

Noisy effects in interferometric quantum gravity tests

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Quantum-enhanced metrology is boosting interferometer sensitivities to extraordinary levels, up to the point where table-top experiments have been proposed to measure Planck-scale effects predicted by quantum gravity theories. In setups involving multiple photon interferometers, as those for measuring the so-called holographic fluctuations, entanglement provides substantial improvements in sensitivity. Entanglement is however a fragile resource and may be endangered by decoherence phenomena. We analyze how noisy effects arising either from the weak coupling to an external environment or from the modification of the canonical commutation relations in photon propagation may affect this entanglement-enhanced gain in sensitivity.

Keywords: Quantum interferometry; decoherence phenomena; quantum gravity effects.

1. Introduction

Most approaches to quantum gravity, either effective or fundamental, generally predict the appearance of non-standard phenomena at the Planck scale, due to the “foamy” structure of spacetime.^a It is however difficult in general to estimate how

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^aThe original idea that at Planck scale quantum fluctuations of the space geometry could destroy the smoothness of the spacetime manifold has been introduced in Ref. 1 and, since then, further discussed by many authors (e.g. see Refs. 2–9); for recent reviews and further details, see Refs. 10 and 11 and references therein.

these necessarily tiny disturbances could become visible in an actual experimental setup.

Interferometric apparatus have emerged as the most suitable setups for such kinds of analysis, in particular, those made of two identical photon interferometers: it has been shown that using a couple of correlated interferometers in specific configurations may allow to efficiently distinguish quantum gravity effects from other spurious signals.^{12–14} The extreme sensitivities that these apparatus need to reach in order to actually measure these minuscule effects is nevertheless challenging.^{15–17}

As in gravitational wave detectors,^{18–20} quantum metrological methods may be employed to enhance the sensitivity of such quantum gravity detectors.²¹ On the one hand, the use of nonclassical states, such as single- and two-mode squeezed states, can be a resource to enhance the sensitivity of optical interferometers,²² allowing, at least in principle, to reach the so-called Heisenberg limit.^{23,24} On the other hand, it has been shown that in the particular setup using two photon interferometers, by feeding them with quantum correlated (entangled) initial photons, the overall sensitivity of the device may be dramatically enlarged, at least in an ideal situation.^{25,26}

Quantum entanglement is however a fragile resource that can be endangered by various decohering phenomena: therefore, it is of utmost importance to investigate to what extent the entanglement enhanced sensitivity of the apparatus is robust against external noise. Indeed, an interferometer is never completely isolated from the external environment, which is in general a source of decohering phenomena. Furthermore, many fundamental theories predict various kinds of spacetime non-commutativity at the undermost, basic level^{27–31}; these phenomena can affect the propagation of the photons inside the interferometers through a modification of the canonical commutation relation, leading to further noisy phenomena. All these unwanted effects may reduce the enhancement in sensitivity obtained by feeding the apparatus with highly nonclassical, entangled light.

The general theory of open quantum systems^{32–38}, i.e. systems in weak interactions with external baths, can be used to estimate the effects produced by the external environment in the double interferometer apparatus. In this framework, the propagation of the photons inside the experimental setup is described by a quantum dynamical semigroup, generalizing the familiar unitary dynamics. On the other hand, the existence of a minimum length, as predicted by most theories based on non-commutative geometry,^{29–31} may lead to a generalized uncertainty principle and as a consequence to a modification of the bosonic canonical commutation relations, the photon mode operators obey.

In the following, we shall discuss in detail how the sensitivity enhancements provided by the use of entangled photons is affected by the presence of both sources of “noise”. In particular, we shall estimate how large the effects of these decohering phenomena should be in order to spoil the enhancement in sensitivity when detecting quantum gravity effects obtained through the use of quantum metrological methods.

2. Detecting Holographic Fluctuations with Entangled Photons

Photon interferometers are among the most accurate devices for detecting tiny effects induced inside the apparatus by external perturbations. In the specific case of quantum gravity, it has been predicted that the “foamy” structure of spacetime at the Planck scale may result in a noncommutativity of spatial coordinates, leading in turn to optical path-length differences between the two arms of the interferometer; the resulting phase shifts have been named *holographic fluctuations*.¹²

As mentioned above, this new kind of fluctuations cannot be detected by a single interferometer: it is nearly impossible to isolate holographic fluctuations from other spurious signals, even using extremely sensitive setups as gravitational antennas. Therefore, specific configurations have been actually designed and built in order to measure the accumulated phases coming from holographic noise.^{15–17}

If two slightly displaced parallel interferometers occupy overlapping spacetime volumes, then they display correlated holographic fluctuations (see the left panel of Fig. 1, “parallel configuration”); on the other hand, by rotating one of the interferometers by 90° , so that one arm of the first interferometer becomes anti-parallel to the one of the other, spacetime overlapping is precluded and, as a consequence, the correlation in holographic fluctuations vanishes (see the right panel of Fig. 1, “orthogonal configuration”). The second configuration can thus be taken as a reference measurement for the background signal, that, once subtracted from the outcome of the first configuration, should allow detecting the quantum gravity-induced holographic fluctuations, provided sufficient sensitivity and statistics are achieved.

