperfect detection and imbalance of the arms of the bipartite state but also its multimode nature. Thus, we need to derive a suitable theoretical description of the detection process involving multimode states, for which all the μ modes in the field are measured shot by shot.

In order to assess the performance of our scheme, we consider a multimode TWB $|\psi_\mu\rangle = \otimes_{k=1}^{\mu}|\psi\rangle_k$ in which each of the μ modes is in the same state, i.e., $|\psi\rangle_k = \sum_n \langle\hat{n}\rangle^n/(1+\langle\hat{n}\rangle)^{n+1}|n\rangle_k \otimes |n\rangle_k$, $\forall k$, and thus is equally populated with the same average number of photons $\langle\hat{n}\rangle = \langle\hat{n}_k\rangle$, $\forall k$ [1]. By exploiting the pairwise correlations, we can state that the overall number of photons in each shot is the same in the two arms, being the sum of μ equal contributions coming from the μ modes. We can thus write the multimode TWB in the following compact form: $|\psi_\mu\rangle = \sum_{n=0}^{\infty}\sqrt{p_n^{\mu}}|n^{\otimes}\rangle \otimes |n^{\otimes}\rangle$, where $|n^{\otimes}\rangle = \delta(n - \sum_{h=1}^{\mu} n_h) \otimes_{k=1}^{\mu} |n\rangle_k$ represents the overall n photons coming from the μ modes that impinge on the detector and

$$p_n^{\mu} = \frac{(n+\mu-1)!}{n!(\mu-1)!(\langle\hat{n}\rangle/\mu+1)^{\mu}(\mu/\langle\hat{n}\rangle+1)^{n}} \tag{3}$$

is the photon-number probability distribution for the multimode TWB. As one may expect, we obtain a final result that depends only on the number of modes μ, the mean value of the number of photons $\langle\hat{n}\rangle$, and the overall detection efficiencies of the two detection chains, η_1 and η_2. In the case of $\eta_1 = \eta_2$ and considering correlation functions up to fourth order, we have (we put $g^{hk} \equiv g_{\hat{m}}^{hk}$)

$$g^{11} = G_\mu^1 + \frac{\eta}{\langle\hat{m}\rangle}, \tag{4a}$$

$$g^{21} = g^{12} = G_\mu^2 + G_\mu^1\frac{1+2\eta}{\langle\hat{m}\rangle} + \frac{\eta}{\langle\hat{m}\rangle^2}, \tag{4b}$$

$$g^{22} = G_\mu^3 + 2G_\mu^2\frac{1+2\eta}{\langle\hat{m}\rangle} + G_\mu^1\frac{1+4\eta+2\eta^2}{\langle\hat{m}\rangle^2} + \frac{\eta}{\langle\hat{m}\rangle^3}, \tag{4c}$$

$$g^{31} = g^{13} = G_\mu^3 + 3G_\mu^2\frac{1+\eta}{\langle\hat{m}\rangle} + G_\mu^1\frac{1+6\eta}{\langle\hat{m}\rangle^2} + \frac{\eta}{\langle\hat{m}\rangle^3}, \tag{4d}$$

where $\langle\hat{m}\rangle$ is the average number of detected photons and $G_\mu^k = \prod_{j=1}^{k}(j+\mu)/\mu$.

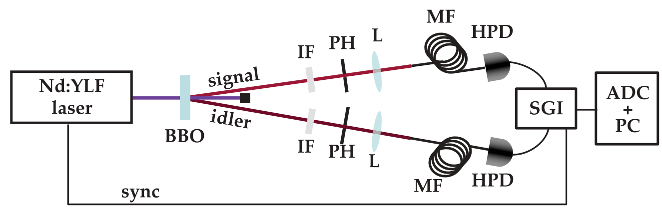

FIG. 1. (Color online) Experimental setup. See the text for details.

