Selective cloning of Gaussian states by linear optics

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(Received 7 May 2007; published 7 August 2007)

We investigate the performance of a selective cloning machine based on linear optical elements and Gaussian measurements, which allows one to clone at will one of the two incoming input states. This machine is a complete generalization of a $1 \rightarrow 2$ cloning scheme demonstrated by Andersen *et al.* [Phys. Rev. Lett. **94**, 240503 (2005)]. The input-output fidelity is studied for a generic Gaussian input state, and the effect of nonunit quantum efficiency is also taken into account. We show that, if the states to be cloned are squeezed states with known squeezing parameter, then the fidelity can be enhanced using a third suitable squeezed state during the final stage of the cloning process. A binary communication protocol based on the selective cloning machine is also discussed.

DOI: 10.1103/PhysRevA.76.022305 PACS number(s): 03.67.Hk, 03.65.Ta, 42.50.Lc

I. INTRODUCTION

Basic laws of quantum mechanics do not allow the generation of exactly alike copies of an unknown quantum state [1–4]. However, approximate copies can be obtained by using devices called quantum cloning machines [5]. The first of such devices was studied to deal with qubits, and then a continuous-variable (CV) [6] analog was developed [7,8]. Thereafter, CV optimal Gaussian cloners of coherent states based on two quite different approaches were proposed: the one relies on a single phase-insensitive parametric amplifier [9,10]; the other, which has been also experimentally realized, is built around a feedforward loop [11]. On the other hand, the latter is much simpler than the former, overcoming the difficulty of implementing an efficient phase-insensitive amplifier operating at the fundamental limit. Since the setup of this device is based only on linear components, throughout this paper we will refer to it as the *linear cloning machine*. Reference [12] investigated the performance of the linear cloning machine when the input state was a single generic Gaussian state (coherent, squeezed coherent, or displaced thermal state), taking into account the effect of fluctuation of the input state covariance matrix, variation in the setup beam splitter ratios, and losses in the detection scheme.

The aim of this paper is to show that the protocol used by the linear cloning machine to clone a single input Gaussian state can be generalized in order to achieve the *selective* cloning of a state chosen between two inputs. The possibility to select one of two states may have useful implementations in binary communication systems, where the two bits are encoded in two quantum states, and the goal of the communication is to send the information from one sender to two receivers. We will address this problem in the final part of the paper.

The paper is structured as follows. In Sec. II, we describe the selective cloning machine and describe the evolution of the input states by means of the characteristic function approach. In Sec. III, the requirements of selective symmetric cloning are exploited and the input-output fidelity is studied. Section IV investigates the possibility of enhancing the cloning fidelity, and in Sec. V a possible application of the selective cloning machine to $1 \rightarrow 2$ binary communication is proposed. Finally, Sec. VI closes the paper with some concluding remarks.

II. THE SELECTIVE LINEAR CLONING MACHINE

The selective cloning machine based on linear optics and Gaussian measurement is schematically depicted in Fig. 1. Two input states, denoted by the density operators ϱ_k, $k=1,2$, are mixed at a beam splitter (BS) with transmissivity τ_1. A Gaussian measurement with quantum efficiency η is performed on one of the outgoing beams, the outcome of the measurement being the complex number z. According to these outcomes, the other beam undergoes a displacement by an amount gz, g being a suitable electronic amplification factor. Finally, the two output states, denoted by the density operators ς_1 and ς_2, are obtained by dividing the displaced state using another BS with transmissivity τ_2. When $\tau_1=\tau_2=1/2$, $g=1$, $\eta=1$, $\varrho_2=\varrho_3=|0\rangle\langle 0|$, and the Gaussian measure-

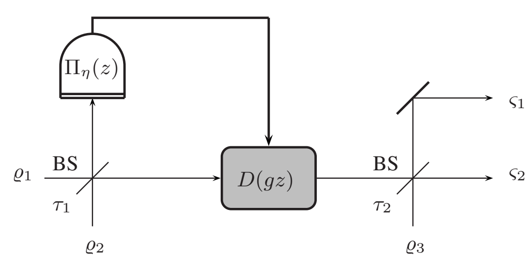

FIG. 1. Selective cloning of Gaussian states by linear optics: the two input states ϱ_k, $k=1,2$, are mixed at a beam splitter (BS) of transmissivity τ_1. One of the two emerging beams is measured by a measurement described by the positive operator valued measure (POVM) $\Pi_\eta(z)$ and the outcome z is forwarded to a modulator, which imposes a displacement gz on the other outgoing beam, g being a suitable amplification factor. Finally, the displaced state is mixed with the state ϱ_3 at a second beam splitter of transmissivity τ_2. The two outputs ς_1 and ς_2 from the beam splitter represents the two clones, which may be made approximately equal to either ϱ_1 or ϱ_2 by changing the gain g from $+1$ to -1.

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ment is an ideal double-homodyne detection, the scheme reduces to that of Ref. [11], which was shown to be optimal for Gaussian cloning of coherent states and has been investigated in Refs. [12–14]. In the following, we carry out a thorough description of the selective cloning machine using the characteristic function approach.

