

Noisy quantum phase communication channels

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Abstract

We address quantum phase channels, i.e. communication schemes where information is encoded in the phase-shift imposed to a given signal, and analyze their performances in the presence of phase diffusion. We evaluate mutual information for coherent and phase-coherent signals, and for both ideal and realistic phase receivers. We show that coherent signals offer better performances than phase-coherent ones, and that realistic phase channels are effective ones in the relevant regime of low energy and large alphabets.

Keywords: quantum communications, quantum phase, optical communication

(Some figures may appear in colour only in the online journal)

1. Introduction

The channel capacity is the upper bound on the rate of information that can be reliably transmitted along a channel, i.e., the number of bits that it can communicate reliably per channel use. For an ideal bosonic channel, i.e. when classical information is encoded onto quantum states of a single-mode field and no noise is present, the capacity is achieved by encoding information onto Fock number states and by retrieving information via the measurement of the number of photons [1, 2]. When amplitude (phase insensitive) noise is present coherent coding may also achieve the ultimate channel capacity [3, 4].

In this paper, we address quantum communication channels in the presence of phase diffusion. In order to investigate in details the effects of phase noise, we consider channels based on phase encoding, where phase diffusion is expected to be most detrimental [5, 6]. In our scheme, information is encoded in the phase-shift imposed to a given seed signal and the information carrier undergoes phase diffusion before arriving at the receiver station. We evaluate mutual information for coherent and phase-coherent seed signals and for both ideal and realistic phase receivers. Our results show that coherent signals offer better performances than phase-coherent ones, and also show that phase channels are effective ones, i.e. they approach the ultimate capacity at least in the low energy regime and for large alphabets.

The paper is organized in the following way. In section 2, we describe the communication protocol and derive a general formula for the mutual information. In section 3 we introduce

the noise model and assess the performances of phase channels in the presence of phase diffusion. In particular, we compare channels employing coherent and phase-coherent seeds using either canonical or double-homodyne phase receivers. Section 4 closes the paper with some concluding remarks.

2. Phase communication channel

A quantum phase communication channel is schematically depicted in figure 1. The sender encodes N letters chosen from a given alphabets onto the N possible values of a phase-shift ϕ_k , where $\phi_k < \phi_l$ if $k < l$ and $0 \leq k < N$. We assume that the letter ϕ_k is encoded onto a seed single-mode state q_0 by applying the unitary phase-shift operator $U(\phi) = \exp(i\phi a^\dagger a)$, a being the annihilation bosonic field operator, $[a, a^\dagger] = 1$, e.g.

$$q_0 \rightarrow q_k \equiv U(\phi_k)q_0U^\dagger(\phi_k). \quad (1)$$

The state q_k is then sent through the channel and arrives at the receiver, who retrieves information by performing a phase measurement and by applying a suitable inference strategy. In practice, the receiver divides the full range of possible values $[0, 2\pi)$ into a fixed number of bins, e.g. the same number N used by the sender, corresponding to the intervals $\mathcal{E}_j = [\phi_j - \Delta, \phi_j + \Delta)$, where $\phi_0 = 0$ and $\bigcup_{j=0}^{N-1} \mathcal{E}_j = [0, 2\pi)$. The width of each bin may be different, though a symmetric choice is often optimal. If ϕ denotes the value of

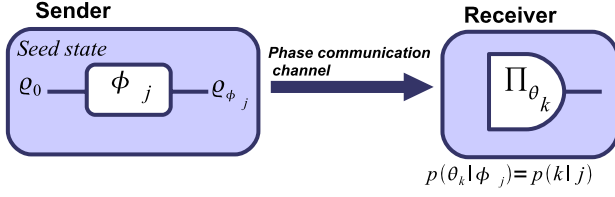


Figure 1. Schematic representation of a phase communication channel.