III. EXPERIMENT

We generated a multimode TWB by the third harmonics (349-nm wavelength) of a mode-locked Nd:yttrium lithium fluoride laser regeneratively amplified at 500 Hz (High-Q Laser Production, Austria) impinging on a type-I beta barium borate (BBO) crystal (β-BaB$_2$O$_4$, Castech, China, cut angle 34°, 4 mm thick). According to the experimental setup sketched in Fig. 1, we adopted a noncollinear interaction geometry in order to avoid possible residues of the pump beam. The nonclassical nature of the generated state can be proven by evaluating the noise-reduction factor [15]. To obtain a good balancing of the quantum efficiencies, we selected two portions of the signal and idler cones close to frequency degeneracy, namely, at 690 and 706 nm, by using two interference filters (IF in Fig. 1). The collection of a single coherence area in the two parties of the TWB state was obtained by inserting two pinholes (PH, 2 mm diameter) at 107 and 109.5 cm from the BBO, respectively. The light passing the pinholes was delivered through two multimode optical fibers to two hybrid photodetectors (HPD, R10467U-40, Hamamatsu, Japan), which are detectors endowed with a quantum efficiency of 50% at 500 nm, a partial photon-counting capability, and a good linear response up to 100 photons. Their outputs were amplified (preamplifier A250 plus amplifier A275, Amptek), synchronously integrated (SGI, SR250, Stanford), and digitized (AT-MIO-16E-1, National Instruments). Each experimental run was performed on 50 000 subsequent laser shots at different values of the pump intensity. The self-consistent procedure to analyze the outputs of each detection chain is explained in detail in Refs. [14,19]; here we note only that the procedure allows us to obtain the mean value of detected photons $\langle\hat{m}\rangle$, the number of modes μ, and the quantum efficiency η directly from the experimental data.

Remarkably, the present detection apparatus can be used to reconstruct both the detected-photon statistics [20] and the shot-by-shot second-order correlation in the number of detected photons [21]. Nevertheless, as our TWB state is intrinsically multimode ($\mu > 150$) and the effective detection efficiency is rather low ($\eta < 4\%$), the experimental characterization of the state in terms of sub-shot-noise correlations becomes challenging. On the contrary, the measurement of high-order correlations offers the possibility to better discriminate the nature of the state under investigation also in critical situations. In Fig. 2 we plot the experimental data (colored solid symbols) obtained by evaluating the high-order correlation functions up to fourth order in the symmetrized form $[g^{hk}]_s = \frac{1}{2}(g^{hk} + g^{kh})$ to take into account all the unavoidable asymmetries of the physical system. Also in Fig. 2 we show the theoretically expected curves (black open symbols) for a

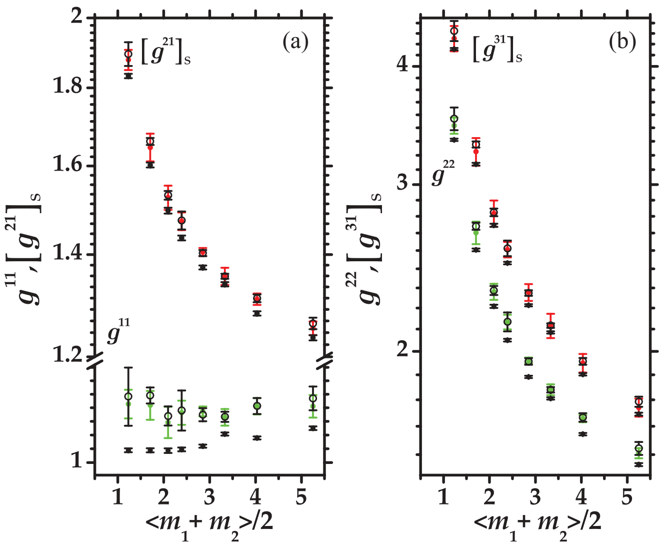

FIG. 2. (Color online) Log-linear plot of high-order correlation functions for a multimode TWB state as functions of the mean value of the number of detected photons. Colored solid symbols, experimental data; black open symbols, theoretical expectations; black solid symbols, theoretical predictions for a multimode thermal state.

