Revealing interference by continuous variable discordant states

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In general, a pair of uncorrelated Gaussian states mixed in a beam splitter (BS) produces a correlated state at the output. However, when the inputs are identical Gaussian states the output state is equal to the input, and no correlations appear, as the interference had not taken place. On the other hand, since physical phenomena do have observable effects, and the BS is there, a question arises on how to reveal the interference between the two beams. We prove theoretically and demonstrate experimentally that this is possible if at least one of the two beams is prepared in a discordant, i.e., Gaussian correlated, state with a third beam. We also apply the same technique to reveal the erasure of polarization information. Our experiment involves thermal states and the results show that Gaussian discordant states, even when they show a positive Glauber P-function, may be useful to achieve specific tasks.

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Understanding the nature of correlations among quantum systems is one of the major task of current research. Quantum correlations, in fact, play a leading role in understanding the very foundations of quantum mechanics, and represent the basic resource for the development of quantum technologies. Different quantities and strategies to discriminate whether correlations have a quantum nature or not [1–4] have been introduced, and it has also been pointed out [5–7] that the criteria based on the informational point of view, such as the quantum discord [8–16], are somehow incompatible with the physical ones based on the Glauber–Sudarshan phase–space approach [17,18]. A paradigmatic example in quantum optics is given by a thermal equilibrium state divided at a beam splitter (BS). This state, which is characterized by Gaussian Wigner functions, is indeed a classical one according to the Glauber approach, however, the bipartite state emerging from the BS displays nonzero Gaussian discord and, thus, from the informational point of view it contains a nonvanishing amount of quantum correlations. It is also worth noting that, for Gaussian states, the only bipartite states with zero Gaussian discord are the factorized ones [11,19] and that there is evidence that the Gaussian discord could be the ultimate quantum discord for Gaussian states [12,20]. In general, if a factorized state $\varrho_{12} = \varrho_1 \otimes \varrho_2$ undergoes a unitary interaction described by the operator U_{12}, then the evolved state $\tilde{\varrho}_{12} = U_{12}\varrho_{12}U_{12}^\dagger$ may be correlated. The total amount of correlations can be quantified by the mutual information $\mathcal{I}[\tilde{\varrho}_{12}] = S[\tilde{\varrho}_1] + S[\tilde{\varrho}_2] - S[\tilde{\varrho}_{12}] = \Delta S_1 + \Delta S_2$, where $\tilde{\varrho}_k = \mathrm{Tr}_h[\tilde{\varrho}_{12}]$, $h \neq k$, $S[\tilde{\varrho}_k] = -\mathrm{Tr}[\tilde{\varrho}_k \ln \tilde{\varrho}_k]$ is the von Neumann entropy and $\Delta S_k = S[\tilde{\varrho}_k] - S[\varrho_k]$. From the above equation we can see the rise of correlation is due to an increase of entropy between the input and output states ϱ_k and $\tilde{\varrho}_k$, respectively. It is clear that if $\varrho_k = \tilde{\varrho}_k$, then $\mathcal{I}[\tilde{\varrho}_{12}] = 0$ (provided that the input is a factorized and thus uncorrelated state). For Gaussian states, this happens when the inputs have the same covariance matrix (CM) and U_{12} corresponds to a bilinear, energy-conserving interaction described by $H_I \propto a^\dagger b + ab^\dagger$, where a and b are bosonic annihilation operators, $[a, a^\dagger] = 1$ and $[b, b^\dagger] = 1$.