the receiver's outcome, we can use the following inference rule:

$$\text{if } \phi \in \Xi_j \Rightarrow \phi \rightarrow \phi_j. \quad (2)$$

If $\{\pi(\theta)\}$, $\theta \in [0, 2\pi)$, represents the positive operator-valued measure (POVM) for a phase measurement, then the POVM associated with the receiver's measurement can be written as:

$$\Pi_j = \int_{\phi_j - \Delta}^{\phi_j + \Delta} \pi(\theta) d\theta, \quad (3)$$

where the bin-width Δ is chosen in order to determine the cardinality of the output alphabet. The probability that the outcome ϕ falls in the bin Ξ_j given the state q_k is:

$$p(\phi \in \Xi_j | q_k) \equiv p(j | k) = \text{Tr}[q_k \Pi_j]. \quad (4)$$

The POVM $\{\pi(\theta)\}$ can be expanded in the photon number basis as

$$\pi(\theta) = \frac{1}{2\pi} \sum_{n,m=0}^{\infty} A_{n,m} e^{-i(n-m)\theta} |n\rangle\langle m|, \quad (5)$$

where $A_{n,m}$ are the elements of the positive and Hermitian matrix \mathbf{A} , which depends on the chosen phase measurement. Note that $U(\phi)\pi(\theta)U^\dagger(\phi) = \pi(\phi + \theta)$, i.e. we are considering covariant phase measurements. In the following we will focus on two particular phase measurements: the canonical phase measurement [7–10] and the marginal phase distribution obtained from the Husimi Q -function [11–15], the latter being a feasible phase measurement achievable, e.g., by heterodyne or double-homodyne detection or by tomographic reconstruction of the quantum state. We have $A_{n,m} = 1$ for the canonical measurement and

$$A_{n,m} = \Gamma\left[1 + \frac{1}{2}(n+m)\right] (n!m!)^{-\frac{1}{2}}$$

for the double homodyne one, $\Gamma[x]$ being the Euler Gamma function. Canonical phase measurement corresponds to the optimal POVM according to quantum estimation theory, but its experimental implementation is still challenging [10]. On the other hand, the marginal distribution of the Q -function may be directly measured by double-homodyne or heterodyne detection, and the corresponding phase distribution has been shown to be invariant under phase-insensitive amplification [16]. We also notice that for mixed signals, i.e. in the presence of noise, the optimal measurements for discrimination, that is, the measurement minimizing the error probability, may differ

from the measurement that maximizes mutual information [17].

Using equations (3) and (5), one can write

$$\Pi_j = \sum_{n,m=0}^{\infty} A_{n,m} f_{n-m}(j) |n\rangle\langle m|, \quad (6)$$

where the structure of the POVM is determined by the resolution function

$$f_d(j) = \frac{1}{2\pi} \int_{\phi_j - \Delta}^{\phi_j + \Delta} e^{-id\theta} d\theta = \frac{\sin \Delta\pi}{\pi d} e^{-id\phi_j}. \quad (7)$$

From now on we focus on the relevant case of equal bin-width $\Delta = \pi/N$, with $\phi_k = 2\pi k/N$ for both the sender and the receiver. We also consider a prior uniform probability distribution for the letters ϕ_k , namely $p(\phi_k) \equiv p(k) = N^{-1}$. The POVM $\{\Pi_j\}$ is covariant, i.e. $\Pi_j = U(\phi_j)\Pi_0 U^\dagger(\phi_j)$, and the resolution function is given by

$$f_d(j) = \frac{1}{N} (-1)^{\frac{jd}{N}} \text{sinc} \frac{\pi d}{N}, \quad (8)$$

where $\text{sinc } x = \sin x/x$. We also have $f_0(j) = N^{-1}$ and $\sum_{j=0}^{N-1} f_d(j) = \delta_{d,0}$, where $\delta_{n,m}$ is the Kronecker delta. Using these relations it is straightforward to prove the completeness of the POVM $\sum_j \Pi_j = \mathbb{1}$.

In order to characterize the performance of the phase communication channel we evaluate the mutual information between the sender and the receiver. This quantity measures the amount of shared information between the two parties and can be written as (we are considering uniform prior distribution):

$$I = \frac{1}{N} \sum_{k,j=0}^{N-1} p(j | k) \log_2 \frac{p(j | k)}{p'(j)}, \quad (9)$$

where $p'(j) \equiv p'(\phi_j) = 1/N$ and

$$\begin{aligned} p(j | k) &= \text{Tr}[q_k \Pi_j] \equiv \text{Tr}[q_0 \Pi_{j-k}] \\ &= \sum_{n,m=0}^{\infty} A_{n,m} f_{n-m}(j-k) q_{n,m} \end{aligned} \quad (10)$$