The characteristic function $\chi_k(\mathbf{\Lambda}_k)\equiv\chi[\varrho_k](\mathbf{\Lambda}_k)$ associated with a Gaussian state ϱ_k of mode $k=1,2,3$ (see Fig. 1) reads

$$\chi_k(\mathbf{\Lambda}_k)=\exp\left(-\frac{1}{2}\mathbf{\Lambda}_k^T\boldsymbol{\sigma}_k\mathbf{\Lambda}_k-i\mathbf{\Lambda}_k^T\boldsymbol{X}_k\right), \tag{1}$$

where $\mathbf{\Lambda}_k=(\mathsf{x}_k,\mathsf{y}_k)^T$, $(\cdots)^T$ denotes the transposition operation, $\boldsymbol{\sigma}_k$ is the covariance matrix, and $\boldsymbol{X}_k=\mathrm{Tr}[\varrho_k(\hat{x},\hat{y})^T]$ is the vector of mean values, $\hat{x}$ and $\hat{y}$ being the quadrature operators $\hat{x}=(1/\sqrt{2})(\hat{a}+\hat{a}^\dagger)$ and $\hat{y}=(i/\sqrt{2})(\hat{a}^\dagger-\hat{a})$, with $\hat{a}$ and $\hat{a}^\dagger$ being the field annihilation and creation operators. In turn, the initial two-mode state $\varrho=\varrho_1\otimes\varrho_2$ is Gaussian, and its two-mode characteristic function reads

$$\chi[\varrho](\mathbf{\Lambda})=\exp\left(-\frac{1}{2}\mathbf{\Lambda}^T\boldsymbol{\sigma}\mathbf{\Lambda}-i\mathbf{\Lambda}^T\boldsymbol{X}\right), \tag{2}$$

with

$$\boldsymbol{\sigma}=\left(\begin{array}{c|c}\boldsymbol{\sigma}_1 & \mathbf{0}\\ \hline \mathbf{0} & \boldsymbol{\sigma}_2\end{array}\right),\quad \boldsymbol{X}=(\boldsymbol{X}_1,\boldsymbol{X}_2)^T, \tag{3}$$

and $\mathbf{\Lambda}=(\mathbf{\Lambda}_1,\mathbf{\Lambda}_2)$. Under the action of the first BS, the state $\chi[\varrho](\mathbf{\Lambda})$ preserves its Gaussian form, namely,

$$\chi[\varrho](\mathbf{\Lambda})\rightsquigarrow\chi[\varrho'](\mathbf{\Lambda})=\exp\left(-\frac{1}{2}\mathbf{\Lambda}^T\boldsymbol{\sigma}\mathbf{\Lambda}-i\mathbf{\Lambda}^T\boldsymbol{X}\right), \tag{4}$$

where $\varrho'=U_{\mathrm{BS},1}\varrho_1\otimes\varrho_2U_{\mathrm{BS},1}^\dagger$, while its covariance matrix and mean values transform as [15]

$$\boldsymbol{\sigma}\rightsquigarrow\tilde{\boldsymbol{\sigma}}\equiv\boldsymbol{S}_{\mathrm{BS},1}^T\boldsymbol{\sigma}\boldsymbol{S}_{\mathrm{BS},1}=\left(\begin{array}{c|c}\boldsymbol{A} & \boldsymbol{C}\\ \hline \boldsymbol{C}^T & \boldsymbol{B}\end{array}\right), \tag{5}$$

$$\boldsymbol{X}\rightsquigarrow\tilde{\boldsymbol{X}}\equiv\boldsymbol{S}_{\mathrm{BS},1}^T\boldsymbol{X}=(\tilde{\boldsymbol{X}}_1,\tilde{\boldsymbol{X}}_2)^T, \tag{6}$$

$\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ are 2×2 matrices, and

$$\boldsymbol{S}_{\mathrm{BS},1}=\left(\begin{array}{c|c}\sqrt{\tau_1}\mathbb{1}_2 & \sqrt{1-\tau_1}\mathbb{1}_2\\ \hline -\sqrt{1-\tau_1}\mathbb{1}_2 & \sqrt{\tau_1}\mathbb{1}_2\end{array}\right) \tag{7}$$

is the symplectic transformation associated with the evolution operator $U_{\mathrm{BS},1}$ of the BS with transmission τ_1. Note that ϱ' is an entangled state if the set of states to be cloned consists of nonclassical states, i.e., states with singular Glauber P function or negative Wigner function [16,17].

The Gaussian measurement with quantum efficiency η (see Fig. 1) is described by the characteristic function

$$\chi[\Pi_\eta(z)](\mathbf{\Lambda}_2)=\frac{1}{\pi}\exp\left(-\frac{1}{2}\mathbf{\Lambda}_2^T\boldsymbol{\sigma}_M\mathbf{\Lambda}_2-i\mathbf{\Lambda}_2^T\boldsymbol{X}_M\right), \tag{8}$$

with $\boldsymbol{X}_M=\sqrt{2}[\mathrm{Re}(z),\mathrm{Im}(z)]^T$ and $\boldsymbol{\sigma}_M\equiv\boldsymbol{\sigma}_M(\eta)$. The probability of obtaining the outcome z is then given by

$$p_\eta(z)=\mathrm{Tr}_{12}[\varrho'\mathbb{I}\otimes\Pi_\eta(z)] \tag{9}$$

$$=\frac{1}{(2\pi)^2}\int_{\mathbb{R}^4}d^4\mathbf{\Lambda}\,\chi(\varrho')(\mathbf{\Lambda})\chi[\mathbb{I}\otimes\Pi_\eta(z)](-\mathbf{\Lambda}) \tag{10}$$

$$=\frac{\exp\left[-\frac{1}{2}(\boldsymbol{X}_M-\tilde{\boldsymbol{X}}_2)^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{X}_M-\tilde{\boldsymbol{X}}_2)\right]}{\pi\sqrt{\mathrm{Det}(\boldsymbol{\Sigma})}}, \tag{11}$$

where $\chi[\mathbb{I}\otimes\Pi_\eta(z)](\mathbf{\Lambda})\equiv\chi[\mathbb{I}](\mathbf{\Lambda}_1)\chi[\Pi_\eta(z)](\mathbf{\Lambda}_2)$, $\chi[\mathbb{I}](\mathbf{\Lambda}_1)=2\pi\delta^{(2)}(\mathbf{\Lambda}_1)$, and $\delta^{(2)}(\zeta)$ is the complex Dirac delta function. We also introduced the 2×2 matrix $\boldsymbol{\Sigma}=\boldsymbol{B}+\boldsymbol{\sigma}_M$.