When the initial state ϱ_{12} and the evolved one $\tilde{\varrho}_{12}$ are exited in the same factorized state, they cannot be discriminated and no correlations appear, as the interference of the two beams had not taken place. On the other hand, since physical phenomena do have observable effects, and the BS is there, a question arises on how to reveal the interference between the two beams. In this Letter, we investigate the dynamics of correlations in this kind of system and demonstrate, both theoretically and experimentally, that revealing interference is possible by adding an ancillary mode 3 correlated with one of the two beams, say beam 2. More explicitly, it is sufficient that the bipartite state ϱ_{23} has nonzero Gaussian discord to reveal the interference between mode 1 and 2 even when the local states $\varrho_2 = \mathrm{Tr}_3[\varrho_{23}] \equiv \varrho_1$ are identical and the interaction at the BS is not creating any correlations between them. Let us consider two generic zero-amplitude Gaussian states [21] $\varrho_k = S(r_k)\nu_{\mathrm{th}}(N_k)S^\dagger(r_k)$, where $\nu_{\mathrm{th}}(N_k) = \sum_{n=0}^{\infty} (N_k)^n/(1+N_k)^{n+1}|n\rangle\langle n|$ is a thermal equilibrium state with N_k thermal photons and $S(r_k) = \exp\{(1/2)r_k[(a_k^\dagger)^2 - a_k^2]\}$ is the squeezing operator, a_k being mode operators, $k = 1, 2$. The 2×2 CM of the state ρ_k can be written as $\sigma_k \equiv \sigma(N_{\mathrm{tot},k}, \beta_k)$, where $\sigma(N,\beta) = \mathrm{Diag}\{f_+(N,\beta), f_-(N,\beta)\}$, $f_\pm(N,\beta) = (1/2) + N \pm \sqrt{\beta N[1 + N(2-\beta)]}$ and we introduced the total number of photons $N_{\mathrm{tot},k} = \mathrm{Tr}[a_k^\dagger a_k \rho_k]$ and the squeezing fraction β, with $N_k = (1-\beta)N_{\mathrm{tot},k}$. We have assumed $r_k > 0$ without loss of generality. With this notation, $\beta = 0$ and $\beta = 1$ correspond to the thermal and the squeezed vacuum state, respectively, while $\sigma(0,0) \equiv \sigma_0$ is the CM of the vacuum state $\varrho_0 = |0\rangle\langle 0|$. Under the action of a BS with transmissivity τ, the initial 4×4 CM $\Sigma_0 = \sigma_1 \oplus \sigma_2$ of the two-mode state $\varrho_1 \otimes \varrho_2$ transforms as $\Sigma_0 \to \Sigma^{(\mathrm{out})} = \left(\begin{smallmatrix}\Sigma_1 & \Sigma_{12}\\ \Sigma_{12} & \Sigma_2\end{smallmatrix}\right)$, where $\Sigma_1 = \tau\sigma_1 + (1-\tau)\sigma_2$, $\Sigma_2 = \tau\sigma_2 + (1-\tau)\sigma_1$, and $\Sigma_{12} = \tau(1-\tau)(\sigma_2 - \sigma_1)$. Note that $\Sigma_{12} \neq \mathbf{0}$ denotes the presence

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of correlation between the outgoing modes. Notice that rewriting $\Sigma_{12} = \tau(1-\tau)[(\sigma_2-\sigma_0)+(\sigma_0-\sigma_1)]$, we can identify two different contributions: the one, $\propto(\sigma_0-\sigma_1)$, which is equal to that obtained by mixing ϱ_1 with the vacuum, i.e., $\varrho_2 \equiv \varrho_0$; similarly, the other, $\propto(\sigma_2-\sigma_0)$, corresponds to that obtained by mixing ϱ_2 with $\varrho_1 \equiv \varrho_0$. On the other hand, interference cannot be seen as the simple sum of two contributions and this will be exploited later on in this Letter, in order to describe the results of our second experiment. As follows from the above analysis, if the input modes are prepared in the same initial state, i.e., $\sigma_1 = \sigma_2$, then the output beams are left in an uncorrelated, factorized state with $\Sigma_0 \equiv \Sigma^{(\mathrm{out})}$ and ($\Sigma_{12} = \mathbf{0}$). In this case, the two above-mentioned contributions cancel each others and the interaction leaves the system unchanged. In order to reveal interference, we correlate mode 2 with a third auxiliary mode 3, i.e., we prepare $\varrho_{23} \neq \varrho_2 \otimes \varrho_3$ such that $\varrho_2 = \mathrm{Tr}_3[\varrho_{23}] = \varrho_1 = \varrho$. Modes 1 and 2 are still left unchanged and uncorrelated after the interference, but now, because of the interaction, part of the correlations shared between modes 2 and 3 are now shared between modes 1 and 3. This monogamy effect [22] can be seen by looking at the evolved CM of the whole state of the three modes. The 6×6 CM of the initial state $\varrho_{123} = \varrho_1 \otimes \varrho_{23}$ reads $\Sigma_{123} = \boldsymbol{\sigma}_1 \oplus \begin{pmatrix} \sigma_2 & \delta_{23} \\ \delta_{23}^T & \sigma_3 \end{pmatrix}$, where $\boldsymbol{\sigma}_k$ is the 2×2 single-mode CM of mode $k = 1, 2, 3$, $\boldsymbol{\sigma}_1 = \boldsymbol{\sigma}_2 = \boldsymbol{\sigma}(N,\beta)$, and N is the total number of photons per mode. The block $\boldsymbol{\delta}_{23} \neq \mathbf{0}$ contains the correlations between modes 2 and 3, which show nonzero Gaussian A- and B-discord [23]. After mixing mode 1 and 2 at the BS we have:

