Spline lineari

coefficienti

valori

```
x = [-5:5];
y = 1./(1+x.^2);
z = linspace(-5,5,101)
    % [-5:0.1:5]
yz = 1./(1+z.^2);
s_1 = interp1(x, y, z, 'method_stringa')
erre = max(abs(s_1 - yz));
```

VALORI ↑

$s_1(z)$ ↓ s_1

'method_stringa':

'lineare': lineare a tratti

'spline': cubica – vedi function spline

COEFFICIENTI ↑

(stessa cosa piecewise:

interp1(x, y, z, 'method', 'pp')

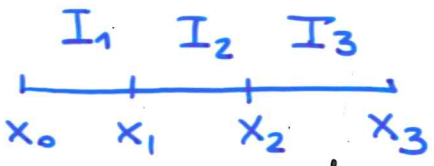
vedi spline cubica

MATLAB E LE SPLINE (CUBICHE)

Matlab: numerati da 1 → 3 ; 1 → 4

$x = [x_0, x_1, x_2, x_3]$

$y = [y_0, y_1, y_2, y_3]$



3 intervalli | 4 nodi

pp-form (piecewise polynomial)

pp = spline (x,y) (not-a-Knot)

VARIABILE STRUTTURATA

form : 'pp'

breaks: $[x_0\ x_1\ x_2\ x_3]$

coefs : [3 x 4 double]

pieces: 3 — 3 intervalli _ RIGHE

order: 4 — 4 coefficienti ($\mathbb{P}_3$) _ COLONNE

dim : 1 ($\mathbb{R} \to \mathbb{R}$)

$$S_j(x) = c_{j,4} + c_{j,3}(x-x_j) + c_{j,2}(x-x_j)^2 + c_{j,1}(x-x_j)^3$$

coef:

$$\begin{bmatrix} c_{1,1} & c_{1,2} & c_{1,3} & c_{1,4} \\ c_{2,1} & c_{2,2} & c_{2,3} & c_{2,4} \\ c_{3,1} & c_{3,2} & c_{3,3} & c_{3,4} \end{bmatrix} \begin{matrix} I_1 \\ I_2 \\ I_3 \end{matrix}$$

$(x-x_j)^3 \quad (x-x_j)^2 \quad (x-x_j)^1 \quad (x-x_j)^0$

$(x-x_j)^0 = 1$

$j = 1,2,3$ (n° intervalli)

x_j nodo sin

PER VALUTARE S_j in x

```
yx = ppval(pp, x)
```

PER ESTRARRE I DATI DA PP:

```
[x, C, l, k] = unmkpp(pp)
```

- coef ↑
- order = grado + 1 ↑
- nodi interp. ↓
- pieces = intervalli ↓

PER RICOSTRUIRE:

```
ppnew = mkpp(x, C)
```

automatico: l = n° righe(C), k = n° colonne(C)

CALCOLO DELLA DERIVATA

$$S_j'(x) = C_{j,3} + 2C_{j,2}(x-x_j) + 3C_{j,1}(x-x_j)^2 \qquad j = 1,2,3$$

spline quadratica ordine 3

$$\begin{bmatrix} 3C_{1,1} & 2C_{1,2} & C_{1,3} \\ 3C_{2,1} & 2C_{2,2} & C_{2,3} \\ 3C_{3,1} & 2C_{3,2} & C_{3,3} \end{bmatrix} \begin{matrix} I_1 \\ I_2 \\ I_3 \end{matrix}$$

$(x-x_j)^2 \quad (x-x_j) \quad (x-x_j)^0$

$S_j'(x)$ $c_{;1}(x-x_j)^3$ $c_{;2}(x-x_j)^2$ $c_{;3}(x-x_j)^1$ ($(x-x_j)^0 = 1$)

Altre spline cubiche

```
xx = linspace(x_a, x_b, n)
yy = f(xx)
```

```
pp = csape(x, y, 'cond_stringa')
```

↳ variabile strutturata piecewise

cond_stringa:

- 'second' naturale $M_0 = M_n = 0$
- 'not-a-Knot'
- 'periodic' $S'(a) = S'(b)$, $S''(a) = S''(b)$

utilizzo della function MATLAB

Spline:

- calcolo diretto dei valori $S(z)$ senza costruzione_output dei coefficienti

  ($\approx$ interp1)

  x : nodi $\{x_0 = a, x_1, \ldots, x_n = b\}$

  y : $f(x)$

  z = linspace (a, b, 1000)

  ↳ valori di S nei 1000 nodi z_k

  S_3 = spline (x, y, z)

  % cubica not-a-Knot

- EQUIVALE A

  >> s3_coef = spline (x, y): var. strutt.

  >> s3_val = ppval (s3_coef, z); valori