The conditional state ϱ_c of the other outgoing beam, obtained when the outcome of the measurement is z, i.e.,

$$\varrho_c=\frac{\mathrm{Tr}_2[\varrho'\Pi_\eta(z)]}{p_\eta(z)}, \tag{12}$$

has the following characteristic function (for the sake of clarity we explicitly write the dependence on $\mathbf{\Lambda}_1$ and $\mathbf{\Lambda}_2$):

$$\chi[\varrho_c](\mathbf{\Lambda}_1)=\int_{\mathbb{R}^2}d^2\mathbf{\Lambda}_2\frac{\chi[\varrho'](\mathbf{\Lambda}_1,\mathbf{\Lambda}_2)\chi[\Pi_\eta(z)](-\mathbf{\Lambda}_2)}{p_\eta(z)} \tag{13}$$

$$=\exp\left\{-\frac{1}{2}\mathbf{\Lambda}_1^T(\boldsymbol{A}-\boldsymbol{C}\boldsymbol{\Sigma}^{-1}\boldsymbol{C}^T)\mathbf{\Lambda}_1 - i\mathbf{\Lambda}_1^T[\boldsymbol{C}\boldsymbol{\Sigma}^{-1}(\boldsymbol{X}_M-\tilde{\boldsymbol{X}}_2)+\tilde{\boldsymbol{X}}_1]\right\}. \tag{14}$$

Now, the conditional state ϱ_c is displaced by the amount gz resulting from the measurement amplified by a factor g. By averaging over all possible outcomes of the double-homodyne detection, we obtain the following output state:

$$\varrho_d=\int_{\mathbb{C}}d^2z\,p_\eta(z)D(gz)\varrho_cD^\dagger(gz), \tag{15}$$

with $D(\zeta)$ being the displacement operator. Since

$$\chi[D(gz)\varrho_cD^\dagger(gz)](\mathbf{\Lambda}_1)=\chi[\varrho_c](\mathbf{\Lambda}_1)\exp(-ig\mathbf{\Lambda}_1^T\boldsymbol{X}_M), \tag{16}$$

we obtain

$$\chi[\varrho_d](\mathbf{\Lambda}_1)=\int_{\mathbb{C}}d^2z\,p_\eta(z)\chi[D(gz)\varrho_cD^\dagger(gz)](\mathbf{\Lambda}_1) \tag{17}$$

$$=\exp\left(-\frac{1}{2}\mathbf{\Lambda}_1^T\boldsymbol{\sigma}_d\mathbf{\Lambda}_1-i\mathbf{\Lambda}_1^T\boldsymbol{X}_d\right), \tag{18}$$

with $\boldsymbol{\sigma}_d=\boldsymbol{A}+g^2\boldsymbol{\Sigma}+g(\boldsymbol{C}+\boldsymbol{C}^T)$ and $\boldsymbol{X}_d=\tilde{\boldsymbol{X}}_1+g\tilde{\boldsymbol{X}}_2$. The integral (17) can be evaluated in the space $\mathbb{R}^2$ by performing the change of variables $z\to\boldsymbol{X}_M=\sqrt{2}[\mathrm{Re}(z),\mathrm{Im}(z)]^T$ and using Eq. (11). The conditional state (15) is then sent to a second BS with transmission τ_2 (see Fig. 1), where it is mixed with the Gaussian state ϱ_3, and finally the two clones are generated. Note that, in practice, the average over all the possible outcomes z in Eq. (15) should be performed at this stage, that is, after the second BS. On the other hand, because of the linearity of the integration, the results are identical, but performing the averaging just before the BS simplifies the calculations. Since ϱ_d is still Gaussian, the two-mode state $\varrho_f = \varrho_d \otimes \varrho_3$ is a Gaussian with covariance matrix and mean given by

$$\boldsymbol{\sigma}_f = \left(\begin{array}{c|c} \boldsymbol{\sigma}_d & \mathbf{0} \\ \hline \mathbf{0} & \boldsymbol{\sigma}_3 \end{array}\right), \; \boldsymbol{X}_f = (\boldsymbol{X}_d, \boldsymbol{X}_3)^T, \tag{19}$$

respectively, which, as in the case of Eqs. (5) and (6), under the action of the BS transform as follows:

$$\boldsymbol{\sigma}_f \rightsquigarrow \boldsymbol{\sigma}_{\text{out}} \equiv \boldsymbol{S}_{\text{BS},2}^T \boldsymbol{\sigma}_f \boldsymbol{S}_{\text{BS},2} = \left(\begin{array}{c|c} \mathcal{A}_1 & \mathcal{C} \\ \hline \mathcal{C}^T & \mathcal{A}_2 \end{array}\right), \tag{20}$$

$$\boldsymbol{X}_f \rightsquigarrow \boldsymbol{X}_{\text{out}} \equiv \boldsymbol{S}_{\text{BS},2}^T \boldsymbol{X}_f = (\mathcal{X}_1, \mathcal{X}_2)^T, \tag{21}$$

where $\mathcal{A}_k$ and $\mathcal{C}$ are 2×2 matrices, and $\boldsymbol{S}_{\text{BS},2}$ is the symplectic matrix given by Eq. (7) with τ_1 replaced by τ_2. Finally, the (Gaussian) characteristic function of the clone ς_k, $k=1,2$, is obtained by integrating over $\boldsymbol{\Lambda}_h$, $h\neq k$, the two-mode characteristic function $\chi(\varrho_{\text{out}})(\boldsymbol{\Lambda}_1,\boldsymbol{\Lambda}_2)$, where $\varrho_{\text{out}} = U_{\text{BS},2}\varrho_f \otimes \varrho_3 U_{\text{BS},2}^\dagger$, i.e.,

$$\chi[\varsigma_k](\boldsymbol{\Lambda}_k) = \frac{1}{2\pi}\int_{\mathbb{R}^2} d^2\boldsymbol{\Lambda}_h \chi[\varrho_{\text{out}}](\boldsymbol{\Lambda}_1,\boldsymbol{\Lambda}_2) \tag{22}$$

$$= \exp\left(-\frac{1}{2}\boldsymbol{\Lambda}_k^T \mathcal{A}_k \boldsymbol{\Lambda}_k - i\boldsymbol{\Lambda}_k^T \mathcal{X}_k\right). \tag{23}$$