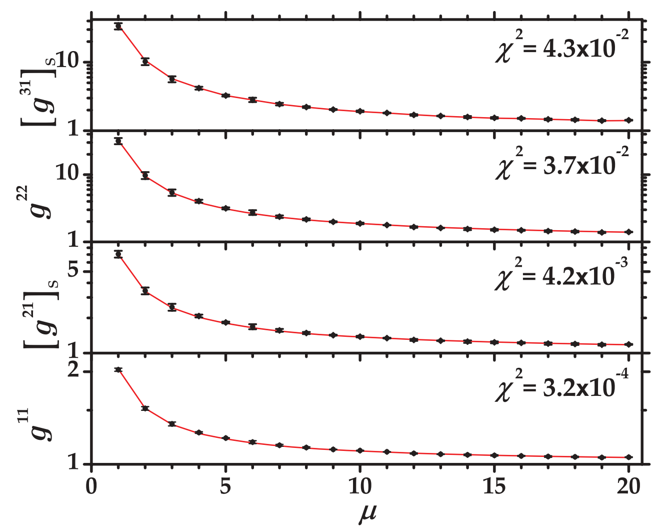

FIG. 3. (Color online) Log-linear plot of high-order correlation functions for a multimode pseudothermal state as a function of the number of modes. Solid circles, experimental data; line, theoretical expectations.

multimode TWB calculated according to Eqs. (4) by using the values of the mean number of detected photons, the number of modes, and the overall detection efficiency as directly obtained from the experimental data, exploiting the self-consistency of our setup [15,18]. In our method, once the calibration is established, the mean values $\langle m \rangle$ are known directly without any fit. For the data presented here, we evaluated the values of η from sub-shot-noise measurements starting from shot-by-shot detected-photon values, while the number of modes μ is evaluated from the fit of detected-photon-number distributions. The results for the parameters are summarized in Table I.

The very high agreement of the experimental data with the theoretical expectation is assessed by the quantity $\chi^2 = \sum (g^{jk}_{\rm exp} - g^{jk}_{\rm th})^2/g^{jk}_{\rm th}$. In particular, we have 2.15×10^{-5}, 1.66×10^{-4}, 8.57×10^{-4}, and 1.16×10^{-4} for g^{11}, $[g^{12}]_{\rm s}$, g^{22}, and $[g^{13}]_{\rm s}$, respectively. For comparison, in Fig. 2 we also plot the theoretical predictions for a classically correlated multimode thermal state having the same mean value and number of modes (black solid symbols): in this case the theoretical predictions can be formally obtained from Eqs. (4)

TABLE I. Values of the parameters for the experimental data obtained with the self-consistent method.

$\langle m_1 \rangle$	$\langle m_2 \rangle$	$\mu = \sqrt{\mu_1 \mu_2}$	η
1.253 ± 0.008	1.207 ± 0.007	202 ± 32	0.031 ± 0.015
1.746 ± 0.010	1.677 ± 0.009	202 ± 34	0.041 ± 0.005
2.129 ± 0.012	2.048 ± 0.011	208 ± 41	0.026 ± 0.008
2.439 ± 0.013	2.356 ± 0.013	187 ± 27	0.037 ± 0.020
2.911 ± 0.015	2.794 ± 0.015	151 ± 14	0.042 ± 0.007
3.376 ± 0.017	3.286 ± 0.017	87 ± 5	0.024 ± 0.007
4.080 ± 0.020	3.967 ± 0.020	101 ± 6	0.057 ± 0.013
5.296 ± 0.026	5.218 ± 0.026	72 ± 4	0.063 ± 0.025

by setting $\eta = 0$ [22]. Note that the error bars of the theoretical curves in Fig. 2 have been obtained by propagating the errors in Table I. It is apparent that quantum correlations are always higher than classical ones, even if the absolute value of the difference is rather low due to the large number of modes and to the low value of quantum efficiency. Again, the values of χ^2 for experimental data and theoretical predictions for a multimode thermal state are 2.06×10^{-3}, 8.73×10^{-3}, 37.71×10^{-3}, and 28.99×10^{-3} for g^{11}, $[g^{12}]_{\rm s}$, g^{22}, and $[g^{13}]_{\rm s}$, respectively, indicating that the difference increases as the order of correlations increases.

In order to explore the capability of our system in different regimes and at any bipartite multimode input states, we also measured a coherent state [in this case the expressions of high-order correlations can be formally obtained from Eqs. (4) by setting $\eta = 0$ and $\mu \to \infty$] and single and multimode pseudothermal states generated by a rotating ground-glass plate and divided by a beam splitter [21,23]. As an example, in Fig. 3 we plot the measured high-order correlations (solid circles) and the corresponding theoretical expectation (line) for a multimode thermal state at a fixed mean value ($\langle \hat{m} \rangle = 3.88$) as a function of the number of modes. The experimental data are in very good agreement with theory, as shown by the χ^2 values reported in Fig. 3.