represents the conditional probability that the outcome of the measurement falls into the j th phase bin when the k th phase value has been encoded onto the signal; $q_{n,m}$ denotes the Fock matrix elements of the initial seed q_0 . Thanks to covariance this probability equals the probability of getting the outcome in the $(j-k)$ th phase bin for the seed signal q_0 . Upon exploiting the symmetries of the resolution function $f_d(j)$, i.e. $f_{-d}(j) = f_d(-j) = f_d^*(j)$, i.e. $f_{-d}(-j) = f_d(j)$, we may introduce the positive quantity $s = |j-k|$, where $0 \leq s \leq N-1$, and rewrite the mutual information as:

$$I \equiv I(N, \bar{n}) = \log_2 N + \sum_{s=0}^{N-1} q(s) \log_2 q(s), \quad (11)$$

where \bar{n} is the average number of photons of the seed signal (see below) and

$$q(s) = \sum_{n,m=0}^{\infty} A_{n,m} f_{n-m}(s) \varrho_{n,m} \quad (12)$$

$$= \frac{1}{N} \left[1 + \sum_{n=0}^{\infty} \sum_{d=1}^{\infty} A_{n,n+d} \left(f_d(s) \varrho_{n+d,d} + c.c. \right) \right] \quad (13)$$

$$= \frac{1}{N} \left[1 + 2 \sum_{n=0}^{\infty} \sum_{d=1}^{\infty} A_{n,n+d} \varrho_{n+d,d} \cos \frac{2\pi ds}{N} \operatorname{sinc} \frac{\pi d}{N} \right], \quad (14)$$

where the last equality is valid for real matrix elements $\varrho_{n,m}$, which is the case by choosing a seed signal with zero phase.

Equation (11) is general enough to cover all the scenarios of quantum phase communication channels we are going to consider. In the next section, we will evaluate the mutual information for two possible classes of seed states in the presence of a phase diffusion process. Before proceeding, however, we introduce the two classes of states we are going to consider as seed state ϱ_0 . The first class is that of coherent states (CSs) of the radiation field, namely, $\varrho_0 = |\alpha\rangle\langle\alpha|$ with $|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \alpha^n / \sqrt{n!} |n\rangle$. Without lack of generality, from now on we assume $\alpha \in \mathbb{R}$. The density matrix ϱ associated with the initial coherent state ϱ_0 has the matrix elements:

$$\varrho_{n,m} = e^{-\bar{n}} \frac{\bar{n}^{(n+m)/2}}{\sqrt{n!m!}}, \quad (15)$$

where $\bar{n} \equiv \alpha^2 = \operatorname{Tr} [\varrho_0 a^\dagger a]$ is the average number of photons of the coherent state ϱ_0 .

The second class of states we will consider consists of the so-called phase-coherent states (PCSs) [18–21]. In this case the seed state is given by $\varrho_0 = |\chi\rangle\langle\chi|$, where $|\chi\rangle = \sqrt{1 - |\chi|^2} \sum_{n=0}^{\infty} \chi^n |n\rangle$ with $|\chi| < 1$. PCSs approach Susskind–Glogower [18] phase states in the limit $|\chi| \rightarrow 1$. This second choice of seed signal is quite interesting because PCSs match the ideal measurement POVM for large $\bar{n} \gg 1$. Moreover, PCSs maintain phase coherence under phase amplification [22], such that they are privileged states for phase-based communication channels. Also in this case we can assume $\chi \in \mathbb{R}$ and the average number $\bar{n} = \langle \chi | a^\dagger a | \chi \rangle = \chi^2 / (1 - \chi^2)$. Since $\chi^2 = \bar{n} / (1 + \bar{n})$, the elements of the density matrix associated with a PCS may be written as

$$\varrho_{n,m} = \frac{1}{1 + \bar{n}} \left(\frac{\bar{n}}{1 + \bar{n}} \right)^{(n+m)/2}. \quad (16)$$