The principal intrinsic limitation in achieving high accuracies in phase determination in such double interferometric devices is due to the shot noise limit. This limitation can be in part circumvented by feeding the apparatus with nonclassical light. Indeed, as in the case of more standard gravitational wave interferometry,^{18–20} also in the case of holographic fluctuation measurements, the use of squeezed light instead of classical coherent one would allow reaching higher sensitivities in phase estimation.

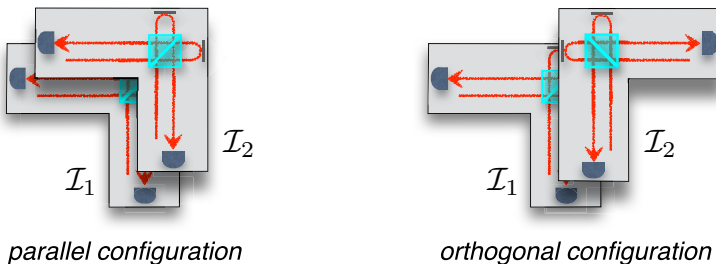


Fig. 1. Left: When two interferometers \mathcal{I}_1 and \mathcal{I}_2 are in the parallel configuration, they display correlated holographic fluctuations. Right: In the orthogonal configuration, the correlation in holographic fluctuations vanishes. More details about the interferometers are given in Fig. 2.

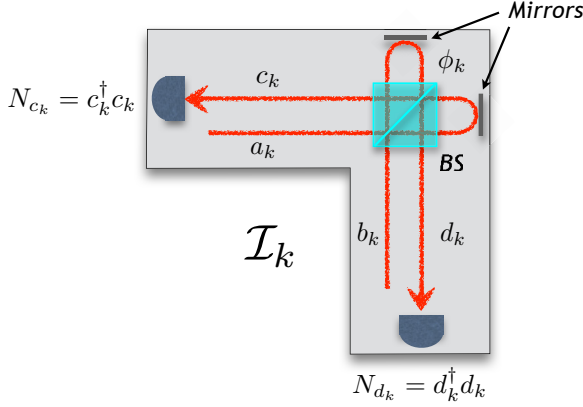


Fig. 2. Schematic configuration of the k th interferometer, $k = 1, 2$, with input a, b and output c, d port modes; BS represents the beam splitter, while the detectors at the output ports measure the photon number N_c and N_d .

However, the major breakthrough in sensitivity enhancement for the measurement of holographic fluctuations was shown to be brought in by feeding the double interferometer with suitable quantum correlated photons.²⁵ The considered setup is made of two identical Michelson-like interferometers, labeled \mathcal{I}_k , $k = 1, 2$ (see Fig. 2). The input fields of the interferometers are described by the creation and annihilation mode operators a_k^\dagger, a_k and b_k^\dagger, b_k , $k = 1, 2$, obeying the standard bosonic commutation relations, $[a_j^\dagger, a_k] = \delta_{jk}$, $[b_j^\dagger, b_k] = \delta_{jk}$; they are combined into a beamsplitter giving rise to the output mode operators c_k^\dagger, c_k and d_k^\dagger, d_k , respectively. The number of photons in the output ports, $N_{c_k} = c_k^\dagger c_k$ and $N_{d_k} = d_k^\dagger d_k$, are measured by means of two photodetectors. As previously mentioned, quantum gravity effects induce an optical path length difference in the interferometers and therefore, a phase shift ϕ_k , so that the relation between input and output modes is given by

$$c_k(\phi_k) = a_k \cos(\phi_k/2) + b_k \sin(\phi_k/2), \quad (1)$$

$$d_k(\phi_k) = b_k \cos(\phi_k/2) - a_k \sin(\phi_k/2). \quad (2)$$

The configuration in which the two interferometers essentially overlap, having the corresponding arms aligned, will be named *parallel* (\parallel), while the second configuration in which one interferometer is rotated with respect to the other, leading to two parallel and two antiparallel arms, will be called *orthogonal* (\perp).

Let us now feed the b -ports of the two interferometers with photons in the same coherent state, i.e. $D_{b_1}(\mu)D_{b_2}(\mu)|0\rangle = |\mu\rangle|\mu\rangle$, where $D_{b_k}(\mu) = \exp(\mu b_k^\dagger - \mu^* b_k)$ is the displacement operator of mode b_k , $k = 1, 2$, and $\mu \in \mathbb{C}$, while the a -ports with photons in an entangled squeezed state; in other terms, the light entering the a -ports is quantum correlated between the two interferometers, while the one entering the b -ports is not. The entangled state is obtained by acting on the vacuum state with the

two-mode squeezing operator

$$S(\zeta) = \exp(\zeta a_1^\dagger a_2^\dagger - \zeta^* a_1 a_2), \quad \zeta = r e^{i\theta}, \quad r, \theta \in \mathbb{R}, \quad (3)$$