$$\Sigma_{123} \to \Sigma_{123}^{(\mathrm{out})} = \begin{pmatrix} \boldsymbol{\sigma}(N,\beta) & \mathbf{0} & \sqrt{1-\tau}\boldsymbol{\delta}_{23} \\ \mathbf{0} & \boldsymbol{\sigma}(N,\beta) & \sqrt{\tau}\boldsymbol{\delta}_{23} \\ \sqrt{1-\tau}\boldsymbol{\delta}_{23} & \sqrt{\tau}\boldsymbol{\delta}_{23} & \boldsymbol{\sigma}_3 \end{pmatrix}. \quad (1)$$

The comparison between input and output CMs shows that while modes 1 and 2 are (locally) left unchanged and uncorrelated, both of them are now correlated with mode 3. Furthermore, the degree of correlations between the modes 2 and 3 is decreased ($\boldsymbol{\delta}_{23} \to \sqrt{\tau}\boldsymbol{\delta}_{23}$) for the benefit of the birth of correlations between the previously uncorrelated modes 1 and 3 ($\mathbf{0} \to \sqrt{1-\tau}\boldsymbol{\delta}_{23}$). It is worth noting that the birth (reduction) of correlation between modes 1 and 3 (modes 2 and 3) is not merely due to the transmission (reflection) of beam 2, but it is due to its interference at the BS: beam 2 evolves in a two-mode correlated state, whose modes are thus correlated with mode 3. For the sake of clarity, we addressed only single-mode beams, but the same results hold also in the presence of multimode Gaussian beams since the phenomenon is essentially due to the tensor product nature of the multimode state, and to the pairwise nature of the interaction at the BS. In the experiment, we exploit correlations among three spatial multimode pseudo-thermal beams. We produce two independent unpolarized beams with thermal statistics addressing 1 ns laser pulses at 532 nm on two independent rotating ground glasses R1 and R2, with inhomogeneities of approximately 1 μm of size. The two speckled beams are collimated with two lenses (L_1 and L_2) of $f = 1.5$ m focal length put at a

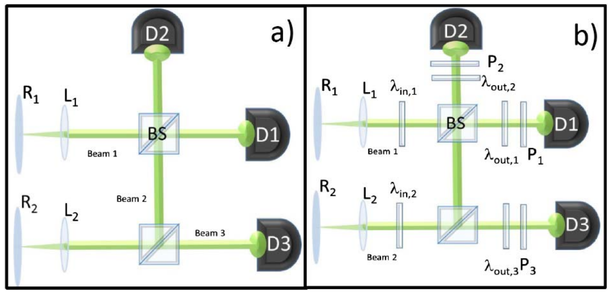

Fig. 1. Revealing interference by continuous variable discordant states: scheme of the two experimental setups.

