

Material conditionals

Sandro Zucchi

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What crows talk about



- ▶ In *Against the mathematicians*, Sextus Empiricus reports that the ancient Greek poet Callimachus wrote an epigram saying: “Even the crows on the roofs caw about the nature of conditionals”.
- ▶ Indeed, the debate on conditionals was very much alive in ancient Greek philosophy.

The debate on conditionals in ancient Greece

For Philo says that a true conditional is “that which does not begin with a truth and end with a falsehood”, as for instance the conditional “If it is day, I converse”, when in fact it is day and I am conversing; but Diodorus defines it as “that which neither was nor is capable of beginning with a truth and ending with a falsehood”; so that according to him the conditional now mentioned seems to be false, since if it is in fact day but I have remained silent it will begin with a truth but end with a falsehood, whereas the conditional “If atomic elements of things do not exist, atomic elements exist” seems true, since it begins with the false clause “atomic elements do not exist” and will end, according to him, with the true clause “atomic elements exist”. And those who introduce “connexion”, or “coherence”, assert that it is a true conditional whenever the opposite of its consequent contradicts its antecedent clause; so that, according to them, the above-mentioned conditionals are false, whereas the conditional “If day exists, day exists” is true. And those who judge by “implication” declare that a conditional is true when its consequent is potentially included in its antecedent; and according to them the conditional “If day exists, day exists”, and every such duplicated conditional, will probably be false for it is not feasible that any object should itself be included in itself.

Outlines of Pyrrhonism, Sextus Empiricus (III sec. d.C.)

Philo on conditionals

- ▶ The thesis attributed by Sextus to Philo of Mégara (IV sec. a.C.) is that a true conditional “does not begin with a truth and end with a falsehood”, namely a true conditional does not have a true antecedent and a false consequent.
- ▶ In modern terms, Philo seems to claim that **natural language conditionals are material conditionals**, namely the logical form of “if φ , then ψ ” is $\lceil \varphi \supset \psi \rceil$.
- ▶ Today’s agenda is to discuss this claim.

A difference in truth value

- ▶ Consider the following conditionals brought up by Adams (1970):
 - (1) If Oswald did not shoot Kennedy, someone else did.
 - (2) If Oswald had not shot Kennedy, someone else would have.
- ▶ Conditional (1) is clearly true (since someone did shoot Kennedy). However, it is doubtful that conditional (2) is true (who knows what would have happened if Oswald had not shot Kennedy).
- ▶ If both conditional (1) and conditional (2) have the logical form " $\sim p \supset q$ ", it is not clear how one can explain their difference in truth value.

Restricting the claim to indicative conditionals

- ▶ In view of conditionals (1) e (2), the claim that natural language conditionals are material conditionals, in such a general form, is hard to defend:
 - (1) If Oswald did not shoot Kennedy, someone else did.
 - (2) If Oswald had not shot Kennedy, someone else would have.
- ▶ Nowadays, the claim that the connective "if... then..." is adequately translated by the connective " \supset " is restricted to *indicative conditionals*, like (1).
- ▶ Conditionals like (2) are called *counterfactual conditionals*, since their antecedent is presupposed to be false.
- ▶ Notice that if we assumed that counterfactual conditionals were material conditionals, we would expect them to be always true, since we take for granted that their antecedent is false. Clearly, this would be an unacceptable result (while it is true that if Caesar had not gone to the Senate on the Ides of March, he would not have died there, it is false that if Caesar had not gone to the Senate on the Ides of March he would have died there).
- ▶ Thus, from now on we will concentrate on the claim that *indicative* conditionals are material conditionals.

An argument for

An argument in favour of the thesis that indicative conditionals are material conditionals is this:

Premise one: Clearly, arguments (3) and (4) are both valid in English:

- (3) If the butler didn't do it, the gardener did. Therefore, either the butler or the gardener did it.
- (4) Either the butler or the gardener did it. Therefore, if the butler didn't do it, the gardener did.

Premise two: If (3)-(4) are both valid in English, then (5) is a material conditional:

- (5) If the butler didn't do it, the gardener did.