giving rise to the two-mode squeezed vacuum state (the so-called twin-beam state):

$$|\text{TWB}\rangle = \frac{1}{\cosh(r)} \sum_{n=0}^{\infty} [\tanh(r)]^n e^{in\theta} |n, n\rangle, \quad (4)$$

where $|n, m\rangle, n, m \in \mathbb{N}$, are standard two-mode Fock states. This state consists of a superposition of paired states with equal number of photons in each mode; as a result, it is a null eigenstate of any moment of the photon number difference operator, namely,

$$(a_1^\dagger a_1 - a_2^\dagger a_2)^p |\text{TWB}\rangle = 0, \quad \forall p \in \mathbb{N}. \quad (5)$$

In order to observe correlated phase-dependent fluctuations, one needs to study the behavior of an observable which depends on both phases $\phi_k, k = 1, 2$; a convenient choice is given by the photon number difference at the output c_k ports of the interferometers, namely,^{25,26}

$$\Delta N(\phi_1, \phi_2) = [N_{c_1}(\phi_1) - N_{c_2}(\phi_2)]^2. \quad (6)$$

In addition, the holographic fluctuations are expected to be a stochastic process and therefore, in order to obtain averages to be compared with experimental outcomes, the expectation of ΔN over the output photon states, hereafter indicated by $\langle \Delta N \rangle$, needs to be further averaged over an appropriate probability distribution $f_\alpha(\phi_1, \phi_2)$:

$$\mathcal{E}_\alpha[\Delta N(\phi_1, \phi_2)] = \int d\phi_1 d\phi_2 f_\alpha(\phi_1, \phi_2) \langle \Delta N(\phi_1, \phi_2) \rangle, \quad \alpha = \parallel, \perp. \quad (7)$$

One can show²⁵ that the holographic fluctuations are actually described by the following phase-shift correlations:

$$\mathcal{E}_\parallel[\delta\phi_1 \delta\phi_2] = \int d\phi_1 d\phi_2 \delta\phi_1 \delta\phi_2 f_\parallel(\phi_1, \phi_2), \quad (8)$$

where $\delta\phi_k = \phi_k - \phi_{k,0}, k = 1, 2$ are phase shift deviations from their corresponding mean central value $\phi_{k,0}$. By making the reasonable assumptions that the distributions $f_\alpha(\phi_1, \phi_2)$ have identical marginals and uncorrelated phase noise in the \perp configuration, one can relate this quantity to the differences of the two averages $\mathcal{E}_\parallel[\Delta N(\phi_1, \phi_2)]$ and $\mathcal{E}_\perp[\Delta N(\phi_1, \phi_2)]$. By expanding the explicit expressions of these expectations in series of $\delta\phi_k$ about the central values $\phi_{k,0}$, one finds, for small $\delta\phi_k$,²⁵

$$\mathcal{E}_\parallel[\delta\phi_1 \delta\phi_2] = \frac{\mathcal{E}_\parallel[\Delta N(\phi_1, \phi_2)] - \mathcal{E}_\perp[\Delta N(\phi_1, \phi_2)]}{\langle \partial_{\phi_1} \partial_{\phi_2} \Delta N(\phi_1, \phi_2) \rangle|_{\phi_k = \phi_{k,0}}}, \quad (9)$$

which holds when the denominator is nonvanishing. We are interested in evaluating the uncertainty $\Delta \mathcal{E}$ with which this quantity can be determined using the double

interferometer apparatus, i.e.

$$\Delta\mathcal{E} = \left[\frac{\text{Var}_{\parallel}[\Delta N(\phi_1, \phi_2)] + \text{Var}_{\perp}[\Delta N(\phi_1, \phi_2)]}{\langle \partial_{\phi_1} \partial_{\phi_2} \Delta N(\phi_1, \phi_2) \rangle^2 |_{\phi_k = \phi_{k,0}}} \right]^{1/2}, \quad (10)$$

where $\text{Var}_{\alpha}[\Delta N(\phi_1, \phi_2)]$ are the variances of $\Delta N(\phi_1, \phi_2)$, in the two configurations $\alpha = \parallel, \perp$. Within the same approximation used above, one finds that, to lowest order in $\delta\phi_k$,

$$\Delta\mathcal{E} \simeq \left[\frac{2\text{Var}_{\parallel}[\Delta N(\phi_1, \phi_2)]}{\langle \partial_{\phi_1} \partial_{\phi_2} \Delta N(\phi_1, \phi_2) \rangle^2 |_{\phi_k = \phi_{k,0}}} \right]^{1/2}. \quad (11)$$

This result was used in Ref. 25 to show that, by feeding the apparatus with the two-mode squeezed vacuum state (4), one can obtain a substantial increase in sensitivity for holographic fluctuations detection with respect to an analogous device using classical light. In particular, in the special case $\phi_{1,0} = \phi_{2,0} = 0$, the interferometers act like two completely transparent media, as one can see from (1) and (2) by setting $\phi_k \equiv \phi_{k,0} = 0$. Therefore, recalling, property (5), the uncertainty $\Delta\mathcal{E}$ vanishes, while with coherent photon input states, the shot noise limits the uncertainty to $\Delta\mathcal{E} \geq \Delta\mathcal{E}_{\text{cl}} \equiv \sqrt{2}/|\mu|^2$.²⁵

This striking result holds only in an ideal setting with interferometers working with perfect efficiency. The robustness of these results against possible setup inefficiencies was also studied in Refs. 25 and 26 by modeling them in terms of photon losses inside the apparatus.