distance f from the disks. Beam 1 is directly sent to a balanced BS while the second is further divided into beams 2 and 3 [Fig. 1(a)]. Each beam $k = 1, 2, 3$, is then sent to the corresponding detector D k, which is a portion of a CCD sensor. The speckled beams are imaged by means of a lens of focal lens $f_I = 25$ cm on the array of pixels. Due to the presence of the lenses L_1 and L_2, each speckle on the CCD array corresponds to a spatial mode of the pseudo-thermal beam. For each beam k we select an area A_k collecting M spatial modes and evaluate the intensity $I_k^{(j)} = \sum_{m=1}^{M} \langle a_{m,k}^\dagger a_{m,k}\rangle$ for each frame j of the CCD where $a_{m,k}$ is the field operator of the m-th mode impinging on the area k. The correlation between the beams h and k is estimated by using the second-order correlation coefficient $c_{h,k} = (\langle I_k I_h\rangle_{\mathrm{fr}} - \langle I_h\rangle_{\mathrm{fr}}\langle I_k\rangle_{\mathrm{fr}})/(\Delta_{\mathrm{fr}}(I_h)\Delta_{\mathrm{fr}}(I_k))$, where $\langle F\rangle_{\mathrm{fr}} = (N_{\mathrm{frame}})^{-1}\sum_{j=1}^{N_{\mathrm{frame}}} F^{(j)}$ is the average over N_{frame} frames and $\Delta_{\mathrm{fr}}(I_k)^2 = \langle I_k^2\rangle_{\mathrm{fr}} - \langle I_k\rangle_{\mathrm{fr}}^2$. It is worth noting that $c_{h,k}$ is independent on the number of modes M, provided that all spatial modes of each beam have the same intensity. In order to align the setup, and to achieve the proper mode matching at the BS, we first realize the superposition of the correlated areas A_1 and A_2 by alternatively stopping beam 1 or beam 2 and maximizing the correlation $c_{1,2}^{(1)}$ and $c_{1,2}^{(2)}$ between the beams outgoing the BS. We obtain $c_{1,2}^{(1)} = 0.97$ and $c_{1,2}^{(2)} = 0.96$.

We then measure the correlations coefficients $c_{1,2}^{(\mathrm{in})}$, $c_{1,3}^{(\mathrm{in})}$, and $c_{2,3}^{(\mathrm{in})}$ of the initial state and $c_{1,2}^{(\mathrm{out})}$, $c_{1,3}^{(\mathrm{out})}$, and $c_{2,3}^{(\mathrm{out})}$ of the states after the mixing of beam 1 and beam 2 in the BS. Experimental results are summarized in Fig. 2 and in Table 1, where we report the measured correlations between the couples of beams before and after

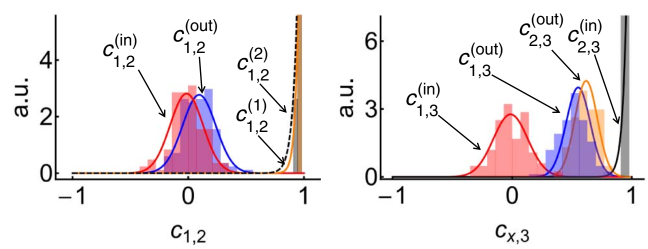

Fig. 2. Correlation coefficients among the three beams before and after mixing of modes 1 and 2. The first plot (left) shows the evolution of the correlation between beams 1 and 2. The second plot (right) refers to correlations among beam 1 and 2 with beam 3.

Table 1. Measured Correlations between Beams h, k Before (in) and After (out) the BS[a]

Beams h, k	$c_{h,k}^{(\mathrm{in})}$	$c_{h,k}^{(\mathrm{out})}$
1, 2	$0.09_{[-0.28;0.46]}$	$-0.01_{[-0.38;0.35]}$
1, 3	$-0.01_{[-0.38;0.36]}$	$0.55_{[0.29;0.81]}$
2, 3	$0.97_{[0.85;1.00]}$	$0.62_{[-0.38;0.85]}$

[a]The subscripts report the confidence intervals at 99%.

the interaction with the BS averaging over $N_{\mathrm{frame}} = 50$ frames. The mean values and the confidence intervals (at 99%) are obtained from the raw data by taking into account the bounded nature of the correlation coefficients $c_{h,k}$ [24]. As it is apparent from Fig. 2 and from Table 1, beams 1 and 2 are not affected by the presence of the BS (the small discrepancies between the measured correlation are due to the slightly imperfect mode matching), whereas the interference between them is revealed by the dramatic change in the correlations with mode 3.