Premise three: If (5) is a material conditional, all conditionals of the same form, namely indicative conditionals, are material conditionals.

Conclusion: Therefore, indicative conditionals are material conditionals.

Validity of the argument

- ▶ The above argument is valid: the conclusion follows from the premises by repeated applications of *modus ponens*.
- ▶ Are the premises true?
- ▶ Let's see what the justification for the premises is.

Justification for the first premise

Premessa uno: Clearly, arguments (3) and (4) are both valid in English:

- (3) If the butler didn't do it, the gardener did. Therefore, either the butler or the gardener did it.
- (4) Either the butler or the gardener did it. Therefore, if the butler didn't do it, the gardener did.

Justification: If (3) e (4) were not valid, there should be a case in which it is plausible to assert the premises, but not the conclusion. But there is no such case.

Justification for the second premise

Premise two: If (3)-(4) are both valid in English, then (5) is a material conditional:

- (3) If the butler didn't do it, the gardener did. Therefore, either the butler or the gardener did it.
- (4) Either the butler or the gardener did it. Therefore, if the butler didn't do it, the gardener did.
- (5) If the butler didn't do it, the gardener did.

Justification: if (3)-(4) are both valid in English, then (by the definition of validity) (5) and (6) are true exactly in the same circumstances:

- (6) Either the butler or the gardener did it.

But (6) is false only in the case it is false that the butler did it and it is false that the gardener did it. Thus, (5) is also false only in the case it is false that the butler did it and it is false that the gardener did it. But if it is false that the butler did it then it is true that the butler didn't do it. Thus, (5) is false only in the case it is true that the butler didn't do it and it is false that the gardener did it. Thus, (5) is false only in case the antecedent is true and the consequent false. Thus, (5) is a material conditional.

Justification for the third premise

Premise three: If (5) is a material conditional, all conditionals of the same form, namely indicative conditionals, are material conditionals.

- (5) If the butler didn't do it, the gardener did.

Justification: plausibly, the truth conditions of sentences with the same grammatical form are determined uniformly. If this were not the case, how could the meaning of indicative conditionals be learned?

A sound argument?

- ▶ The above argument for the thesis that indicative conditionals are material conditionals was presented and discussed in Stalnaker (1975).
- ▶ Stalnaker, by the way, rejects first premise, by which (4) is a valid argument:
 - (4) Either the butler or the gardener did it. Therefore, if the butler didn't do it, the gardener did.
- ▶ Stalnaker provides an elaborate analysis to explain why (4) looks valid but it isn't. I won't present it here.

Arguments against

- ▶ We shall now examine some arguments against the thesis that indicative conditionals are material conditionals.
- ▶ There are at least seven such arguments.

First argument

- ▶ In LP, a sentence of the form $\lceil \varphi \supset \psi \rceil$ is true under these conditions: φ is false or ψ is true.
- ▶ Thus, if indicative conditionals are material conditionals, the following conditionals should be true (the examples are by Graham Priest). In fact, they are false:

- (7) If New York is in New Zealand then $2 + 2 = 4$.
- (8) If New York is in the United States then World War II ended in 1945.
- (9) If World War II ended in 1941 then gold is an acid.

The paradoxes of material implication

- ▶ Notice that, if indicative conditionals are material conditionals, arguments (10)-(11) should be valid in English (but they are not):

- (10) New York is not in New Zealand. Thus, if New York is in New Zealand then $2 + 2 = 4$.
- (11) World War II did not end in 1941. Thus, if World War II ended in 1941 then gold is an acid.

- ▶ Arguments (10)-(11) should be valid in English if indicative conditionals are material conditionals, since the semantics of “ \supset ” has the following consequences:

- (12) $\psi \models_{LP} \varphi \supset \psi$, for every formula φ e ψ .
- (13) $\sim \varphi \models_{LP} \varphi \supset \psi$, for every formula φ e ψ .

- ▶ Consequences (12)-(13) are called “paradoxes of material implication”.

