

THE BALANCE OF QUANTUM CORRELATIONS FOR A CLASS OF FEASIBLE TRIPARTITE CONTINUOUS VARIABLE STATES

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We address the balance of quantum correlations for continuous variable (CV) states. In particular, we consider a class of feasible tripartite CV pure states and explicitly prove two Koashi–Winter-like conservation laws involving Gaussian entanglement of formation (EoF), Gaussian quantum discord and sub-system Von Neumann entropies. We also address the class of tripartite CV mixed states resulting from the propagation in a noisy environment, and discuss how the previous equalities evolve into inequalities.

Keywords: Continuous variables; quantum correlations; entanglement and quantum discord.

1. Introduction

Multipartite quantum correlations in continuous variable (CV) systems are valuable resources for quantum technology.^{1–8} Fully inseparable three-mode Gaussian states^{9–11} have been proposed to realize cloning at distance,^{12,13} and experimental schemes to generate multimode CV entangled states have been already suggested and demonstrated.^{14–20}

More recently, more general quantum correlations in bipartite Gaussian states have been analyzed: Gaussian quantum discord has been introduced^{21,22} and experimentally investigated.^{23–25} For discrete variables several interesting relations among the different measures of quantum correlations have been introduced^{26–33} with the aim of clarifying their different meaning and role.³⁴

The dimension of the Hilbert space does not appear to play a crucial role and, indeed, balance of correlations for CV systems has been derived in terms of the Renyi entropy.³⁵ In addition, balance of correlations in quantum measurements,

and during the propagation in noisy channels, has been discussed both for discrete and CVs.^{36–41}

Motivated by these results, in this paper we investigate whether some form of conservation laws for quantum correlations may be established for CV systems and Gaussian measures of correlations. In particular, we consider a specific class of feasible pure CV tripartite states, and analyze in detail the balance of Gaussian correlations among the bipartite subsystems. We found that Koashi–Winter-like relations, involving EoF, quantum discord and system entropies, may be generalized to CVs using Gaussian measures of quantum correlations, at least for the class of feasible states that we took into account. The results are encouraging enough to suggest a direct experimental verification and to foster investigations of correlation balance in more general Gaussian states. Our results may also be taken as an argument in favor of the conjecture that Gaussian quantum discord is the quantum discord for Gaussian states⁴² and contribute to the ongoing discussion on the nature of quantum correlations in CV systems.⁴³

The paper is structured as follows. In Sec. 2 we introduce the class of feasible tripartite Gaussian states we are going to consider. In Sec. 3 we present the conservation laws for quantum correlations, whereas in Sec. 4 we consider the propagation in a noisy environment, and discuss how the previous equalities evolve into inequalities. Section 5 closes the paper with some concluding remarks.

2. A Class of Feasible Tripartite Gaussian States

Let us consider three-modes of a bosonic field, coupled through the interaction Hamiltonian

$$H = \gamma_1 a^\dagger c^\dagger + \gamma_2 b^\dagger c + \text{h.c.} \quad (1)$$

The Hamiltonian H describes, e.g., two interlinked bilinear interactions taking place among three-modes of the radiation field. In fact, it has been studied long ago^{44,45} for the description of parametric processes in $\chi^{(2)}$ media, assuming a suitable configuration satisfying phase-matching conditions. Interaction schemes described by H has been realized using a single $\chi^{(2)}$ nonlinear crystal where the two interactions take place simultaneously.¹⁹ The three-mode entanglement generated by H may be indeed exploited to realize optimal cloning at distance of coherent states.²⁰ Analogue interaction schemes may be realized in condensate systems.^{46,47}

The Hamiltonian in Eq. (1) admits the constant of motion

$$K(t) \equiv N_A(t) - N_B(t) - N_C(t) \equiv K(0),$$

where $N_j(t)$ denotes the average number of photons in the j th mode. Starting from the vacuum $|\mathbf{0}\rangle \equiv |0\rangle_1 \otimes |0\rangle_2 \otimes |0\rangle_3$ we have $K(0) = 0$ and thus $N_A(t) = N_B(t) + N_C(t) \forall t$. The expressions for $N_j(t)$ can be obtained by the Heisenberg evolution of the field operators; upon introducing $\Omega = \sqrt{|\gamma_2|^2 - |\gamma_1|^2}$ we have

$$N_A = N_B + N_C, \quad N_B = \frac{|\gamma_1|^2 |\gamma_2|^2}{\Omega^4} [\cos \Omega t - 1]^2, \quad N_C = \frac{|\gamma_1|^2}{\Omega^2} \sin^2(\Omega t).$$