The significant role of discord in our protocol is illustrated in Fig. 3, where we show the behavior of the output correlations $c_{1,3}^{(\mathrm{out})}$ and $c_{2,3}^{(\mathrm{out})}$ as functions of the discord between modes 2 and 3 at the input. As it is apparent from the plots, correlations at the output are monotone functions of the initial discord. Nonzero correlations are created for any value of initial discord. The three lines in both panels of Fig. 3 correspond to three different values of the transmissivity of the BS creating discord between modes 2 and 3. As expected from the form of the CM in Eq. (1), increasing the transmissivity of the BS increases the output correlations between modes 2 and 3 at the expense of correlations between modes 1 and 3.

In order to further clarify the role of the ancillary mode 3 we now consider a different scenario, where the two input beams do not interact at the BS. As depicted in Fig. 1(b), this is achieved by two half-wave plates $\lambda_{\mathrm{in},1}$ and $\lambda_{\mathrm{in},2}$, which set horizontal polarization (H) for beam 1, i.e., $\varrho_1^{(H)}$, and vertical polarization (V) for beam 2, $\varrho_2^{(V)}$. We assume that mode 2 and 3 have the same polarization. Due to the different polarizations, modes 1 and 2 no longer interfere at the BS. Rather, they both interact with a vacuum mode with the same polarization entering the other port of the beam, thus giving rise to two couples of collinear, superimposed correlated beams one with V polarization, the other with H polarization. Overall, we have four modes, and the two states at the output are

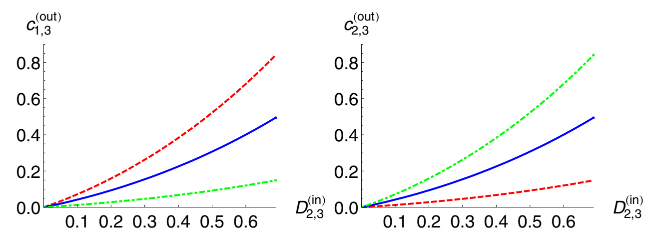

Fig. 3. Output correlations $c_{1,3}^{(\mathrm{out})}$ (left) and $c_{2,3}^{(\mathrm{out})}$ (right) as a function of the initial discord between modes 2 and 3. In both panels the red-dashed lines denote the curves for transmissivity equal to 15%, the solid blue lines are for the balanced case, and the green dot-sashed lines for transmissivity 85%.

