

Geometry of Schemes - Second part

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Academic Year 2023/2024

Exercise 1. Let k be an algebraically closed field and consider the projective space \mathbb{P}_k^n for $n \geq 1$. Let X be the zero locus of an irreducible homogeneous polynomial of degree $d > 0$ in \mathbb{P}_k^n . Compute the dimension of $H^i(X, \mathcal{O}_X)$.

Exercise 2. Let k be a field. Let X be a Noetherian integral separated scheme over k which is regular in codimension 1. Show that the same properties hold for $X \times \mathbb{P}_k^n$ and that $\text{Cl}(X \times \mathbb{P}_k^n) \cong \text{Cl}(X) \times \mathbb{Z}$.

Exercise 3. Let k be an algebraically closed field and consider a smooth projective irreducible curve C over k . Set $S := C \times C$. Show that $p_1^* \oplus p_2^*: \text{Pic}(C) \oplus \text{Pic}(C) \rightarrow \text{Pic}(S)$ is injective, where $p_i: S \rightarrow C$ is the i -th projection.

Exercise 4. Consider the smooth quadric surface $Q := \mathbb{P}_k^1 \times \mathbb{P}_k^1$, where k is a field with $\text{char}(k) \neq 2$. Compute the canonical sheaf ω_Q .

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