$$\int_0^{\infty} x e^{-2x}\, dx$$

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$t = e^{-x}$ $\quad \ln t = -x \quad x = -\ln t$

$x \to 0 \quad t \to 1$

$x \to \infty \quad t \to 0$

$dt = -e^{-x} dx \quad \Rightarrow \quad dx = -\frac{1}{e^{-x}} dt = -\frac{1}{t} dt$

$$\int_1^0 (-\ln t)\, t^2 \left(-\frac{1}{t}\right) dt = \int_1^0 t \ln t \, dt = -\int_0^1 t \ln t \, dt$$

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20 giugno 2013 n° 2

$I_k = \int_0^1 x^k e^{x-1}\, dx$ $\qquad I_1 = \int_0^1 x e^{x-1} dx = x e^{x-1}\Big|_0^1 - \int_0^1 e^{x-1} dx$

$\qquad = 1 - \left[e^{x-1} \right]_0^1 = 1 - 1 + \frac{1}{e} = \frac{1}{e}$

$I_k = 1 - k I_{k-1}$

$I_k + k I_{k-1} = 1$

$$\int_0^1 x^k e^{x-1} dx + k \int_0^1 x^{k-1} e^{x-1} dx =$$

$$x^k e^{x-1}\Big]_0^1 \cancel{- k \int_0^1 x^{k-1} e^{x-1} dx} \cancel{+ k \int_0^1 x^{k-1} e^{x-1} dx} = 1$$