The evolved state $|\mathbf{T}\rangle = U_t|\mathbf{0}\rangle$ reads as follows

$$|\mathbf{T}\rangle = \frac{1}{\sqrt{1+N_A}} \sum_{nm} \left(\frac{N_B}{1+N_A}\right)^{n/2} \left(\frac{N_C}{1+N_A}\right)^{m/2} \sqrt{\frac{(n+m)!}{n!m!}} |n+m, n, m\rangle, \quad (2)$$

where $U_t = \exp(-iHt)$ is the evolution operator. The states $|\mathbf{T}\rangle$ are Gaussian states, since they are generated from the vacuum by a bilinear Hamiltonian. Upon introducing the canonical operators q_k and p_k , $k = A, B, C$, and the vector $R = (q_A, p_A, q_B, p_B, q_C, p_C)^T$, it is straightforward to prove that the mean values of the canonical operators are zero $\langle R \rangle \equiv \langle \mathbf{T}|R|\mathbf{T}\rangle = 0$, whereas the covariance matrix (CM), whose elements are given by

$$\Sigma_{hk} = \frac{1}{2} \langle \{R_k, R_h\} \rangle - \langle R_k \rangle \langle R_h \rangle,$$

can be written in the following block form:

$$\Sigma_{\mathbf{T}} = \begin{pmatrix} \mathbf{D}_A & \mathbf{O}_{AB} & \mathbf{O}_{AC} \\ \mathbf{O}_{AB} & \mathbf{D}_B & \mathbf{O}_{BC} \\ \mathbf{O}_{AC} & \mathbf{O}_{BC} & \mathbf{D}_C \end{pmatrix}. \quad (3)$$

where:

$$\mathbf{D}_A = \left(N_A + \frac{1}{2}\right) \mathbb{1}, \quad \mathbf{D}_B = \left(N_B + \frac{1}{2}\right) \mathbb{1}, \quad \mathbf{D}_C = \left(N_C + \frac{1}{2}\right) \mathbb{1}, \quad (4a)$$

$$\mathbf{O}_{AB} = \sqrt{N_B(N_A+1)} \mathbb{P}, \quad \mathbf{O}_{AC} = \sqrt{N_C(N_A+1)} \mathbb{P}, \quad \mathbf{O}_{CB} = \sqrt{N_B N_C} \mathbb{1}, \quad (4b)$$

with $N_A = N_B + N_C$, $\mathbb{1} = \text{Diag}(1, 1)$ and $\mathbb{P} = \text{Diag}(1, -1)$. Since partial trace is a Gaussian operation the single-mode partial traces:

$$\varrho_A = \text{Tr}_{BC} [|\mathbf{T}\rangle\langle\mathbf{T}|], \quad \varrho_B = \text{Tr}_{AC} [|\mathbf{T}\rangle\langle\mathbf{T}|], \quad \varrho_C = \text{Tr}_{AB} [|\mathbf{T}\rangle\langle\mathbf{T}|],$$

and the two-mode ones

$$\varrho_{AB} = \text{Tr}_C [|\mathbf{T}\rangle\langle\mathbf{T}|], \quad \varrho_{AC} = \text{Tr}_B [|\mathbf{T}\rangle\langle\mathbf{T}|], \quad \varrho_{BC} = \text{Tr}_A [|\mathbf{T}\rangle\langle\mathbf{T}|],$$

are Gaussian states as well. The corresponding CM may be obtained by dropping the corresponding lines in $\Sigma_{\mathbf{T}}$. The single-mode partial traces thus have diagonal CM $\sigma_k = \mathbf{D}_k$, corresponding to thermal states:

$$\nu(N_k) = \sum_p \frac{N_k^p}{(N_k+1)^{p+1}} |p\rangle\langle p|, \quad k = A, B, C, \quad (5)$$

and the corresponding von Neumann entropies are given by

$$\mathcal{S}_k = f\left(\sqrt{\det \sigma_k}\right) = f\left(N_k + \frac{1}{2}\right),$$

where:

$$f(x) = \left(x + \frac{1}{2}\right) \ln\left(x + \frac{1}{2}\right) - \left(x - \frac{1}{2}\right) \ln\left(x - \frac{1}{2}\right) \quad \text{for } x > \frac{1}{2}. \quad (6)$$