In the following, we shall study how the evaluation of $\Delta\mathcal{E}$ may be affected by the presence of a weakly coupled external environment and by a gravity-induced modification of the bosonic commutation relations obeyed by the photon modes.

3. Noise Induced by an External Environment

In a realistic scenario, the photons traveling inside the two interferometers inevitably feel the presence of the surrounding, external environment, leading to noisy effects that might endanger the accuracy in phase determination. In general, it is hard to estimate the form and magnitude of these unwanted effects due to the complexity of the photon dynamics inside the apparatus; however, in the specific situation at hand, the coupling between the photons and the environment can be assumed to be very weak and in such a case, their behavior can be effectively described using the well established theory of quantum open systems.^{32–38}

Quite in general, the environment, which is made of an infinite number of microscopic degrees of freedom, can be modeled as a free bosonic bath in equilibrium at a given inverse temperature β . The total system, photons plus bath, can be initially prepared in a separable state of the form $\rho \otimes \rho_{\beta}$, where ρ is the density matrix describing the photon initial state, while ρ_{β} is the Gibbs density matrix describing the

equilibrium state of the environment. For rather generic (bilinear) interactions between photons and environment, in the limit of weak coupling, the reduced photon dynamics of the photons inside the interferometric setup, obtained by tracing over the bath degrees of freedom, can be described by a master equation in Kossakowski–Lindblad form.³²

The relevant equation describing the time evolution of the a_1 and a_2 -mode photon states can then be cast in the following form:

$$\frac{\partial \rho(t)}{\partial t} = -i[H, \rho] + \sum_{i,j=1}^4 C_{ij} \left(V_j \rho V_i^\dagger - \frac{1}{2} \{V_i^\dagger V_j, \rho\} \right), \quad (12)$$

where $V_i, i = 1, 2, 3, 4$ represent the components of the four-vector $(a_1, a_1^\dagger, a_2, a_2^\dagger)$, while $H = \omega_\gamma \sum_k a_k^\dagger a_k$ is the free photon Hamiltonian, with ω_γ the photon energy; in addition, $\{, \}$ signifies anticommutation. The coefficient matrix C_{ij} , known as Kossakowski matrix, contains the information about the environment. In the case of a free bath of bosons at energy ω and temperature $T = 1/\beta$, it can be taken as the following simple form^{39,40}:

$$C_{ij} = \lambda \begin{pmatrix} 1 + M & 0 & 0 & 0 \\ 0 & M & 0 & 0 \\ 0 & 0 & 1 + M & 0 \\ 0 & 0 & 0 & M \end{pmatrix}, \quad (13)$$

where $M = (e^{\beta\omega} - 1)^{-1}$ is the usual Boltzmann factor, while λ is the photon-environment coupling constant.

In the master equation (12), one can distinguish two contributions: the first one is the standard Hamiltonian term leading to a unitary evolution, while the other is responsible for noisy effects due to the presence of the environment: the part containing the anticommutator produces dissipation, while the remaining term leads to decohering effects. Due to these effects, the finite-time dynamics generated by (12) is no longer unitary, but of semigroup type, with composition holding only forward in time.

As described in the previous section, the states of the photons at the a_k ports are prepared in an entangled twin-beam state. In order to evaluate the environmental disturbances on phase estimation, one now has to propagate in time this state according to the evolution equation (12) up to the time $\tau = 4L/c$, where L is the length of the interferometer arms and c is the speed of light.

This can be more easily obtained by passing to a phase-space description, i.e. by introducing the two-mode Wigner function corresponding to the photon state ρ :

$$W(z_1, z_2) = \frac{1}{\pi^2} \int d^2\xi_1 d^2\xi_2 e^{z_1 \xi_1^* - z_1^* \xi_1 + z_2 \xi_2^* - z_2^* \xi_2} \text{Tr}[\rho D_1(\xi_1) D_2(\xi_2)], \quad z_1, z_2 \in \mathbb{C}, \quad (14)$$

where $D_k(\xi_k) = \exp(\xi_k a_k^\dagger - \xi_k^* a_k)$, $k = 1, 2$ are the so-called displacement operators. This description is completely equivalent to the one in terms of density matrices; in particular, it allows computing expectation values of monomials in the mode operators a_1, a_2 by means of the following general formula⁴¹:

$$\begin{aligned} & \text{Tr}[\rho(a_1^\dagger)^{n_1} a_1^{m_1} (a_2^\dagger)^{n_2} a_2^{m_2}] \\ &= n_1! n_2! \left(-\frac{1}{2}\right)^{n_1+n_2} \iint d^2 z_1 d^2 z_2 \\ & \quad \times z_1^{m_1-n_1} z_2^{m_2-n_2} L_{n_1}^{(m_1-n_1)}(2|z_1|^2) L_{n_2}^{(m_2-n_2)}(2|z_2|^2) W(z_1, z_2), \end{aligned} \quad (15)$$

where $L_n^\alpha(x)$ are the generalized Laguerre polynomials.