The good quality of the experimental results suggests that high-order correlations can be used to infer the very nature of bipartite multimode states. A more direct way to establish nonclassicality is to address a suitable parameter satisfying boundary conditions [24–26]. Here we consider two of those conditions in our experimental situation, namely, the Schwarz inequality,

$$\langle m_1 m_2 \rangle \Big/ \sqrt{\langle m_1^2 \rangle \langle m_2^2 \rangle} > 1 , \tag{5}$$

and the noise-reduction factor,

$$[\langle (m_1 - m_2)^2 \rangle - \langle m_1 - m_2 \rangle^2]/\langle m_1 + m_2 \rangle < 1 , \tag{6}$$

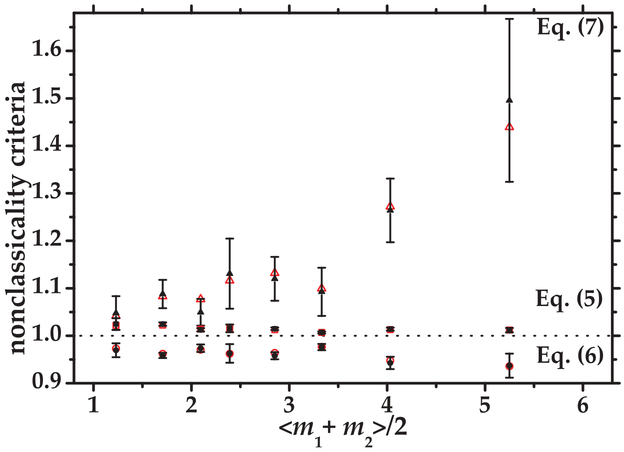

FIG. 4. (Color online) Nonclassicality criteria for a multimode TWB as a function of the mean value of the number of detected photons. Solid symbols, experimental data; open symbols, theoretical predictions. See the text for details.

and we introduce a new inequality,

$$\langle m_1\rangle\langle m_2\rangle \frac{g^{22}-[g^{13}]_{\rm s}}{g^{11}} + \sqrt{\langle m_1\rangle\langle m_2\rangle}\,\frac{[g^{12}]_{\rm s}}{g^{11}} > 1\,, \qquad (7)$$

in which we have used again the symmetrized quantities $[g^{hk}]_{\rm s}$. Indeed, all the above inequalities must be fulfilled by nonclassical light. Note that inequality (7) is equivalent to requiring $g_{\hat{n}}^{22} - g_{\hat{n}}^{31} > 0$ in the case of perfectly balanced efficiencies. In Fig. 4 we plot the results for the three inequalities above for our experimental data on multimode TWB as functions of the mean intensity. It is worth noting that, in our setup, changing the mean intensity of the output corresponds to changing both the size of the coherence areas and the number of modes. Figure 4 thus actually compares the experimental results with the theory evaluated at each point in the very values of the experimental parameters as calculated from the data. This explains the irregularities in the behavior of the data from point to point. As expected, the measured state is nonclassical according to all the three inequalities, but the amount of nonclassicality that we can quantify with respect to the boundary unitary value is much larger for the condition in inequality (7). This indicates that using high-order correlations can be helpful in all the situations in which the nonclassical nature is critical to be proven by other criteria, such as the case of a TWB state endowed with a high number of modes and/or a low quantum efficiency, for which we expect very small values of the noise-reduction factor.

IV. CONCLUSIONS

In conclusion, we have investigated the performances of a direct detection scheme to measure high-order correlations. Thanks to its compactness, it can be easily embedded for state characterization in experiments involving multimode pulsed optical states. We defined correlation functions at any order by means of quantities that can be experimentally accessed by direct detection with an experimental apparatus that allows accessing sizable mean photon values. We have introduced a nonclassicality criterion based on these high-order correlations that represents a useful discriminating tool of the nature of the state in critical cases, in which other criteria are violated by a limited amount: in some sense this criterion acts as an "amplifier" of nonclassicality violation. We have tested our theoretical and experimental procedure on a multimode TWB state by developing a multimode description that makes the calculation of high-order correlations straightforward. For the sake of completeness, we have also presented the direct comparison with an equally populated multimode pseudothermal state. The experimental results are in very good agreement with the theory, thus encouraging the exploitation of our scheme for reliable experimental characterization of quantum states to be used for quantum technology up to the applicative level.

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