The explicit expressions of $\mathcal{X}_1$ and $\mathcal{X}_2$ are

$$\mathcal{X}_1 = \sqrt{\tau_2}(f_1\boldsymbol{X}_1 + f_2\boldsymbol{X}_2) - \sqrt{1-\tau_2}\boldsymbol{X}_3, \tag{24a}$$

$$\mathcal{X}_2 = \sqrt{1-\tau_2}(f_1\boldsymbol{X}_1 + f_2\boldsymbol{X}_2) + \sqrt{\tau_2}\boldsymbol{X}_3, \tag{24b}$$

with

$$f_1 \equiv f_1(\tau_1,\tau_2,g) = \sqrt{\tau_1} + g\sqrt{1-\tau_1}, \tag{25}$$

$$f_2 \equiv f_2(\tau_1,\tau_2,g) = g\sqrt{\tau_1} - \sqrt{1-\tau_1}, \tag{26}$$

whereas $\mathcal{A}_1$ and $\mathcal{A}_2$ can be written in a compact form as follows:

$$\mathcal{A}_1 = \tau_2(f_1^2\boldsymbol{\sigma}_1 + f_2^2\boldsymbol{\sigma}_2 + g^2\boldsymbol{\sigma}_M) + (1-\tau_2)\boldsymbol{\sigma}_3, \tag{27a}$$

$$\mathcal{A}_2 = (1-\tau_2)(f_1^2\boldsymbol{\sigma}_1 + f_2^2\boldsymbol{\sigma}_2 + g^2\boldsymbol{\sigma}_M) + \tau_2\boldsymbol{\sigma}_3. \tag{27b}$$

III. SELECTIVE CLONING

From Eqs. (24) and (27), we see that the two outgoing states ς_1 and ς_2 are generally different. In this paper, we will consider the case in which the clones are equal, therefore, in order to make them exactly alike, we have to put $\tau_2=1/2$ and $\boldsymbol{X}_3=\mathbf{0}$: in this case, $\mathcal{X}_1=\mathcal{X}_2$ and $\mathcal{A}_1=\mathcal{A}_2$. A further inspection of Eqs. (24) and (27) with $\tau_2=1/2$ shows that the states ς_k could be quite different from both the input states, the covariance matrices and the mean value vectors being linear combinations of the input ones. On the other hand, if f_2 (or f_1) vanishes, then the Gaussian output states depend only on $\boldsymbol{\sigma}_1$, $\boldsymbol{X}_1$ (or $\boldsymbol{\sigma}_2$, $\boldsymbol{X}_2$), $\boldsymbol{\sigma}_3$, and $\boldsymbol{\sigma}_M$. In the following, we will investigate this scenario thoroughly.

After we have chosen the symmetric output setup, i.e., $\tau_2=1/2$ and $\boldsymbol{X}_3=\mathbf{0}$, we are interested in removing the dependence on the state, e.g., ϱ_2 from the output states, namely, we want to let f_2 vanish; this is achieved when

$$g \equiv g_1(\tau_1) = \sqrt{(1-\tau_1)/\tau_1}, \tag{28}$$

which gives $f_1=\tau_1^{-1/2}$, and leads to

$$\mathcal{X}_1 = \mathcal{X}_2 = (2\tau_1)^{-1/2}\boldsymbol{X}_1 \tag{29}$$

$$\mathcal{A}_1 = \mathcal{A}_2 = \frac{1}{2}\left(\frac{1}{\tau_1}\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_3 + \frac{1-\tau_1}{\tau_1}\boldsymbol{\sigma}_M\right). \tag{30}$$

It is now clear that, if the first BS is balanced ($\tau_1=1/2$), we obtain

$$\mathcal{X}_1 = \mathcal{X}_2 = \boldsymbol{X}_1, \tag{31a}$$

$$\mathcal{A}_1 = \mathcal{A}_2 = \boldsymbol{\sigma}_1 + \frac{1}{2}(\boldsymbol{\sigma}_3 + \boldsymbol{\sigma}_M). \tag{31b}$$

This is the $1\to 2$ symmetric cloning of the state ϱ_1. This configuration has been experimentally implemented to optimally clone coherent states [11,12]. Notice that $g_1(1/2)=1$.

On the contrary, in order to eliminate the dependence on the state ϱ_1, one needs (we are assuming again $\tau_2=1/2$ and $\boldsymbol{X}_3=\mathbf{0}$)

$$g \equiv g_2(\tau_1) = -\sqrt{\tau_1/(1-\tau_1)}, \tag{32}$$

which gives $f_2=-(1-\tau_1)^{-1/2}$ and leads to

$$\mathcal{X}_1 = \mathcal{X}_2 = -[2(1-\tau_1)]^{-1/2}\boldsymbol{X}_2, \tag{33}$$

$$\mathcal{A}_1 = \mathcal{A}_2 = \frac{1}{2}\left(\frac{1}{1-\tau_1}\boldsymbol{\sigma}_2 + \boldsymbol{\sigma}_3 + \frac{\tau_1}{1-\tau_1}\boldsymbol{\sigma}_M\right), \tag{34}$$

and if $\tau_1=1/2$ one has

$$\mathcal{X}_1 = \mathcal{X}_2 = -\boldsymbol{X}_2, \tag{35a}$$

$$\mathcal{A}_1 = \mathcal{A}_2 = \boldsymbol{\sigma}_2 + \frac{1}{2}(\boldsymbol{\sigma}_3 + \boldsymbol{\sigma}_M). \tag{35b}$$

As a matter of fact, to obtain the actual symmetric cloning of the state ϱ_2, we have to implement a unitary transformation to change the phase of the output states as follows: $\mathcal{X}_h \to -\mathcal{X}_h$. Notice that $g_2(1/2)=-1$.