The master equation (12) generating the dissipative time evolution of the photon density matrix $\rho(t)$ becomes a Fokker–Planck equation for the corresponding Wigner function; explicitly, one finds

$$\partial_t W_t(z_1, z_2) = \frac{\lambda}{2} \left[\sum_j (\partial_{x_j} x_j + \partial_{y_j} y_j) + (2M + 1) \sum_j (\partial_{x_j}^2 + \partial_{y_j}^2) \right] W_t(\alpha_1, \alpha_2), \quad (16)$$

where x_j and y_j are the real and imaginary parts of z_j , $j = 1, 2$.

The initial a -port photon state is the entangled twin-beam state (4), and its corresponding Wigner function is of Gaussian form. Since Eq. (16) contains at most second-order derivative, it preserves its Gaussian form; indeed, the time evolved Wigner function takes the explicit form⁴²:

$$W_t(z_1, z_2) = \frac{1}{4\pi^2 \Sigma_+^2 \Sigma_-^2} \exp \left\{ -\frac{(x_1 + x_2)^2}{4\Sigma_+^2} - \frac{(y_1 + y_2)^2}{4\Sigma_-^2} - \frac{(x_1 - x_2)^2}{4\Sigma_-^2} - \frac{(y_1 - y_2)^2}{4\Sigma_+^2} \right\}, \quad (17)$$

where the functions,

$$\Sigma_\pm = \frac{1}{2} \left(M + \frac{1}{2} \right) (1 - e^{-\lambda t}) + \sigma_\pm e^{-\lambda t}, \quad (18)$$

give the explicit time dependence, while the coefficients $\sigma_\pm = e^{\pm 2|r|}$ contain the dependence on the initial squeezing parameter r (cf. (3)).

Using this result with $t = \tau$, the photon flight time inside the interferometers, and the general formula (15), one can now evaluate how the uncertainty $\Delta\mathcal{E}$ in the determination of the holographic fluctuations in (11) is altered by the presence of the environment. As at the end of the previous section, we shall consider the case in which the central values of the phase shifts vanish, $\phi_{1,0} = \phi_{2,0} = 0$, so that any deviation from the ideal result $\Delta\mathcal{E} = 0$ there obtained is due to the noisy effects induced by the environment. In this situation, recalling (6) and (11), the expression of the uncertainty $\Delta\mathcal{E}$ reduces to

$$\Delta\mathcal{E}/\Delta\mathcal{E}_{\text{cl}} = 2 \frac{(\langle \Delta\mathcal{N}^4 \rangle - \langle \Delta\mathcal{N}^2 \rangle^2)^{1/2}}{\langle (a_1^\dagger + a_1)(a_2^\dagger + a_2) \rangle}, \quad \Delta\mathcal{N} = a_1^\dagger a_1 - a_2^\dagger a_2, \quad (19)$$

where $\Delta\mathcal{E}_{\text{cl}}$ is the uncertainty obtained by feeding the apparatus with classical coherent light.

The explicit evaluation of this ratio is cumbersome but straightforward, and it amounts to the computation of integrals of the form (15) with monomials up to order four.⁴³ In lowest order situated in the small parameter $\lambda\tau$, one finds:

$$\Delta\mathcal{E}/\Delta\mathcal{E}_{\text{cl}} \simeq \frac{8\sqrt{\lambda\tau}}{\sinh(2r)} [(2M + 1) \cosh(2r) - 1]^{1/2}. \quad (20)$$

In general, also the coherent states $|\mu\rangle$ entering the other two ports of the apparatus are affected by damping, so that they are modified by additional terms of order $\lambda\tau$ or smaller. However, their contribution to the uncertainty involves in practice only the denominator of (19); since the numerator turns out to be proportional to $\sqrt{\lambda\tau}$, one can compute the denominator in the zeroth order approximation, i.e. with ordinary coherent states.

The behavior of the ratio $\Delta\mathcal{E}/\Delta\mathcal{E}_{\text{cl}}$ as a function of the dimensionless parameter $\lambda\tau$ is reported in Fig. 3, for different values of the parameter M . As discussed before, this parameter describes the bath properties, and in particular, it contains the dependence on the bath temperature. However, imperfections in the preparation of the initial twin-beam state could result in additional, effective “thermal” noise that can further contribute to M .⁴⁴ It is thus preferable to study the behavior of the ratio in (20) for different values of M instead of directly the bath temperature, treating M as an effective thermal parameter.