3. Quantum phase communication channels with phase diffusion

The effect of the phase diffusion process on the evolution of a single-mode state ϱ may be accurately described by the

master equation [6, 23]:

$$\frac{d}{dt} \varrho = \frac{\Gamma}{2} \mathcal{L} [a^\dagger a] \varrho, \quad (17)$$

where $\mathcal{L} [O] \varrho = 2O\varrho O^\dagger - O^\dagger O \varrho - \varrho O^\dagger O$ and Γ is the phase noise amplitude. An initial state $\varrho(0) = \sum_{n,m} \varrho_{n,m} |n\rangle\langle m|$ evolves with time as [6] $\varrho(\tau) = \sum_{n,m=0}^{\infty} e^{-(n-m)^2 \tau/2} \varrho_{n,m} |n\rangle\langle m|$, where $\tau \equiv \Gamma t$ is the *dephasing parameter*. One can easily see that the diagonal elements are unaffected by the phase noise, thus, energy is conserved, whereas the off-diagonal elements decay exponentially. Upon inserting the density matrix elements (15) and (16) into the above relation we may evaluate the mutual information also in the presence of phase noise. Results will be discussed in the following subsections.

3.1. Ideal phase receiver

In this case the POVM describing the ideal (canonical) measurement is obtained from equation (3) with $A_{n,m} = 1$, $\forall n, m$. In turn, after the phase diffusion process the conditional probability $q(s)$ of equation (12) reads:

$$q_{\text{CS}}(s) = \frac{1}{N} \left[1 + 2e^{-\bar{n}} \sum_{d=1}^{\infty} \operatorname{sinc} \frac{\pi d}{N} \cos \frac{2\pi s d}{N} \times e^{-\frac{1}{2}d^2\tau} \sum_{n=0}^{\infty} \frac{\bar{n}^{n+\frac{1}{2}d}}{\sqrt{n!(n+d)!}} \right], \quad (18)$$

and

$$q_{\text{PCS}}(s) = \frac{1}{N} \left[1 + 2 \sum_{d=1}^{\infty} \operatorname{sinc} \frac{\pi d}{N} \cos \frac{2\pi s d}{N} \left(\frac{\bar{n}}{1 + \bar{n}} \right)^{d/2} e^{-\frac{1}{2}d^2\tau} \right], \quad (19)$$

for CSs and PCSs, respectively. The mutual information $I^{(\text{ID})}$ is then evaluated using equation (11) for both CS- and PCS-based channels. The results are shown in figure 2 where $I^{(\text{ID})}$ is reported as a function of τ for different value of \bar{n} (top panels on the left) and as a function of \bar{n} and different values of τ (bottom panels on the left) for $N = 20$. As it is apparent from the plots, $I^{(\text{ID})}$ increases with the average number of photons \bar{n} and decreases with the dephasing parameter. Coherent signals offer better performances for any value of both the average number of photons and the noise parameter as one can also see from the three-dimensional plot on the right of figure 2, where we show the ratio $R^{(\text{ID})} = I_{\text{PCS}}^{(\text{ID})} / I_{\text{CS}}^{(\text{ID})}$ as a function of \bar{n} and τ for $N = 20$.

3.2. Q phase receiver

In this subsection we address channels with phase receivers achieving the marginal Q -function phase distribution, which correspond to feasible measurements in quantum optics and may be also reconstructed by quantum tomography [24, 25]. In particular, experimental data from double-homodyne or heterodyne detection are distributed according to the Husimi Q -function of the signals, and the marginal distribution