The results of this section are summarized in Table I: in the case of symmetric cloning ($\tau_1=\tau_2=1/2$ and $\boldsymbol{X}_3=\mathbf{0}$), one can select the state to clone simply change the value of the gain g from $+1$ to -1.

IV. ENHANCEMENT OF LINEAR CLONING FIDELITY

The similarity between the input state ϱ_k and the clone ς_h, k, $h=1,2$, can be quantified by means of the fidelity [18]

TABLE I. Selective symmetric cloning ($\tau_1=\tau_2=1/2$ and $\boldsymbol{X}_3=\boldsymbol{0}$): by changing the value of the electronic gain from +1 to −1, one can choose to clone the state ϱ_1 or ϱ_2, respectively. Notice that, if $g=-1$, a unitary transformation at the output is needed in order to obtain the right sign of the amplitude $\mathcal{X}_k$, $k=1,2$.

g	$\mathcal{A}_1=\mathcal{A}_2$	$\mathcal{X}_1=\mathcal{X}_2$
+1	$\boldsymbol{\sigma}_1+\frac{1}{2}(\boldsymbol{\sigma}_3+\boldsymbol{\sigma}_M)$	$\boldsymbol{X}_1$
−1	$\boldsymbol{\sigma}_2+\frac{1}{2}(\boldsymbol{\sigma}_3+\boldsymbol{\sigma}_M)$	$-\boldsymbol{X}_2$

$$F(\varrho_k,\varsigma_h)=\left[\mathrm{Tr}\left(\sqrt{\sqrt{\varrho_k}\varsigma_h\sqrt{\varrho_k}}\right)\right]^2, \tag{36}$$

which, for Gaussian states, reduces to [12,19,20]

$$F_\eta\equiv F(\varrho_k,\varsigma_h)=\frac{1}{\sqrt{\mathrm{Det}(\boldsymbol{\sigma}_k+\mathcal{A}_h)+\delta}-\sqrt{\delta}} \times\exp\left[-\frac{1}{2}(\boldsymbol{X}_k-\mathcal{X}_h)^T(\boldsymbol{\sigma}_k+\mathcal{A}_h)^{-1}(\boldsymbol{X}_k-\mathcal{X}_h)\right], \tag{37}$$

where $\delta=4\left[\mathrm{Det}(\boldsymbol{\sigma}_k)-\frac{1}{4}\right]\left[\mathrm{Det}(\mathcal{A}_h)-\frac{1}{4}\right]$. Note that, for pure Gaussian states, $\mathrm{Det}(\boldsymbol{\sigma}_k)=\frac{1}{4}$, and in turn $\delta=0$. In the case of symmetric cloning $\boldsymbol{X}_k=\mathcal{X}_h$, the fidelity (37) reduces to

$$F_\eta(\boldsymbol{\sigma}_k,\boldsymbol{\sigma}_3,\boldsymbol{\sigma}_M)\equiv\frac{1}{\sqrt{\mathrm{Det}(\boldsymbol{\sigma}_k+\mathcal{A}_h)+\delta}-\sqrt{\delta}}, \tag{38}$$

and the cloning machine is said to be *universal* because of its invariance with respect to displacement of the input states.

It is a matter of fact that we can now maximize Eq. (38) by a suitable choice of the state ϱ_3 ($\boldsymbol{\sigma}_1$, $\boldsymbol{\sigma}_2$, and $\boldsymbol{\sigma}_M$ being fixed). Without loss of generality, we assume that the covariance matrix associated with ϱ_3 has the following diagonal form:

$$\boldsymbol{\sigma}_3=\begin{pmatrix}\omega_{11} & 0\\ 0 & \omega_{22}\end{pmatrix}, \tag{39}$$

with

$$\omega_{11}=\frac{2\mathcal{J}+1}{2}e^{2s},\quad \omega_{22}=\frac{2\mathcal{J}+1}{2}e^{-2s}, \tag{40}$$

i.e., a squeezed thermal state with $\mathcal{J}$ mean thermal photons and squeezing parameter s. We recall that $\boldsymbol{X}_3=\boldsymbol{0}$ in order to satisfy the symmetric cloning requirements. Now, if

$$\boldsymbol{\sigma}_k=\begin{pmatrix}\gamma_{11}^{(k)} & \gamma_{12}^{(k)}\\ \gamma_{12}^{(k)} & \gamma_{22}^{(k)}\end{pmatrix},\quad \boldsymbol{\sigma}_M=\begin{pmatrix}\Delta_{11}^2 & \Delta_{12}\\ \Delta_{12} & \Delta_{22}^2\end{pmatrix}. \tag{41}$$

are the explicit forms of the covariance matrices of ϱ_k, $k=1,2$, and of the measurement $\Pi_\eta(z)$, respectively, then we find that the fidelity reaches a maximum for (for the sake of simplicity, we do not report explicitly the dependence of $\gamma_{mn}^{(k)}$ on k, since it is clear what is the input state ϱ_k under consideration)

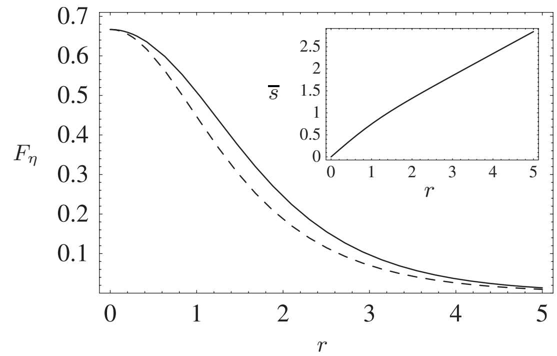

FIG. 2. Fidelity $F_\eta(\boldsymbol{\sigma}_k,\boldsymbol{\sigma}_3,\boldsymbol{\sigma}_M)$ in the case of symmetric cloning, when ϱ_1 is a squeezed state with real squeezing parameter r; $\boldsymbol{\sigma}_3$ is chosen to be the covariance matrix $\boldsymbol{\sigma}_{\bar{s}}$ of a vacuum squeezed state (solid line) or $\boldsymbol{\sigma}_0$ of the vacuum state (dashed line). The inset shows the optimal squeezing parameter $\bar{s}$ as a function of r. See text for details. We set $\eta=1$.