The CM of the two-mode partial traces are given by

$$\sigma_{AB} = \begin{pmatrix} \mathbf{D}_A & \mathbf{O}_{AB} \\ \mathbf{O}_{AB} & \mathbf{D}_B \end{pmatrix}, \quad \sigma_{AC} = \begin{pmatrix} \mathbf{D}_A & \mathbf{O}_{AC} \\ \mathbf{O}_{AC} & \mathbf{D}_B \end{pmatrix}, \quad \sigma_{BC} = \begin{pmatrix} \mathbf{D}_B & \mathbf{O}_{BC} \\ \mathbf{O}_{BC} & \mathbf{D}_C \end{pmatrix} \quad (7)$$

and are already in the so-called standard block form (to which any CM may be brought by means of local symplectic transformations)

$$\sigma = \begin{pmatrix} \mathbf{A} & \mathbf{C} \\ \mathbf{C} & \mathbf{B} \end{pmatrix} \quad \mathbf{A} = \text{Diag}(a, a), \quad \mathbf{B} = \text{Diag}(b, b), \quad \mathbf{C} = \text{Diag}(c_1, c_2).$$

For σ_{AB} and σ_{AC} we have $c_1 = -c_2$ whereas for σ_{BC} we have $c_1 = c_2$. This means that ϱ_{AB} and ϱ_{AC} are squeezed thermal states (STS) of the form:

$$\varrho_{AB} = S(r)\nu(N_C) \otimes |0\rangle\langle 0|S^\dagger(r), \quad \sinh^2 r = \frac{N_B}{1 + N_C}, \quad (8)$$

$$\varrho_{AC} = S(r)\nu(N_B) \otimes |0\rangle\langle 0|S^\dagger(r), \quad \sinh^2 r = \frac{N_C}{1 + N_B}, \quad (9)$$

respectively, $S(r) = \exp\{r(a^\dagger b^\dagger - ab)\}$ being the two-mode squeezing operator, whereas ϱ_{BC} corresponds to a mixed thermal state (MTS) that may equivalently expressed as (recall that $N_A = N_B + N_C$):

$$\varrho_{BC} = U(\phi)\nu(N_A) \otimes |0\rangle\langle 0|U^\dagger(\phi), \quad \cos^2 \phi = \frac{N_B}{N_B + N_C}, \quad (10)$$

$$\varrho_{BC} = U(\phi) |0\rangle\langle 0| \otimes \nu(N_A)U^\dagger(\phi), \quad \cos^2 \phi = \frac{N_C}{N_B + N_C}, \quad (11)$$

where $U(\phi) = \exp\{\phi(a^\dagger b - ab^\dagger)\}$ is a bilinear mixing operator describing e.g., the action of a beam splitter.

3. Balance of Correlations

In order to investigate the correlations between the modes of the state $|\mathbf{T}\rangle$, we introduce the symplectic eigenvalues $\lambda_\pm^{(hk)}$ of the two-mode state ϱ_{hk} and the minimum symplectic eigenvalue $\tilde{\lambda}_-^{(hk)}$ of its partial transpose, which may be obtained as⁴⁸:

$$\lambda_\pm^{(hk)} = 2^{-1/2} \sqrt{I_h + I_k + 2I_{hk} \pm \sqrt{(I_h + I_k + 2I_{hk})^2 - 4J_{hk}}}, \quad (12)$$

$$\tilde{\lambda}_-^{(hk)} = 2^{-1/2} \sqrt{I_h + I_k - 2I_{hk} - \sqrt{(I_h + I_k - 2I_{hk})^2 - 4J_{hk}}},$$

respectively, where

$$I_k = \det \mathbf{D}_k, \quad I_{hk} = \det \mathbf{O}_{hk}, \quad J_{hk} = \det \sigma_{hk}.$$