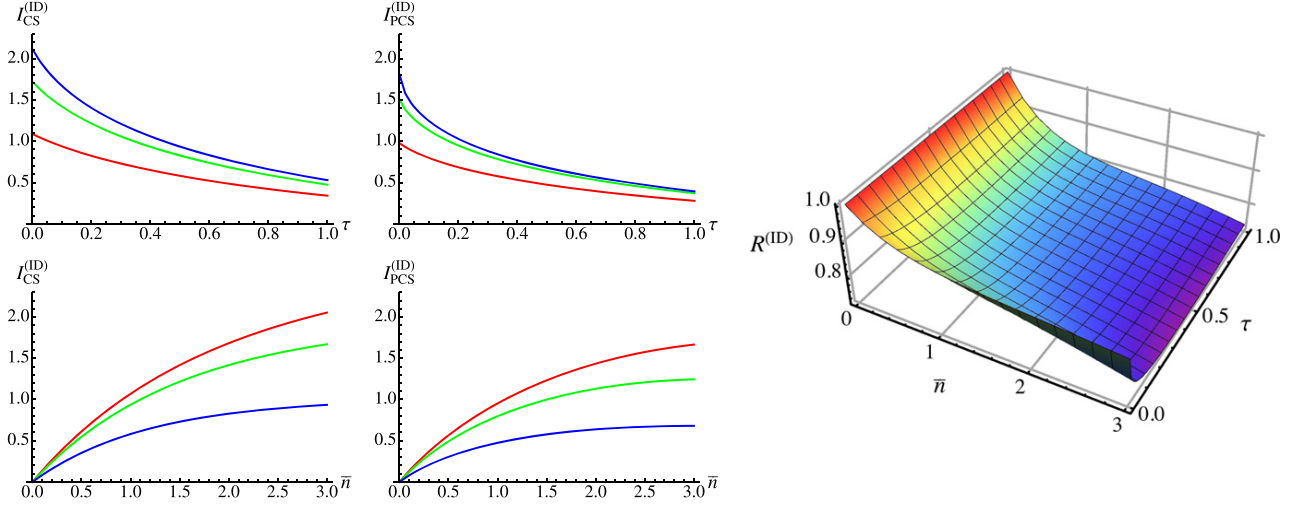


Figure 2. Performances of noisy phase communication channels with ideal phase receivers. The two panels on the top left show the mutual information $I^{(ID)}$ as a function of the the noise parameter τ for CSs (left) and PCSs (right) for different values of the average number of photons: from bottom to top $\bar{n} = 1, 2, 3$. The two bottom panels show the mutual information $I^{(ID)}$ as a function of the average number of photons \bar{n} for CSs (left) and PCSs (right) for different values of the noise parameter: from top to bottom $\tau = 10^{-2}, 10^{-1}, 5 \times 10^{-1}$. The three-dimensional plot on the right side of the figure shows the ratio $R^{(ID)} = I_{PCS}^{(ID)} / I_{CS}^{(ID)}$ as a function of \bar{n} and τ . In all the plots we set $N = 20$.

provides experimentally accessible phase information. The phase POVM is given by equation (5) with $A_{n,m} = \Gamma[1 + \frac{1}{2}(n+m)](n!m!)^{-\frac{1}{2}}$, such that the corresponding phase distribution is broader than the corresponding canonical one for any signal. The conditional probabilities $q(s)$ for the CSs and PCS are now given by:

$$q_{CS} = \frac{1}{N} \left[1 + 2e^{-\bar{n}} \sum_{d=1}^{\infty} \text{sinc} \frac{\pi d}{N} \cos \frac{2s\pi d}{N} \times e^{-\frac{1}{2}d^2\tau} \sum_{n=0}^{\infty} \frac{\Gamma\left(1+n+\frac{d}{2}\right)}{n!(n+d)!} \bar{n}^{n+\frac{1}{2}d} \right] \quad (20)$$

and

$$q_{PCS}(s) = \frac{1}{N} \left[1 + 2 \sum_{d=1}^{\infty} \text{sinc} \frac{\pi d}{N} \cos \frac{2s\pi d}{N} \times e^{-\frac{1}{2}d^2\tau} \sum_{n=0}^{\infty} \frac{\Gamma\left(1+n+\frac{d}{2}\right)}{\sqrt{n!(n+d)!}} \frac{\bar{n}^{n+\frac{1}{2}d}}{(1+\bar{n})^{n+\frac{1}{2}d+1}} \right] \quad (21)$$

respectively. The mutual information $I^{(Q)}$ is still obtained using equation (11). In figure 3 we plot the mutual information $I^{(Q)}$ as a function of τ (panels on the top left) and of \bar{n} (bottom left panel) for both CSs and PCSs at fixed values of the other involved parameters. The behavior of $I^{(Q)}$ is similar to that obtained for ideal phase receivers (see figure 2). However, its value is slightly smaller for both input

states. As we found for the ideal receiver, also in this case coherent signals outperform phase coherent ones in terms of mutual information, see the three-dimensional plot on the right of figure 3, where we show the ratio $R^{(Q)} = I_{PCS}^{(Q)} / I_{CS}^{(Q)}$ as a function of \bar{n} and τ .

3.3. Discussion

We will close the section by comparing the performances of the two receivers in the relevant quantum regime of low number of photons, $\bar{n} \ll 1$. In particular, we are interested in assessing the performances for large alphabets, $N \gg 1$. In this limit, the mutual information for the two classes of seed signals coincides, up to the first order, for both receivers, and thus we address the sole case of CSs.