$$s=\bar{s}\equiv\frac{1}{4}\ln\left(\frac{4\gamma_{11}+\Delta_{11}^2}{4\gamma_{22}+\Delta_{22}^2}\right),\quad \mathcal{J}=0, \tag{42}$$

i.e., ϱ_3 should be a squeezed vacuum state with covariance matrix $\boldsymbol{\sigma}_3\equiv\boldsymbol{\sigma}_{\bar{s}}=\frac{1}{2}\,\mathrm{Diag}(e^{2\bar{s}},e^{-2\bar{s}})$. Indeed, such a maximization of the fidelity requires knowledge of γ_{11} and γ_{22}.

The result obtained above generalizes the conclusions given in Ref. [12]. The linear cloning machine described in [12], used to perform $1\to2$ cloning of the state ϱ_1, follows from the present scheme choosing $\varrho_2=\varrho_3=|0\rangle\langle0|$, corresponding to $\boldsymbol{\sigma}_1=\boldsymbol{\sigma}_3=\boldsymbol{\sigma}_0\equiv\frac{1}{2}\mathbb{1}_2$, and $\boldsymbol{\sigma}_M=[(2-\eta)/(2\eta)]\mathbb{1}_2$, which describes the covariance matrix of the double-homodyne detection with quantum efficiency η. From Eq. (42), we see that sending the vacuum into the second BS is the best choice only if ϱ_1 is a coherent state or a displaced thermal state [12] (in both cases $s=0$ and $\boldsymbol{\sigma}_3$ reduces to the vacuum state covariance matrix, since $\boldsymbol{X}_3=\boldsymbol{0}$). On the contrary, when $\boldsymbol{\sigma}_k$ is the covariance matrix associated with the squeezed state $D(\alpha)S(r)|0\rangle=|\alpha,r\rangle$, where $D(\alpha)=\exp(\alpha a^\dagger-\alpha^* a)$ and $S(r)=\exp\left[\frac{1}{2}r(a^{\dagger 2}-a^2)\right]$ are the displacement and squeezing operators, respectively, r being the real squeezing parameter, then $2\gamma_{11}=2\gamma_{22}^{-1}=e^{2r}$, and the cloning fidelity is optimized if ϱ_3 is a squeezed state with squeezing parameter given by Eq. (42). Figure 2 shows the enhancement of the fidelity in the case of squeezed state $1\to2$ cloning, when a suitable squeezed vacuum state with squeezing parameter $\bar{s}$ given in Eq. (42) is used, instead of the vacuum state, as input ϱ_3 (see Fig. 1). In the present case, we have

$$\bar{s}\equiv\bar{s}(r,\eta)=\frac{1}{4}\ln\left(\frac{4\eta e^{2r}+2-\eta}{4\eta e^{-2r}+2-\eta}\right), \tag{43}$$

which is plotted as a function of r with $\eta=1$ in the inset of Fig. 2. The effect of nonunit quantum efficiency can be seen in Fig. 3, where we plot the quantity

$$G(r,\eta)=\frac{F_\eta(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_{\bar{s}},\boldsymbol{\sigma}_M)-F_\eta(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_0,\boldsymbol{\sigma}_M)}{F_\eta(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_0,\boldsymbol{\sigma}_M)} \tag{44}$$

as a function of r for different values of η. $G(r,\eta)$ expresses the relative improvement of cloning fidelity. As is apparent

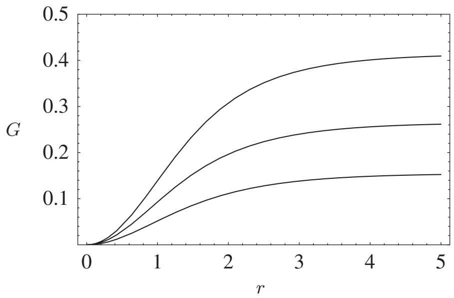

FIG. 3. $G(r,\eta)$ given in Eq. (44) as a function of the squeezing parameter of the input state for different values of η; from top to bottom, $\eta=1.0$, 0.75, and 0.5. See the text for details.

from the plot, one has enhancement of fidelity for any value of η as long as the signals show nonzero squeezing.

V. 1→2 BINARY COMMUNICATION

In this section we address an application of the selective cloning machine to a $1\rightarrow 2$ binary communication protocol. The goal is to encode a classical sequence (string) $\mathcal{S}$ of two classical symbols, e.g., -1 and $+1$, into a quantum sequence $\mathcal{S}'$ of two quantum states, e.g., ϱ_1 and ϱ_2, eventually unknown, and to send it to two receivers, which are interested not only in the classical message but also in the quantum states encoding it. In this case a cloning machine is necessary to generate the copies $\mathcal{R}_1$ and $\mathcal{R}_2$ of $\mathcal{S}'$. Let us now assume that the *sender*, which possesses the string $\mathcal{S}$, is not able to generate $\mathcal{S}'$ itself, so it needs a *service provider* that provides a communication channel based on the states ϱ_1 and ϱ_2. However, since the service provider does not know $\mathcal{S}$, the communication channel should be independent on the message the sender wants to send. In this scenario the selective cloning machine (operating in the symmetric cloning regime) presented above can be a useful tool.