Note that:

$$S_k = f(\sqrt{I_k}). \quad (13)$$

whereas the von Neumann entropies of the two-mode partial traces are given by

$$\mathcal{S}_{hk} = f(\lambda_+^{(hk)}) + f(\lambda_-^{(hk)}). \quad (14)$$

The positivity of the state is equivalent to $\lambda_{\pm}^{(hk)} \geq (1/2)$ and the positivity of the partially transposed state to $\tilde{\lambda}_{\pm}^{(hk)} \geq (1/2)$. The bipartite Gaussian state ϱ_{hk} is thus entangled if and only if $0 \leq \tilde{\lambda}_{-}^{(hk)} < (1/2)$. In our case we have

$$\lambda_{-}^{(hk)} = \frac{1}{2} \quad \text{and} \quad \lambda_{+}^{(hk)} = \frac{1}{2} + N_j \quad (15)$$

and thus the von Neumann entropy of any two-mode partial trace equals the von Neumann entropy of the remaining (single) mode, in formula:

$$\mathcal{S}_{hk} = \mathcal{S}_j \quad (16)$$

where, in both Eqs. (15) and (16), we have $h \neq k \neq j$ and $h, k, j = A, B, C$.

For the symplectic eigenvalues of the partial transposes we have

$$\begin{aligned} \tilde{\lambda}_{-}^{AB} = \frac{1}{\sqrt{2}} \left[\left(N_A + \frac{1}{2} \right)^2 + 2N_B(N_A + 1) + \left(N_B + \frac{1}{2} \right)^2 \right. \\ \left. - (N_A + N_B + 1) \sqrt{4N_B(N_A + 1) + N_C^2} \right]^{1/2}, \end{aligned} \quad (17)$$

$$\tilde{\lambda}_{-}^{AC} = \tilde{\lambda}_{-}^{AB}(N_B \leftrightarrow N_C), \quad (18)$$

$$\tilde{\lambda}_{-}^{BC} = \left[(N_B - N_C)^2 + N_B + N_C + \frac{1}{2} - |N_B - N_C| \sqrt{(N_A + 1)^2 - 4N_B N_C} \right]^{1/2}, \quad (19)$$

from which it is easy to see that for any value of N_B and N_C we have

$$0 \leq \tilde{\lambda}_{-}^{(AB)} \leq \frac{1}{2}, \quad 0 \leq \tilde{\lambda}_{-}^{(AC)} \leq \frac{1}{2}, \quad \tilde{\lambda}_{-}^{(BC)} \geq \frac{1}{2},$$

i.e., ϱ_{AB} and ϱ_{AC} are entangled states whereas ϱ_{BC} is separable. If $0 \leq \tilde{\lambda}_{-}^{(hk)} < (1/2)$, i.e., ϱ_{hk} is entangled, then the Gaussian EoF is given by

$$E_{hk}(\varrho) = f(y_{hk}),$$

where^{49,50}:

$$y_{hk} = \frac{(\sqrt{I_h} + \sqrt{I_k})(\sqrt{I_h I_k} - |I_{hk}| + 1/4) - 2\sqrt{|I_{hk}| \tilde{J}_{hk}}}{(\sqrt{I_h} + \sqrt{I_k})^2 - 4|I_{hk}|}, \quad (20)$$

with $\tilde{J}_{hk} = \det(\sigma_{hk} + (1/2)\Omega)$, and the symplectic form Ω given by

$$\Omega = \omega \oplus \omega, \quad \omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

In our case, ϱ_{AB} and ϱ_{AC} are entangled and we have:

$$y_{AB} = y_{BA} = \frac{1}{2} + \frac{N_B}{1 + N_C}, \quad y_{AC} = y_{CA} = \frac{1}{2} + \frac{N_C}{1 + N_B}, \quad (21)$$

respectively. On the other hand, being the state ϱ_{BC} separable, $y_{BC} = y_{CB} = 1/2$ and thus its EoF is zero.