In figure 4 the behavior of the ratio $R_{CS}^{(Q/ID)} = I_{CS}^{(Q)} / I_{CS}^{(ID)}$ is shown as a function of \bar{n} for different values of N and three values of the noise parameter. We find that the value of $R_{CS}^{(Q/ID)}$ is always larger than $\pi/4$, approaching this value in the limit $\bar{n} \ll 1$. It is also worth noting that for increasing τ and for $\bar{n} < 1$, the large alphabet limit $N \gg 1$, i.e. the dashed lines in the panels figure 4, is achieved for not too large values of N (say, $\simeq 10$). As it is apparent from the plots of figure 4, the ratio $R_{CS}^{(Q/ID)}$ is proportional to the average number of photons for $\bar{n} \ll 1$ and this resembles the ultimate channel capacity obtained in the presence of amplitude noise. Overall, this means the realistic phase channels, based on CSs and feasible receivers, offer good performances in terms of mutual information, not too far from the ultimate capacity. These findings may be confirmed by expanding the mutual information up to first order in the average photon number of

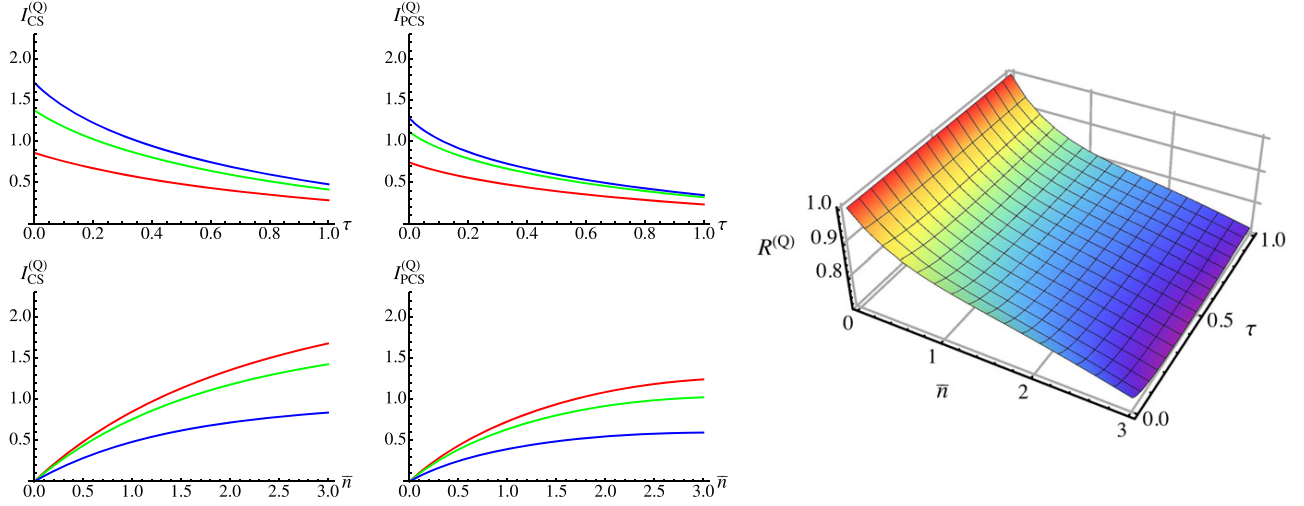


Figure 3. Performances of noisy phase communication channels with Q phase receivers. The two panels on the top left show the mutual information $I^{(Q)}$ as a function of the noise parameter τ for CSs (left) and PCSs (right) for different values of the average number of photons: from bottom to top $\bar{n} = 1, 2, 3$. The two panels on the bottom right shows the mutual information $I^{(Q)}$ as a function of the average number of photons \bar{n} for CSs (left) and PCSs (right) and for different values of the noise parameter: from top to bottom $\tau = 10^{-2}, 10^{-1}, 5 \times 10^{-1}$. The three-dimensional plot in the right side of the figure illustrates the ratio $R^{(Q)} = I_{\text{PCS}}^{(Q)} / I_{\text{CS}}^{(Q)}$ as a function of \bar{n} and τ . In all the plots we set $N = 20$.