distinguishable. If we put two polarization filters after the BS, we can select beams with a fixed polarization $\alpha = H, V$, which are Gaussian states with CM given by (we set $\tau = 1/2$) $\Sigma_{\mathrm{out}}^{(H)} = (1/2)\begin{pmatrix} \boldsymbol{\sigma}_1^{(H)}+\boldsymbol{\sigma}_0 & \boldsymbol{\sigma}_0-\boldsymbol{\sigma}_1^{(H)} \\ \boldsymbol{\sigma}_0-\boldsymbol{\sigma}_1^{(H)} & \boldsymbol{\sigma}_1^{(H)}+\boldsymbol{\sigma}_0 \end{pmatrix}$ and $\Sigma_{\mathrm{out}}^{(V)} = (1/2)\begin{pmatrix} \boldsymbol{\sigma}_2^{(V)}+\boldsymbol{\sigma}_0 & \boldsymbol{\sigma}_2^{(V)}-\boldsymbol{\sigma}_0 \\ \boldsymbol{\sigma}_2^{(H)}-\boldsymbol{\sigma}_0 & \boldsymbol{\sigma}_2^{(V)}+\boldsymbol{\sigma}_0 \end{pmatrix}$, respectively, where $\sigma_k^{(\alpha)}$ are the same as σ_k, $k = 1, 2$, but now we emphasize the polarization dependence $\alpha = H, V$. Thanks to the polarization, we can clearly distinguish the correlations coming from the off diagonal $\propto (\boldsymbol{\sigma}_0 - \boldsymbol{\sigma}_1^{(H)})$ and $\propto (\boldsymbol{\sigma}_2^{(V)} - \boldsymbol{\sigma}_0)$. In this experiment, the physical action that we want to reveal is the erasure of the information about the polarization. This is done as in the quantum erasure protocol for discrete variables [25]: we insert two polarization rotators set at 45° after the BS and on the path of mode 3. After filtering, the resulting three H-polarized (V-polarized) modes have the same CM as in Eq. (1) for a suitable choice of the input total energy and squeezing fraction. We measured the second-order correlation coefficient between the two beams before and after the BS without acting on their polarizations, obtaining $c_{1,2}^{(H,V,\mathrm{in})} = -0.01$ and $c_{1,2}^{(H,V,\mathrm{out})} = 0.97$, respectively. In this case, because of the orthogonal polarizations, the beams do not interfere each other, and each input is divided into two correlated parties. After the interaction, all the beams are projected to the 45° polarization basis by means of three half-wave plates $\lambda_{\mathrm{out},k}$ and three polarizers P_k oriented in the H direction, $k = 1, 2, 3$ [see Fig. 1(b)]. Again, we measure correlation $c_{1,2}^{(\mathrm{out})}(@45^\circ)$, $c_{1,3}^{(\mathrm{out})}(@45^\circ)$, and $c_{2,3}^{(\mathrm{out})}(@45^\circ)$ between the corresponding beams. We then perform the same measurement projecting the modes onto the vertical basis removing the half-wave plates [$c_{1,2}^{(\mathrm{out})}(@V)$, $c_{1,3}^{(\mathrm{out})}(@V)$, and $c_{2,3}^{(\mathrm{out})}(@V)$]. In fact, the erasure of information about polarization affects correlations between beam 1 and 2 (see Table 2). The correlations $c_{1,2}^{(H,V,\mathrm{out})} = 0.97$ reduce to $c_{1,2}^{(\mathrm{out})}(@45^\circ) = 0.10$ when the information about initial polarization is lost. Analogously, beams 2 and 3, which show high correlations in V basis, $c_{2,3}^{(\mathrm{out})}(@V) = 0.97$, loose correlation in the 45° basis [$c_{2,3}^{(\mathrm{out})}(@45^\circ) = 0.53$], while the uncorrelated beam 1 and 3 gain correlation. Also in this case, the use of discordant states for beams 2 and 3 reveals the physical action, here the erasure, performed on beams 1 and 2, despite the fact that this cannot be done by inspecting the involved beams only.

In summary, while a pair of uncorrelated Gaussian states mixed in a BS produce, in general, a correlated

Table 2. Measured Correlations between the Beams h, k after the BS with Polarizers[a] at 45° and at V

$c_{1,2}^{(\mathrm{out})}(@45^\circ)$	$c_{1,3}^{(\mathrm{out})}(@45^\circ)$	$c_{2,3}^{(\mathrm{out})}(@45^\circ)$
$0.10_{[-0.25;0.46]}$	$0.54_{[0.27;0.80]}$	$0.53_{[0.24;0.81]}$
$c_{1,2}^{(\mathrm{out})}(@V)$	$c_{1,3}^{(\mathrm{out})}(@V)$	$c_{2,3}^{(\mathrm{out})}(@V)$
$0.97_{[0.87;1.00]}$	$-0.01_{[-0.38;0.36]}$	$0.97_{[0.84;1.00]}$

[a]Without polarization selection we have $c_{1,2}^{(H,V,\mathrm{in})} = -0.01_{[-0.38;0.35]}$ and $c_{1,2}^{(H,V,\mathrm{out})} = 0.97_{[0.86;1.00]}$, see text for details.

bipartite state, two equal Gaussian states do not. No correlations appear at the output, and the interference cannot be detected looking at the two beams only. We have proved theoretically and experimentally that this task may be pursued using an ancillary beam, prepared in a discordant state with one of the two inferring beams, thus confirming that discord can be consumed to encode information that can only be accessed by coherent quantum interactions [26]. Our experiment involves thermal states and the results show that Gaussian discordant states, even when they show a positive Glauber P-function, may be useful to achieve specific tasks.

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