For MTS and STS the Gaussian quantum discords $\mathcal{D}^\rightarrow(\varrho_{hk}) \equiv \mathcal{D}_{hk}$ and $\mathcal{D}^\leftarrow(\varrho_{hk}) \equiv \mathcal{D}_{kh}$ (measurements performed on the first and second mode respectively) may be expressed as²¹:

$$\mathcal{D}_{hk} = f(\sqrt{I_h}) + f\left(\sqrt{I_k} - \frac{2|I_{hk}|}{1 + 2\sqrt{I_h}}\right) - f(\lambda_-^{(hk)}) - f(\lambda_+^{(hk)}), \quad (22a)$$

$$\mathcal{D}_{kh} = f(\sqrt{I_k}) + f\left(\sqrt{I_h} - \frac{2|I_{hk}|}{1 + 2\sqrt{I_k}}\right) - f(\lambda_-^{(hk)}) - f(\lambda_+^{(hk)}). \quad (22b)$$

Starting from Eqs. (22) we can write a conservation law for the quantum correlations of the two-modes state ϱ_{hk} . At first, we rewrite Eqs. (22) as follows:

$$\mathcal{D}_{hk} + h(\lambda_+^{(hk)}) = \mathcal{S}_h + f\left(\sqrt{I_k} - \frac{2|I_{hk}|}{1 + 2\sqrt{I_h}}\right) - f(\lambda_-^{(hk)}), \quad (23a)$$

$$\mathcal{D}_{kh} + h(\lambda_+^{(hk)}) = \mathcal{S}_k + f\left(\sqrt{I_h} - \frac{2|I_{hk}|}{1 + 2\sqrt{I_k}}\right) - f(\lambda_-^{(hk)}), \quad (23b)$$

where we used Eq. (13). Besides, for the state (2), Eqs. (15) say that $f(\lambda_-^{(hk)}) = 0$ and $f(\lambda_+^{(hk)}) = \mathcal{S}_j$, $h \neq k \neq j$, where we used the identity $\lambda_+^{(hk)} = \sqrt{I_j}$. Furthermore, it is straightforward to verify that:

$$\sqrt{I_h} - \frac{2|I_{hk}|}{1 + 2\sqrt{I_k}} = y_{hj} \quad (h \neq k \neq j) \quad (24)$$

with y_{hj} given in Eqs. (21). Summarizing, for the state $|\mathbf{T}\rangle$ in Eq. (2), Eqs. (23) leads to the following conservation laws

$$\mathcal{D}_{hk} + \mathcal{S}_j = \mathcal{S}_h + E_{kj}, \quad (25a)$$

$$\mathcal{D}_{hk} + \mathcal{S}_{hk} = \mathcal{S}_h + E_{kj}, \quad (25b)$$

with $h \neq k \neq j$ and $h, k, j = A, B, C$.

The above equalities are explicitly showing that Koashi–Winter-like conservation laws for quantum correlations may be written for CV systems using Gaussian measures for quantum correlations, at least for the class of feasible states described by Eq. (2). Equation (25b) is the direct counterpart of the balance of correlations obtained for discrete variables, and thus we may expect, or conjecture, that it is of general validity for CV systems. On the other hand, Eq. (25a) is due to the specific properties of the state $|\mathbf{T}\rangle$. Notice also that the validity of Eq. (25b) may represent an argument in favor of the conjecture that Gaussian quantum discord is the quantum discord for Gaussian states.⁴² Finally, we notice that both Eqs. (25a)

and (25b) represent an independent check of the formula for the EoF in the case of nonsymmetric states.⁵⁰

4. Balance of Correlations in the Presence of Noise

In this section we investigate how, and to which extent, the conservation laws (25) addressed in Sec. 3 are modified by the propagation of the state in noisy channels. We assume that the three-modes evolve through three identical uncorrelated noisy channels and that the Markovian approximation is valid. The propagation is thus described by the following Master equation

$$\dot{\varrho}_t = \frac{\gamma}{2} \sum_{k=1,2,3} \{(N_{\text{th}} + 1)\mathcal{L}[a_k] + N_{\text{th}}\mathcal{L}[a_k^\dagger]\} \varrho_t, \quad (26)$$

where ϱ_t is the density matrix of the tripartite system described by the field operators $a_1 = a$, $a_2 = b$ and $a_3 = c$, $\mathcal{L}[a]\varrho_t = 2a\varrho_t a^\dagger - a^\dagger a\varrho_t - \varrho_t a^\dagger a$ is the Lindblad superoperator, γ is the overall damping rate, while N_{th} represents the effective number of photons of the noisy channels.² Upon preparing the three-modes in the initial state $|\mathbf{T}\rangle$, the Gaussian nature is preserved during the evolution and the evolved CM Σ_t can be written as⁴⁸:

$$\Sigma_\tau = e^{-\tau} \Sigma_{\mathbf{T}} + (1 - e^{-\tau}) \Sigma_\infty \quad (27)$$

where we introduced $\tau = \gamma t$, $\Sigma_{\mathbf{T}}$ is the CM of the initial state given in Eq. (3) and $\Sigma_\infty = (1/2)(1 + 2N_{\text{th}})\mathbb{1}_4$ is the asymptotic CM, $\mathbb{1}_4$ being the 4×4 identity matrix.