Second argument

- ▶ In LP, if a material conditional is false, the antecedent is true. Thus:

- (14) $\sim (\varphi \supset \psi) \models_{LP} \varphi$, for every formula φ and ψ .

- ▶ Thus, if indicative conditionals are material conditionals, argument (15) should be valid in English, but it's not:

- (15) It is not true that if Mélenchon is the president of France, Macron won the elections. Thus, Mélanchon is the president of France.

Third argument

- ▶ In LP, if a material conditional is false, the consequent is false. Thus:

$$(16) \quad \sim(\varphi \supset \psi) \models_{LP} \sim\psi, \text{ for every formula } \varphi \text{ and } \psi.$$

- ▶ Thus, if indicative conditionals are material conditionals, argument (17) should be valid in English:

(17) It is not the case that, if I go to the party tonight, I shall get drunk tonight. So, I shall not get drunk tonight. (Sainsbury)

- ▶ But (17) is not a valid argument in English. Suppose that, when I go to a party at night, I always stay sober (because I don't feel lonely). However, when I stay at home at night, I always get drunk (because I feel lonely). In this case, the premise of (17) is true but the conclusion false.

Fourth argument

- ▶ It's easy to show that (18) is true (in this sense, the connective “ \supset ” is *transitive*):

$$(18) \quad \psi \supset \zeta, \varphi \supset \psi \models_{LP} \varphi \supset \zeta, \\ \text{for every formula } \varphi, \psi \text{ e } \zeta.$$

- ▶ Thus, if indicative conditionals are material conditionals, argument (19) should be valid in English. But it isn't (clearly, the conclusion does not follow from the premises):

(19) If Trump will not run for the 2024 elections, he will flee the country. If Trump is in jail, he will not run for the 2024 elections. Thus, if Trump is in jail, he will flee the country. (Stalnaker 1968, updated example)

Fifth argument

- ▶ It's easy to show that (20) is true:

$$(20) \quad \varphi \supset \psi \models_{LP} (\varphi \wedge \zeta) \supset \psi, \\ \text{for every formula } \varphi, \psi \text{ and } \zeta.$$

- ▶ Thus, if indicative conditionals are material conditionals, argument (21) should be valid in English. But it isn't:

(21) If I put a pinch of salt in the soup, it'll taste good. Thus, if I put a pinch of salt in the soup and I put some gasoline in the soup, it'll taste good.

- ▶ An inference of the form in (20) is called “strengthening of the antecedent”.
- ▶ (Stalnaker 1968 provides an example similar to (21) for counterfactual conditionals).

Sixth argument

- ▶ It's easy to show that (22) is true:

$$(22) \quad \varphi \supset \sim\psi \models_{LP} \psi \supset \sim\varphi, \text{ for every formula } \varphi \text{ e } \psi.$$

- ▶ But argument (23) is not valid in English:

(23) If it rains, it is not the case that will rain a lot. Therefore, if it rains a lot, it is not the case that it will rain. (Jackson)

- ▶ This argument was proposed by Stalnaker (1968).

Seventh argument

- ▶ It's easy to show that (24) is true:

(24) $\models_{LP} (\varphi \supset \psi) \vee (\psi \supset \varphi)$, for every formula $\varphi \in \psi$.

- ▶ But the same is not true for English indicative conditionals. Imagine a discussion in which I claim that Herzog was the first to summit the Annapurna and you claim that it's not true. In this case, (25) and (26) are both false. Therefore, (27) is false:

(25) If I am right, you are right.

(26) If you are right I am right.

(27) If I am right, you are right or if you are right I am right.

- ▶ (This problem was raised by Read 1988).

The pragmatic defense

- ▶ The counterexamples to the thesis that indicative conditionals are material conditionals are daunting.
- ▶ How is it possible to defend the thesis in the light of these counterexamples?
- ▶ The philosophers endorsing the thesis appeal to *pragmatic* considerations, namely considerations concerning the rules that govern communicative exchanges.
- ▶ I won't try to show how this strategy is applied to every problem we described.
- ▶ I'll try to give a general idea of how the strategy works, and I'll show how it allows to deal with some of the counterexamples.