Although the presence of the bath now makes the uncertainty nonvanishing, the advantage of feeding the apparatus with entangled photon state is still apparent, provided the couplings with the external environment is kept small. Note that the uncertainty indeed approaches zero in the case of vanishingly small coupling $\lambda\tau$.

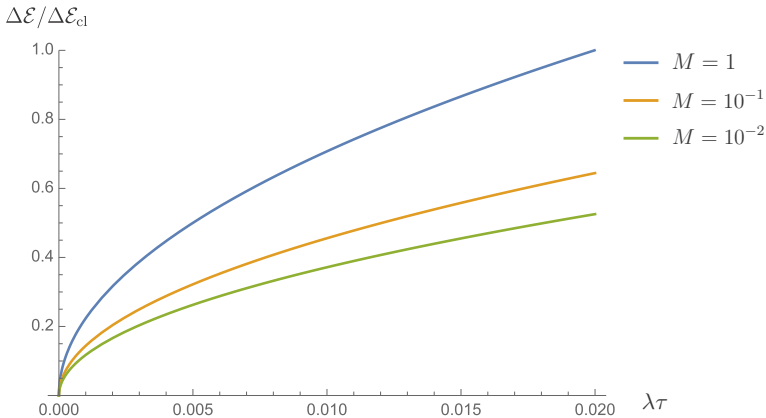


Fig. 3. Behavior of the uncertainty, normalized to its classical value, in the presence of an external bath, as a function of the coupling parameter $\lambda\tau$, for different values of the parameter M , with squeezing $r = 2$.

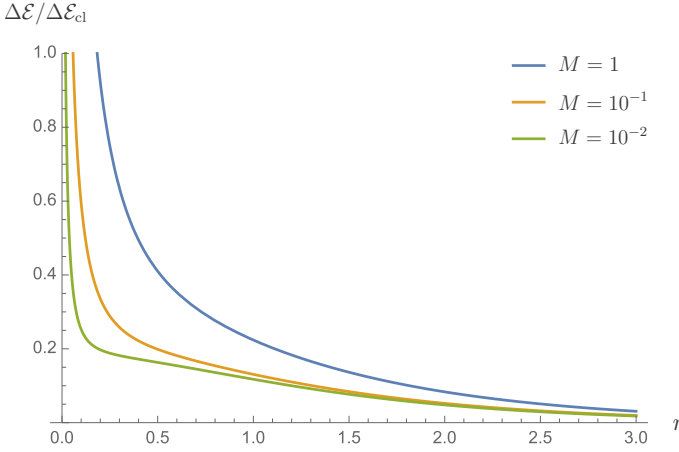


Fig. 4. Behavior of the uncertainty, normalized to its classical value, in the presence of an external bath as a function of the squeezing parameter r , for different values of the parameter M , in a regime of weak coupling, $\lambda\tau = 10^{-3}$.

The behavior of the uncertainty as a function of the squeezing parameter is instead reported in Fig. 4: one realizes that by increasing r , one can effectively contrast the noisy action of the bath. According to these plots, the ratio $\Delta\mathcal{E}/\Delta\mathcal{E}_{\text{cl}}$ appears to become infinitely large for vanishing squeezing: this is due to the approximation used in deriving the formula in (9) which ceases to be reliable for vanishingly small r , as its denominator becomes zero.^b

The setup design proposed in Ref. 25 that uses entangled photons therefore appears rather robust against environmental noise: the apparatus still retains a better sensitivity in holographic fluctuations determination than the one attainable using classical coherent light.

As a final remark, note that the “foamy” structure of spacetime at the Planck scale can itself effectively act as a noisy environment for the propagating photons.^{45,46} In this case, on rough dimensional grounds, one can estimate the dimensionless coupling parameter $\lambda\tau$ to be suppressed by at least an inverse power of the Planck mass M_P , i.e. $\lambda\tau \simeq \omega_\gamma/M_P$, with ω_γ the mean photon energy (see Refs. 45 and 46 for further discussions). For typical photon energy used in experiments ($\omega_\gamma \simeq 1$ eV) and squeezing parameter $r \simeq 1$, the normalized uncertainty $\Delta\mathcal{E}/\Delta\mathcal{E}_{\text{cl}}$ is found to be as small as 10^{-15} . Therefore, the decohering effects generated by quantum gravity-induced environments can be safely ignored. However, as discussed in the following section, other Planck scale phenomena can still influence the behavior of the traveling photons inside the interferometers and therefore affect the estimation of the uncertainty $\Delta\mathcal{E}$.

^bOnly for a zero temperature bath ($M = 0$), the expression in (20) is still valid even for vanishing squeezing: in this case, no advantage in sensitivity should be gained with respect to a “classical” apparatus, as the light entering all ports of the double interferometer is coherent, and indeed, one finds $\Delta\mathcal{E} \simeq \Delta\mathcal{E}_{\text{cl}}$.