By using the evolved CM (27) and its time-dependent local symplectic invariants, we can calculate the quantities appearing in Eqs. (23). In particular, now one has:

$$f(\lambda_-^{(hk)}) \geq 0 \quad \text{and} \quad f(\lambda_+^{(hk)}) \leq \mathcal{S}_j \quad (28)$$

and, in turn, the equalities (25) changes into inequalities. The analytic expressions of the evolved conservation laws may be evaluated analytically, but they are quite clumsy and are not reported here explicitly.

In order to study the evolution of the law (25a) we define the following function:

$$\Delta_{hk}(\tau) = [\mathcal{D}_{hk}(\tau) + \mathcal{S}_j(\tau)] - [\mathcal{S}_h(\tau) + E_{kj}(\tau)], \quad (29)$$

where all the involved quantities are calculated starting from the results of the Sec. 3 but with the evolved state ϱ_t . As an example, Fig. 1 is a region-plot $\Delta_{hk}(\tau)$ as functions of τ and $\delta = N_B - N_C$ for given N_B and N_{th} : depending on the values of the involved parameters, $\Delta_{hk}(\tau)$ can be positive or negative. In particular, Δ_{AB} and Δ_{AC} are always negative, Δ_{BA} whereas Δ_{CA} can change the sign (remarkably, this holds true also for $N_{\text{th}} = 0$). For the case of symmetric states. i.e., $\delta = 0$, one can write the following set of inequalities:

$$\mathcal{D}_{AB} + \mathcal{S}_C \leq \mathcal{S}_A, \quad \mathcal{D}_{BA} + \mathcal{S}_C \geq \mathcal{S}_B + E_{AC}, \quad (30a)$$

$$\mathcal{D}_{AC} + \mathcal{S}_B \leq \mathcal{S}_A, \quad \mathcal{D}_{CA} + \mathcal{S}_B \geq \mathcal{S}_C + E_{AB}, \quad (30b)$$

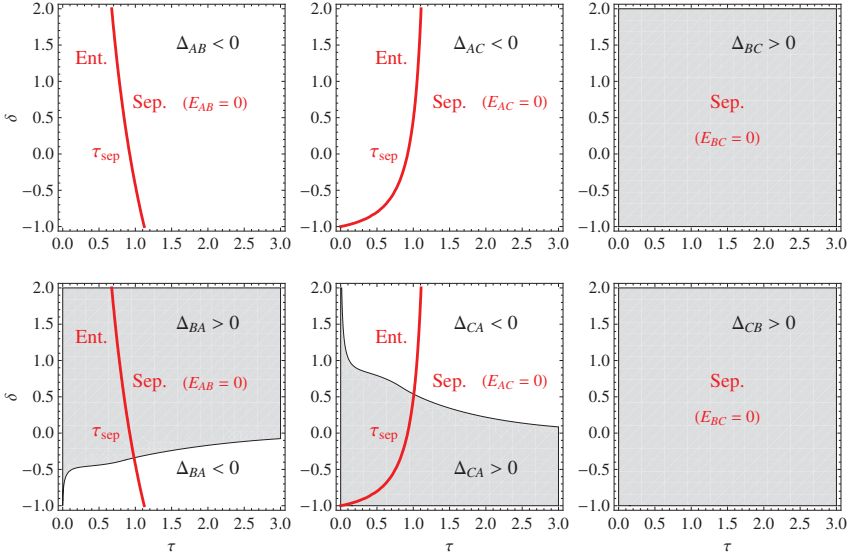


Fig. 1. (Color online) Balance of the correlations in the presence of dissipation and thermal noise. Region plots of Δ_{hk} , $h \neq k$ with $h, k = A, B, C$, as functions of $\delta = N_B - N_C$ of $\tau = \gamma t$. We set $N_B = 1$ and $N_{th} = 0.2$. Gray and white regions refer to the corresponding $\Delta_{hk} > 0$ and $\Delta_{hk} < 0$, respectively. The red line represents the separability time τ_{sep} : if $\tau \geq \tau_{sep}$ then $\tilde{\lambda}_{-}^{(hk)} \geq (1/2)$ and the state $\varrho_{hk}(t)$ is no longer entangled (Ent.) and becomes separable (Sep.), i.e., $E_{hk} = 0$. See the text for details.

