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TOWARD A FULL RECONSTRUCTION OF DENSITY MATRIX BY ON/OFF MEASUREMENTS

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The knowledge of density matrix is fundamental for several applications, ranging from quantum information to the foundations of quantum mechanics and quantum optics. Nevertheless, quantum tomography based on homodyne detection is a rather complicated technique when applied to short pulses in photocounting regime. In this paper, we present an experimental work addressed to test an innovative scheme for a full reconstruction of the density matrix by using on/off detection coupled to phase measurements respect to a local oscillator.

Keywords: Quantum optical states; density matrix.

1. Introduction

The knowledge of density matrix is fundamental for several applications, ranging from quantum information[1] to the foundations of quantum mechanics[2] and quantum optics.[3,4] Nevertheless, quantum tomography[5–7] based on homodyne detection is a rather complicated method, indeed homodyne detection is a not trivial technique particularly when fast pump lasers are used (requiring temporal and spatial mode matching). Since also detectors with photon-number resolving capabilities and of practical use are not available,[8–13] various theoretical studies[14–17] have been addressed to achieve the reconstruction of the (diagonal) elements of the density matrix exploiting the information achievable with realistic detectors, finding a favourable experimental test in Refs. 18–23, where a very satisfactory reconstruction of the statistics of mono-partite and bi-partite quantum optical states was obtained by using on/off detectors following the method of Refs. 16 and 17.

In this paper, we present a preliminary work addressed to extend this method to a full reconstruction of the density matrix by using on/off detection coupled to phase measurements respect to a local oscillator. The paper is structured as follows: Sec. 2 reviews the method to reconstruct the density matrix and gives a simple description of the experiment. In Sec. 3 we explain in details our setup and presents the results concerning a coherent signal. Secction 4 closes the paper with some concluding remarks.

2. Reconstruction Method

In this section we briefly review the density matrix reconstruction method proposed in Ref. 24. Let ρ be the density matrix of the state we want to reconstruct. The reconstruction relies on the measurement of the photon distribution of the displaced state $\rho(\beta) \equiv D(\beta)\,\rho\,D^\dagger(\beta)$, $\beta = |\beta|\,e^{i\varphi}$, for fixed $|\beta|$ and different values of φ, i.e. $p_n(\varphi) = \langle n|\rho(\beta)|n\rangle$. The density matrix elements of the reconstructed state ϱ are then given by[24]:

$$\langle m+s|\varrho|m\rangle = \frac{1}{\mathcal{N}_\varphi}\sum_{l=1}^{\mathcal{N}_\varphi}\sum_{n=0}^{\overline{n}} F_{m,n}^{(s)}\,e^{is\varphi_l}\,p_n(\varphi_l), \tag{1}$$

where $\mathcal{N}_\varphi$ is the number of phase φ_l considered, $l = 1,\ldots,\mathcal{N}_\varphi$, $\overline{n}$ is such that $\varrho_{kh} \equiv \langle h|\varrho|k\rangle$ can be neglected if $h,k > \overline{n}$, and $F_{m,n}^{(s)}$ are functions of $|\beta|$ whose analytic expression is reported in Ref. 24.

The challenging task is now the measurement of the photon statistics, since this requires photodetectors resolving the number of photons. An alternative method has been given so far in Ref. 16, 17, where it was shown that the photon distribution p_n for a quantum optical state can be retrieved via on/off detectors. Assuming negligible dark counts, the "off" probability is related to the p_n's thanks to the relation:

$$P_0(\eta) = \sum_n (1-\eta)^n p_n, \tag{2}$$

η being the quantum efficiency of the detector. Measuring a given signal for different quantum efficiencies η_ν, $\nu = 1,\ldots,N$, starting form Eq. (2) we obtain a statistical model for the positive parameters p_n, which can be solved by *Expectation Maximization Algorithm* (EM).[16,17]

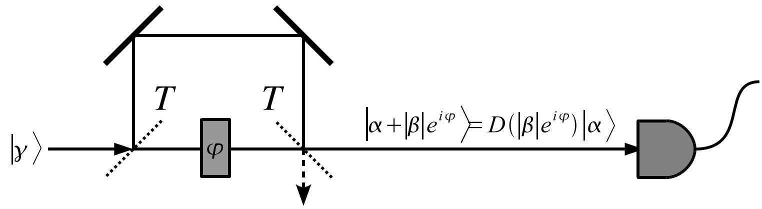

Fig. 1. Simplified sketch of the experiment aimed to reconstruct the density matrix of a coherent state $|\alpha\rangle$. A signal excited to the coherent state $|\gamma\rangle$, γ real, enters a Mach-Zehnder interferometer with two beam splitters with transmissivity T. The output filed can be written as $D(\beta|e^{i\varphi})|\alpha\rangle$, where $|\beta| = T\gamma$, $\alpha = R\gamma$ and φ is the phase difference between the signals added by the interferometer. The other output is discharged.