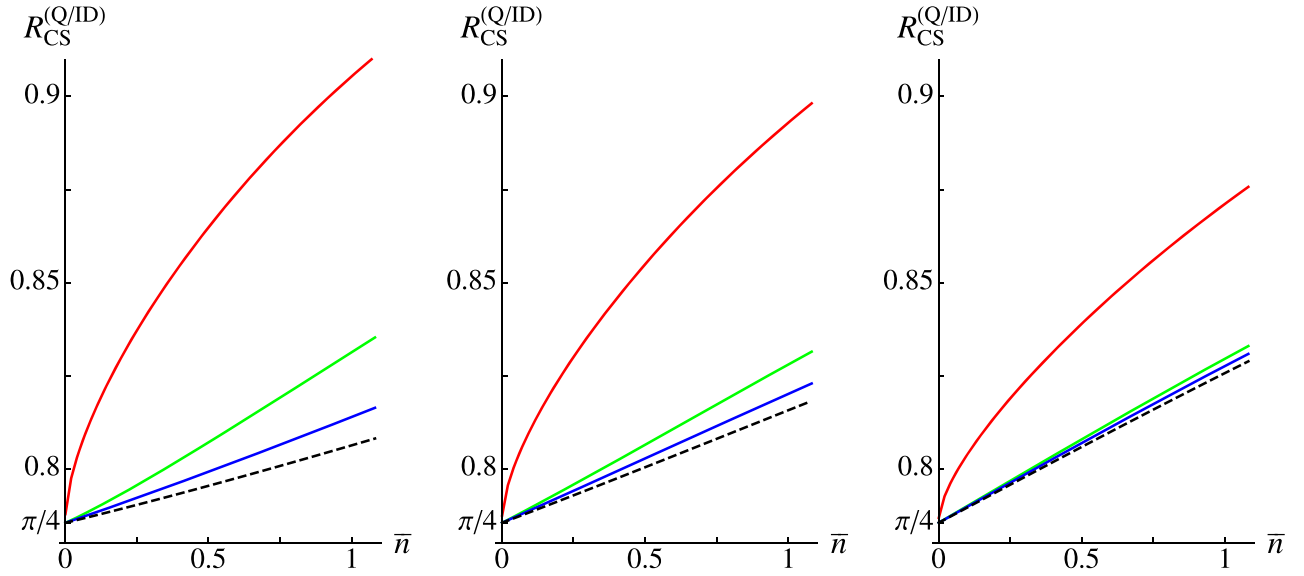


Figure 4. Plots of the ratio $R_{\text{CS}}^{(Q/ID)} = I_{\text{CS}}^{(Q)} / I_{\text{CS}}^{(ID)}$ as a function of the average number of photons \bar{n} and noise parameter $\tau = 0.1$ (left), $\tau = 0.2$ (center), and $\tau = 0.4$ (right). In all the plots the solid lines refer to (from top to bottom) $N = 3, 5, 7$, whereas the dashed lines are the corresponding limits for $N \gg 1$.

the seed signals. For both CS and PCS we have

$$I_{\text{ID}} \bar{n} \lesssim 1 \frac{\bar{n} \text{sinc}^2\left(\frac{\pi}{N}\right) e^{-\tau}}{\log 2} N \gg 1 \frac{\bar{n} e^{-\tau}}{\log 2} \quad (22)$$

for the ideal receiver and

$$I_Q \bar{n} \lesssim 1 \frac{\pi}{4} \frac{\bar{n} \text{sinc}^2\left(\frac{\pi}{N}\right) e^{-\tau}}{\log 2} N \gg 1 \frac{\pi \bar{n} e^{-\tau}}{4 \log 2} \quad (23)$$

for the Q -function one.

4. Conclusions

We have analyzed quantum phase communication channels based on phase modulation of coherent and PCs and have assessed their performances in the presence of phase noise. More precisely, we have evaluated the mutual information between the sender and the receiver in the presence of phase

diffusion both for ideal and realistic phase receivers. Our results show the robustness of phase communication channels, especially in the regime of low energy and (moderately) large alphabets. Besides, we have also shown that this results may be obtained with customary coherent signals, which outperform phase-coherent ones in terms of mutual information and robustness against noise.

Acknowledgments

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