The $1\rightarrow 2$ communication protocol based on the selective cloning machine is sketched in Fig. 4 and can be summarized in these steps: (a) the service provider mixes ϱ_1 and ϱ_2 at the balanced BS and addresses the outputs to the sender; (b) the sender performs double-homodyne detection onto one of the two beams and displaces the other one by an amount gz, z being the outcome of the measurement and g being chosen according to the entries ± 1 of $\mathcal{S}$; (c) the displaced beam is divided into the two clones $\tilde{\varsigma}_1(z)=\tilde{\varsigma}_2(z)\equiv\tilde{\varsigma}^{(k)}(z)$ by means of another balanced BS, with $k=1(2)$ if $g=+1(-1)$.

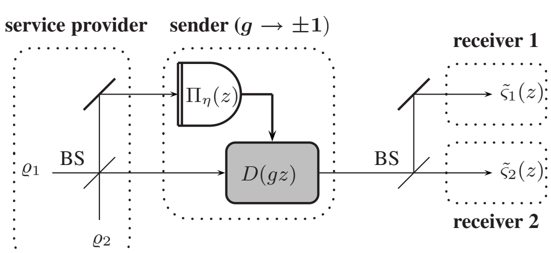

FIG. 4. $1\rightarrow 2$ binary communication assisted by the selective cloning machine. The "service provider" provides the communication channel by mixing the two states ϱ_1 and ϱ_2 at a balanced BS and by addressing the outgoing beams to the "sender." The sender performs a measurement on one of the beams and displaces the other by an amount gz, z being the measurement's outcome and g being chosen according to the bit the sender wants to encode. Finally, the message is split into two clones by means of a second BS. See text for detalis.

It is worth noting that the selective cloning machine is now operating in a "single-shot" regime, namely, each clone is obtained after a single outcome z of the double-homodyne detection and not after a complete measurement onto a state. In turn, each clone actually depends on z. Once the receivers get the single clone, they need a strategy to decide if the bit was $+1$, corresponding to ϱ_1, or -1, corresponding to ϱ_2.

In order to illustrate the protocol, in the following we address the simple case in which

$$\varrho_1 = \varrho_2 = |\alpha\rangle\langle\alpha| \tag{45}$$

are coherent states, i.e., $\boldsymbol{\sigma}_k=\frac{1}{2}\mathbb{1}_2$ and $\boldsymbol{X}_1=\boldsymbol{X}_2=\sqrt{2}(\alpha,0)$ (for the sake of simplicity we take α as real and positive). We recall that the clones of ϱ_2 have the amplitude with a π phase shift (see Table I) with respect to input one: in this way it is possible to distinguish between $\tilde{\varsigma}^{(1)}(z)$ and $\tilde{\varsigma}^{(2)}(z)$. Note that one has

$$U_{\mathrm{BS},1}\varrho_1 \otimes \varrho_2 U_{\mathrm{BS},1}^{\dagger} = |0\rangle\langle 0| \otimes |\sqrt{2}\alpha\rangle\langle\sqrt{2}\alpha|. \tag{46}$$

One of the possible strategies to distinguish between $\tilde{\varsigma}^{(1)}(z)$ and $\tilde{\varsigma}^{(2)}(z)$ is performing a homodyne detection, which is described by the POVM [21]

$$\Pi_x(\varepsilon) = \frac{1}{\sqrt{2\pi\sigma_\varepsilon^2}}\int_{\mathbb{R}} dy\, \exp\left(-\frac{(y-x)^2}{2\sigma_\varepsilon^2}\right)\Pi_y, \tag{47}$$

where $\sigma_\varepsilon^2=(1-\varepsilon)/(4\varepsilon)$, ε is the detection quantum efficiency, and $\Pi_y=|y\rangle\langle y|$, with

$$|y\rangle = \frac{e^{-y^2/2}}{\pi^{1/4}}\sum_{n=0}^{\infty}\frac{H_n(y)}{\sqrt{n!2^n}}|n\rangle \tag{48}$$

being an eigenstate of the quadrature operator $\hat{y}=(1/\sqrt{2})(a+a^\dagger)$ of the measured mode. In Eq. (48), $H_n(y)$ denotes the nth Hermite polynomial. Finally, the decision is taken according to the following rule: if $x\geqslant\bar{x}\Rightarrow k=1$, otherwise $k=2$, $\bar{x}$ being a threshold value. On the other hand, $\tilde{\varsigma}^{(1)}(z)$ and $\tilde{\varsigma}^{(2)}(z)$ are not orthogonal, and then we have to evaluate the probability of inferring the wrong state, namely, the *error probability*, defined as follows:

$$H_e(z) = \frac{1}{2}[P_z(2|1) + P_z(1|2)], \tag{49}$$

where $P_z(h|k)$ is the probability of inferring the state $\tilde{\varsigma}^{(h)}(z)$ when the actual state was $\tilde{\varsigma}^{(k)}(z)$, $h\neq k$. In writing Eq. (49), we assumed that the two states are sent with the same *a priori* probability $p=1/2$. The explicit expressions of $P_z(2|1)$ and $P_z(1|2)$ read as follows:

$$P_z(2|1) = \int_{-\infty}^{\bar{x}} dx\, \mathrm{Tr}[\tilde{\varsigma}^{(1)}(z)\Pi_x(\varepsilon)], \tag{50a}$$

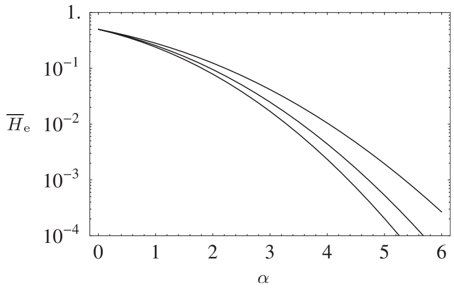

FIG. 5. Average error probability $\bar{H}_e(\alpha,\eta,\varepsilon)$ as a function of the amplitude α and different values of the quantum efficiencies: we set $\varepsilon=1.0$ and, from bottom to top, $\eta=1.0$, 0.5, and 0.75.