The additional phase information needed by Eq. (1) can be obtained following the interferometric scheme proposed in Ref. 24. As a first step toward a test of that measurement scheme, here we analyze the reconstruction of the density matrix of a coherent state $|\alpha\rangle$ and, without loss of generality, we assume the amplitude α as real. Figure 1 shows a simplified version of the scheme used in our experiment. The amplitude of the coherent state and of the displacement can be also modified attenuating the optical paths. This scheme allows to vary φ by suitably tuning the interferometer and, then, the $p_n(\varphi)$ in Eq. (1) are finally retrieved by on/off measurements onto the output field and EM algorithm.

3. Experimental Setup and Results

The actual implementation of our experiment is depicted in Fig. 2.

The output power of a He-Ne laser ($\lambda = 632.8\,\text{nm}$) is attenuated to single photon regime by neutral filters. The spatial profile of the beam is reshaped by a spatial filter realized by two converging lenses and a 100 μm diameter wide pinhole. The laser cavity is also preserved by backreflections, which may cause instability by means of an optical isolator consisting in a Faraday rotator between two orthogonal polarizers.

A beam-splitter is used to pick up part of the beam in order to monitor the laser amplitude fluctuations. The remaining part of the beam, which is the signal to be reconstructed, is sent to the interferometer. The unbalanced Mach-Zender interferometer has been realised on single block of low thermal expansion coefficient material (Invar) in order to guarantee long term stability of the set-up. A piezoelectric-movement system allows to change the phase between the "short" and "long" paths by driving the position of the reflecting prism with resolution of the order of nanometer.

For each position of the prism, the "no-click" frequencies are collected for different values of attenuation of the ND filter in front a Perkin-Elmer Single Photon Avalanche Photodiode (SPCM-AQR).

The detector is gated with a repetition rate of 200 kHz by a 20 ns wide time window. In order to obtain a reasonable statistics, a single run consists of 5 repetitions of 4 second acquisitions. Events are recorded by a NI-6602 PCI counting module.

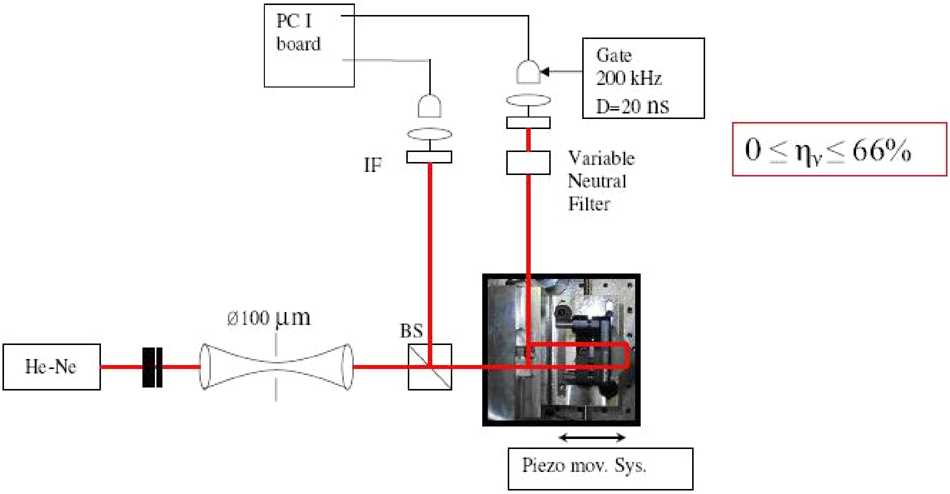

Fig. 2. Setup for the reconstruction of the density matrix for a coherent state. The emission of a He-Ne laser ($\lambda = 632.8\,\text{nm}$) is lowered to single photon regime by neutral filters. A spatial filter realized by two converging lenses and a $100\,\mu$m diameter-wide pinhole purifies the shape of the beam and allows to select a single spatial mode. A beam-splitter reflects part of the beam to a control detector used to monitor the laser amplitude fluctuations, while the remaining part, which is the signal to be reconstructed, is sent to the interferometer. The phase between the "short" and "long" paths in the interferometer can be changed by driving the position of the reflecting prism by means of a PI piezo-movement system. A set of variable neutral filters allows to collect photons for different values of the quantum efficiency. The detectors used are Perkin-Elmer Single Photon Avalanche Photodiode(SPCM-AQR) gated by a 20 ns wide time window width (repetition rate 200 kHz). A single run consists of 5 repetitions of 4 seconds acquisitions and events are recorded by a NI-6602 PCI counting board.