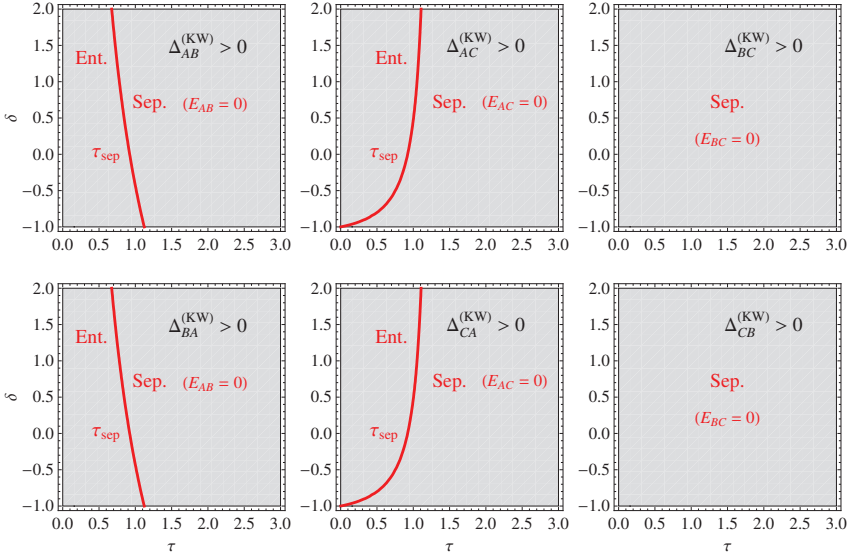


Fig. 2. (Color online) Balance of the correlations in the presence of dissipation and thermal noise. Region plots of $\Delta_{hk}^{(KW)}$, $h \neq k$ with $h, k = A, B, C$, as functions of $\delta = N_B - N_C$ of $\tau = \gamma t$, for the same choice of the other involved parameters as in Fig. 1. Note that one always has $\Delta_{hk} > 0$ (gray regions). The red line still represents the separability time τ_{sep} . See the text for details.

$$\mathcal{D}_{BC} + \mathcal{S}_A \geq \mathcal{S}_B + E_{CA}, \quad \mathcal{D}_{CB} + \mathcal{S}_A \geq \mathcal{S}_C + E_{BA}, \quad (30c)$$

where we used $E_{BC} = E_{CB} = 0$.

Analogously, to address the evolution of the conservation law (25b), we introduce the function:

$$\Delta_{hk}^{(KW)}(\tau) = [\mathcal{D}_{hk}(\tau) + \mathcal{S}_{hk}(\tau)] - [\mathcal{S}_h(\tau) + E_{kj}(\tau)]. \quad (31)$$

As one can see in Fig. 2, where we plot $\Delta_{hk}^{(KW)}(\tau)$ for the same choice of parameters as in Fig. 1, now one finds the following inequality holding for all the bipartitions, and any value of the interaction time,

$$\mathcal{D}_{hk}(\tau) + \mathcal{S}_{hk}(\tau) \geq \mathcal{S}_h(\tau) + E_{kj}(\tau), \quad (32)$$

$h \neq k \neq j$ and $h, k, j = A, B, C$. Inequality (32) generalizes to CV and Gaussian measures of correlation, the inequality discussed in Ref. 32 for the discrete case.

5. Conclusion

In conclusion, we have proved that the balance of correlations originally investigated for three-qubit systems, involving EoF, quantum discord and single-system entropies is valid also for a feasible class of tripartite Gaussian CV states, upon using Gaussian measures of quantum correlations. Furthermore, in the presence of dissipation and thermal noise, the balance turns into inequalities between the previous quantities, depending on the actual values of the involved parameters. The results are encouraging enough to suggest a direct experimental verification and to foster investigations of correlation balance in more general Gaussian states.

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