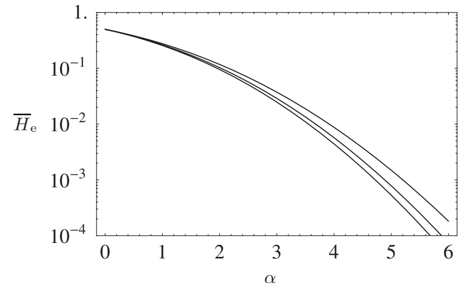

FIG. 6. Average error probability $\bar{H}_e(\alpha,\eta,\varepsilon)$ as a function of the amplitude α and different values of the quantum efficiencies: we set $\eta=0.75$ and, from bottom to top, $\varepsilon=1.0$, 0.5, and 0.75.

$$P_z(1|2) = \int_{\bar{x}}^{+\infty} dx\, \mathrm{Tr}[\tilde{\varsigma}^{(2)}(z)\Pi_x(\varepsilon)]. \tag{50b}$$

It is easy to see that, because of the choice of the states ϱ_1 and ϱ_2, the probability $H_e(z)$ is minimum when $\bar{x}=0$. The average error probability is then given by

$$\bar{H}_e(\alpha,\eta,\varepsilon) = \int_{\mathbb{C}} d^2z\, p_\eta(z) H_e(z), \tag{51}$$

where $p_\eta(z)$ is the double-homodyne detection probability given by Eq. (9). In Figs. 5 and 6, we plot Eq. (51) as a function of the amplitude α and different values of η and ε: as one may expect, the effect of nonunit quantum efficiencies is to increase $\bar{H}_e$, since inferring the right state becomes more difficult; on the other hand, in order to reduce the error probability, one has to increase the amplitude of the input coherent states.

It is worth mentioning that, when ϱ_1 and ϱ_2 are nonclassical states, then $U_{\mathrm{BS},1}\varrho_1 \otimes \varrho_2 U_{\mathrm{BS},1}^\dagger$ is entangled [16,17], and such correlations can be used to reveal the presence of an eavesdropper along the communication line by means a suitable nonlocality test [22,23].

VI. CONCLUDING REMARKS

We have addressed the performance of a $1\to 2$ selective cloning machine based on linear optics and Gaussian measurement, which allows us to clone one of two incoming input states. We have shown that this is achieved by simply changing the gain of a feedforward loop. Moreover, a third Gaussian state can be used in the final stage of the cloning process in order to enhance the input-output fidelity. We have found that for coherent or thermal states this state reduces to the vacuum state. On the contrary, for squeezed input states, a suitable squeezed vacuum should be considered, depending on the squeezing parameter of the input and on the measurement quantum efficiency. Finally, a protocol for $1\to 2$ binary communication involving the selective cloning machine has been proposed, and the average error probability has been evaluated for a particular choice of the states involved.

ACKNOWLEDGMENTS

Fruitful discussions with M. G. A. Paris, A. R. Rossi, and U. L. Andersen are kindly acknowledged. This work has been supported by MIUR through Project No. PRIN-2005024254-002.

[1] W. K. Wootters and W. H. Zurek, Nature (London) **299**, 802 (1982).
[2] D. Dieks, Phys. Lett. **92A**, 271 (1982).
[3] G. C. Ghirardi and T. Weber, Nuovo Cimento Soc. Ital. Fis., B **78**, 9 (1983).
[4] H. P. Yuen, Phys. Lett. **113A**, 405 (1986).
[5] V. Buzek and M. Hillery, Phys. Rev. A **54**, 1844 (1996).
[6] S. Braunstein and P. van Loock, Rev. Mod. Phys. **77**, 513 (2005).
[7] N. J. Cerf, A. Ipe, and X. Rottenberg, Phys. Rev. Lett. **85**, 1754 (2000).
[8] N. J. Cerf and S. Iblisdir, Phys. Rev. A **62**, 040301(R) (2000).
[9] S. L. Braunstein, N. J. Cerf, S. Iblisdir, P. van Loock, and S. Massar, Phys. Rev. Lett. **86**, 4938 (2001).
[10] J. Fiurášek, Phys. Rev. Lett. **86**, 4942 (2001).
[11] U. L. Andersen, V. Josse, and G. Leuchs, Phys. Rev. Lett. **94**, 240503 (2005).
[12] S. Olivares, M. G. A. Paris, and U. L. Andersen, Phys. Rev. A **73**, 062330 (2006).
[13] S. Olivares, M. G. A. Paris, and U. L. Andersen, Acta Phys. Hung. New Ser.: Heavy Ion Phys. **26**, 293 (2006).
[14] S. Olivares and M. G. A. Paris, J. Phys. A: Math. Theor. **40**, 7945 (2007).
[15] A. Ferraro, S. Olivares, and M. G. A. Paris, *Gaussian States in Quantum Information* (Bibliopolis, Napoli, 2005).
[16] M. G. A. Paris, Phys. Rev. A **59**, 1615 (1999).
[17] Xiang-bin Wang, Phys. Rev. A **66**, 024303 (2002).
[18] A. Uhlmann, Rep. Math. Phys. **9**, 273 (1976); R. Jozsa, J. Mod. Opt. **41** 2315 (1994).
[19] H. Scutaru, J. Math. Phys. **31**, 3659 (1998).
[20] H. Nha and H. J. Carmichael, Phys. Rev. A **71**, 032336 (2005).
[21] S. Olivares and M. G. A. Paris, J. Opt. B: Quantum Semiclassical Opt. **6**, 69 (2004).
[22] K. Banaszek and K. Wódkiewicz, Phys. Rev. A **58**, 4345 (1998).
[23] H. Nha and H. J. Carmichael, Phys. Rev. Lett. **93**, 020401 (2004).