We point out that our scheme allows to estimate the error in real time, that it is suitable both for pure and mixed states and that statistic information is obtained by sampling a discrete matrix rather than measuring a continuous distribution in phase space as homodyne quantum tomography schemes.

In Fig. 3 we show the interference fringes at the output of the interferometer. We have chosen $\mathcal{N}_\varphi = 11$ different phases and the average energy E has been obtained from the reconstructed $p_n(\varphi)$. Even if the visibility is not very high, for coherent signals this is not an issue, since this can be controlled by rescaling the experimental data without affecting the nature of the reconstructed density matrix.

Due to the small number of phases ($\mathcal{N}_\varphi = 11$) considered in this preliminary experiment, only the diagonal and the first off-diagonal elements of the density matrix ϱ can be reconstructed. The results are shown in Fig. 4 where we plot the reconstructed (ϱ_{nm}), the expected (ρ_{nm}) density matrix elements and the absolute difference $\Delta_{nm} = |\rho_{nm} - \varrho_{nm}|$. The reconstructed ϱ corresponds to a coherent state $|\alpha\rangle$ with energy $|\alpha|^2 = 3.06$. Considering the small amount of data, the agreement between ρ and ϱ is good.

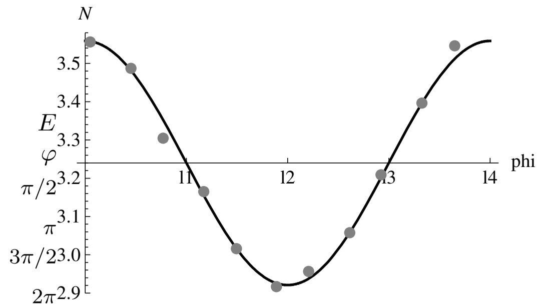

Fig. 3. Interference fringes: experimental data (gray disks) and fit (solid line).

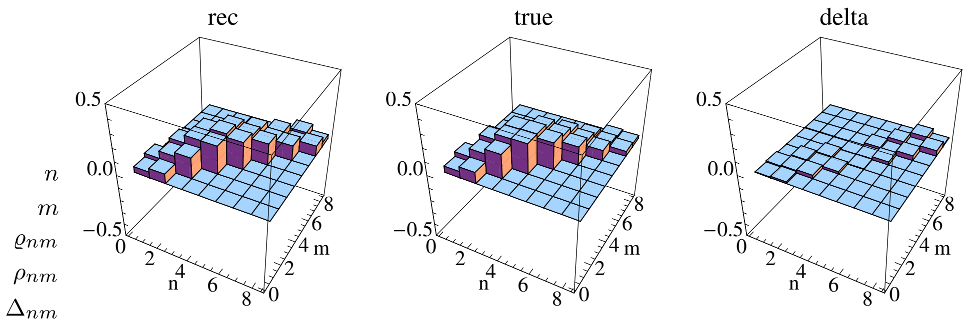

Fig. 4. Reconstructed density matrix elements (ϱ_{nm}) which agree with those (ρ_{nm}) of a coherent state $|\alpha\rangle$ with $|\alpha|^2 = 3.06$. We also plot the quantity $\Delta_{nm} = |\rho_{nm} - \varrho_{nm}|$. Only the diagonal and the first off-diagonal elements have been reconstructed due to the small number of phases considered in the experiment.

In order to improve the efficiency of the reconstruction scheme, the next efforts will be aimed in the directions of optimizing the visibility of the interference fringes and of extending the reconstruction to elements in density matrix farther from the diagonal by increasing the number of phase steps.

In the future the method is to be used to also characterize different kinds of optical fields, the next planned application being the reconstruction of the density matrix for the radiation emitted by a pseudo-thermal source.

4. Concluding Remarks

We demonstrated the partial reconstruction of the density matrix of a coherent state following Ref. 24. Our results show the validity of the method and promptus to further efforts in order to provide complete reconstruction of the density matrix for arbitrary quantum optical sources by using on/off detection coupled to phase measurements.

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