4. Noise Induced by Modified Commutation Relations

As mentioned in the introductory remarks, many approaches to fundamental physics predict the existence of a minimum length, leading in turn to a modification of the usual canonical commutation relations. Taking an effective approach, the most general extension of the coordinate-momentum commutation relations involves additional terms^{27–31}:

$$[x_i, p_j] = i\delta_{ij} + g_{ij}, \quad [x_i, x_j] = \ell_{ij}, \quad [p_i, p_j] = h_{ij}. \quad (21)$$

As a result of this modification, also the behavior of the photons inside the interferometers, especially those that are prepared in a highly nonclassical, entangled state, may be altered as well. Although in general the quantities g_{ij}, ℓ_{ij} and h_{ij} may themselves be functions of the coordinates x_i and momenta p_i , we shall hereafter consider a simplified model where only the x - p commutation relations are modified by a constant contribution, i.e.

$$[x_i, p_i] = i(1 + \varepsilon), \quad (22)$$

while $[x_1, x_2] = [p_1, p_2] = 0$, and $[x_i, p_j] = 0$ for $i \neq j$. By passing from the phase space to mode operators, one easily sees that the standard canonical commutation relations can be altered as follows:

$$[a_1, a_2] = \varepsilon, \quad [a_1, a_2^\dagger] = \varepsilon, \quad [a_i, a_i^\dagger] = 1 + \varepsilon, \quad (23)$$

by the introduction of a real, adimensional, phenomenological parameter ε , assumed to be small $\varepsilon \ll 1$.^c

The above modified commutation relations can be expressed in terms of standard mode oscillators $A_i, A_i^\dagger, i = 1, 2$,

$$\begin{aligned} [A_i, A_i^\dagger] &= 1, & [A_i, A_j] &= 0, & [A_i, A_j^\dagger] &= 0 \quad i \neq j, \\ A_i|0\rangle &= 0 \quad i = 1, 2, \end{aligned} \quad (24)$$

through the following relations:

$$\begin{aligned} a_1 &= A_1\sqrt{1 + \varepsilon} + \frac{\varepsilon}{2\sqrt{1 + \varepsilon}}(A_2 - A_2^\dagger), \\ a_2 &= A_2\sqrt{1 + \varepsilon} + \frac{\varepsilon}{2\sqrt{1 + \varepsilon}}(A_1 + A_1^\dagger), \end{aligned} \quad (25)$$

^c Assuming that the noncommutative effects originate at a fundamental energy scale M_F , one can express this parameter as $\varepsilon = \alpha(\omega_\gamma/M_F)^\delta$, with α an adimensional constant and $\delta = 1, 2$; the value $\delta = 2$ is favored by string theory models and black hole physics, while $\delta = 1$ can be motivated by more abstract group and algebraic considerations.⁴⁷ Experimental efforts try to set bounds on the parameter α in both these scenarios, using both astrophysical systems and table-top experiments (e.g. see Refs. 47–53), assuming for simplicity M_F of order of the Planck mass. A safe upper bound on the possible value of ε that can be deduced from these studies is of the order $10^{-1} - 10^{-2}$.

which indeed reproduce (23). It should be stressed that A_i, A_i^\dagger are just auxiliary operators, useful for performing actual computations as they obey standard canonical commutation relations; instead, photon states must now be constructed and described through the commutators in (23).

Note that the algebra generated by (23) does not admit a Fock representation, i.e. a representation based on a lowest weight state, as defined by the condition $a_i|0\rangle = 0$. In such cases, one defines the vacuum state through the auxiliary A -modes as in (24). As a consequence, the two-mode squeezing operator $S(\zeta)$, constructed with the a -modes as in (3), no longer generates the twin-beam state (4) when acting on the vacuum, rather a modified one $|\text{TWB}'\rangle = S(\zeta)|0\rangle$. Since the parameter ε is assumed to be very small, it will be sufficient to compute the new state to first order in ε .

The new input state for the a -ports of the apparatus can then be obtained by first expanding

$$\zeta a_1^\dagger a_2^\dagger - \zeta^* a_1 a_2 = \mathcal{A} + \varepsilon \mathcal{B}, \quad (26)$$

with

$$\begin{aligned} \mathcal{A} &= \zeta A_1^\dagger A_2^\dagger - \zeta^* A_1 A_2, \\ \mathcal{B} &= \zeta \left\{ \frac{1}{2} [(A_1^\dagger + A_2^\dagger)^2 + A_1^\dagger A_1 - A_2 A_2^\dagger] \right\} - \text{h.c.}, \end{aligned}$$

and then using

$$e^{\mathcal{A} + \varepsilon \mathcal{B}} = e^{\mathcal{A}} \left\{ 1 + \int_0^1 du \frac{d}{du} [e^{-u\mathcal{A}} e^{u(\mathcal{A} + \varepsilon \mathcal{B})}] \right\}, \quad (27)$$

$$\approx e^{\mathcal{A}} \left\{ 1 + \varepsilon \int_0^1 du e^{-u\mathcal{A}} \mathcal{B} e^{u\mathcal{A}} \right\}, \quad (28)$$

to compute to first order in ε the action of the squeezing operator $S(\zeta)$ on the vacuum. Assuming for simplicity a real squeezing parameter $\zeta \equiv r \in \mathbb{R}$, and recalling that

$$e^{-uA} A_{1,2} e^{uA} = \cosh(ru) A_{1,2} + \sinh(ru) A_{2,1}^\dagger,$$

one finally gets that the modified input state becomes

$$|\text{TWB}'\rangle = |\text{TWB}\rangle + \varepsilon r e^{rA} \left\{ \frac{1}{2} (A_1^\dagger + A_2^\dagger)^2 - 1 \right\} |0\rangle, \quad (29)$$

where, now, $|\text{TWB}\rangle = e^{r(A_1^\dagger A_2^\dagger - A_1 A_2)} |0\rangle$; the state $|\text{TWB}'\rangle$ is a combination of entangled states.

Using similar techniques and approximations, one can now evaluate the uncertainty $\Delta \mathcal{E}$ in holographic fluctuations estimation modified by the presence of the parameter ε . As in the case of environmental noise discussed in the previous section, we shall assume zero central values of the phase shifts, $\phi_{1,0} = \phi_{2,0} = 0$, so that the result (19)

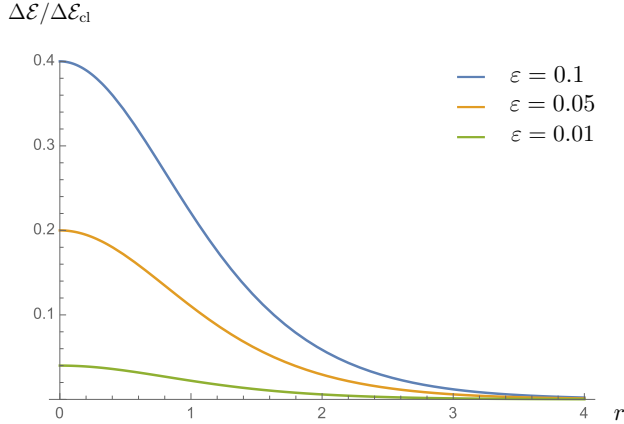


Fig. 5. Behavior of the uncertainty, normalized to its classical value, in the presence of modified photon-mode commutation relations, as a function of the squeezing parameter r , for various values of the deformation parameter ε .

still holds, since as mentioned before, in this condition, the interferometers work as completely transparent media. Indeed, (19) is the result of algebraic manipulations and does not depend on specific properties of the states. As a result, also in this case, a non vanishing $\Delta\mathcal{E}$ can only be ascribable to the Planck scale modified commutation relations (23).

The explicit calculation gives, to first order in ε ,

$$\Delta\mathcal{E}/\Delta\mathcal{E}_{\text{cl}} = \frac{8r\varepsilon}{\sinh(2r)}. \quad (30)$$

Note that also the coherent states entering the other two ports of the apparatus should be defined using the modified mode operators in (23), so that, to first order in ε , similar to (29), one can write: $|\mu'\rangle = |\mu\rangle + \varepsilon$ - correction. However, since they contribute only to the denominator of (19) and the numerator is proportional to ε , one can compute the denominator in the zeroth order approximation, i.e. with ordinary coherent states.

The behavior of this ratio as a function of the squeezing parameter r is plotted in Fig. 5. One can clearly see that the enhancement in sensitivity for the detection of holographic noise due to the presence of entangled initial photons is still present, even for relatively large values of ε .

5. Concluding Remarks and Outlooks

The spacetime noncommutativity at the Planck scale that most quantum gravity theory predicts can in principle be detected using suitable photon interferometric apparatus. The idea is that the noncommutativity in space position can induce quantum fluctuations on the optical components of an interferometer: these disturbances modify the length of the optical path of the photons traveling inside the

setup, causing a measurable change in the overall optical phase shift. This signal, dubbed holographic fluctuation, is predicted to be extremely small, but might be in the reach of setups using two interferometers, especially if fed with highly nonclassical, entangled light.

These conclusions hold for an ideal apparatus, perfectly isolated from its environment. Instead, we have here analyzed to what extent the entanglement enhanced sensitivity in detecting holographic fluctuations proves to be robust against decohering effects. In fact, the photons travelling inside the interferometers inevitably interact with their environment, and this leads to noise and dissipation; furthermore, Planck scale noncommutativity, whose effects we want to detect, may itself act as a decohering mechanism via a modification of the canonical commutation relations obeyed by the photon mode creation and annihilation operators.

We find that, if the coupling of the photons with the external environment is weak, a constraint in general very well satisfied in common experimental conditions and the violations of the standard photon mode commutation relations are small, a phenomenologically sensible assumption, the examined decohering effects will not be able to completely nullify the advantages brought in by the use of entangled light. In other terms, our results seem to confirm the validity of the approach employing quantum-enhanced metrology for detecting quantum gravity Planck scale effects.

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