Truth and assertability

- ▶ The pragmatic defense is based on the distinction between **truth** and **assertability**.
- ▶ We may illustrate this distinction with an example by Lewis (1976):

We are gathering mushrooms; I say to you "You won't eat that one and live." A dirty trick: I thought that one was safe and especially delicious, I wanted it myself, so I hoped to dissuade you from taking it without actually lying. I thought it highly probable that my trick would work, that you would not eat the mushroom, and therefore that I would turn out to have told the truth. But though **what I said had a high subjective probability of truth, it had a low assertability** and it was a misdeed to assert it.

A true sentence that should not be asserted

- ▶ In Lewis's example, sentence (28) is **true** if you won't eat the mushroom, since the negation of a conjunction is true if one of the conjuncts is false:

(28) You won't eat that one and live.
- ▶ However, although it is true, (28) **should not be asserted**, since it suggests misleadingly that the mushroom is poisonous.
- ▶ (After the speaker eats the mushroom, the hearer might rightly complain that he had been deceived).

How is the misleading suggestion generated

a Gricean account

- ▶ Why does the assertion of (28) misleadingly suggest that the mushroom is poisonous?
(28) You won't eat that one and live.
- ▶ Grice (1967) would explain it in this way. By asserting (28) the speaker violated one of the rules that should govern rational communication, namely the maxim: make the strongest (most informative) assertion you can (for the purposes of the conversation you are engaged in).
- ▶ Indeed, in Lewis's story, the speaker asserts the negation of a conjunction when he could have been more informative by asserting the negation of the first conjunct ("You won't eat that one").
- ▶ Since the speaker gives no indication he is violating the rule, the hearer assumes that the speaker is not in a position to deny the first conjunct and is asserting that either the hearer will not eat the mushroom or he will eat it and not live.
- ▶ So, the hearer infers that the mushroom is poisonous.

Grice's theory of conversation

- ▶ Grice's technical term for an inference generated via the conversational maxims is "conversational implicature".
- ▶ The conversational implicature of an assertion is not a necessary consequence of the assertion, since it doesn't follow from the assertion alone, but the maxims of the conversation play a crucial role in deriving the inference.
- ▶ The maxim "make the strongest (most informative) assertion you can (for the purposes of the conversation you are engaged in)" is called "maxim of quantity".

Another example of conversational implicature

- ▶ If one utters (29) out of the blue, this suggests that (30) is true:
(29) Some students passed the exam.
(30) Not all the students passed the exam.
- ▶ However, although (29) suggests that (30) is true, (30) is not a necessary consequence of (29).
- ▶ We can convince ourselves that (30) is not a necessary consequence of (29), by observing that the inference of (30) from (29) is *cancelable*. Indeed, it is not contradictory to assert (31):
(31) Some students passed the exam, indeed all of them passed.
- ▶ This suggests that the inference of (30) from (29) is another case of conversational implicature.

How is the implicature generated

- ▶ The implicature from (29) to (30) is another case in which the maxim of quantity is at work:
(29) Some students passed the exam.
(30) Not all the students passed the exam.
- ▶ Given this maxim, a speaker who knows that all the students passed the exam, should assert (32) rather than (29), since (32) is more informative than (29):
(32) All the students passed the exam.
- ▶ Since the speaker has given no indication that she is violating this maxim of quantity, the hearer, unless she has reason to believe that the speaker is not fully informed of the facts, concludes that the speaker knows that (30) is true.

Informativity and logical strength

- ▶ Deciding when an assertion is more informative than another may not be a trivial matter.
- ▶ For our purposes, let's agree that an assertion A is *stronger (or more informative)* than an assertion B if B is a necessary consequence of A, but A is not a necessary consequence of B.
- ▶ In Lewis's story, the speaker asserts (33) instead of asserting (34):

(33) $\sim (p \wedge q)$
(you won't eat that one and live)

(34) $\sim p$
(you won't eat that one)

- ▶ Since the speaker is in a position to assert (34) and (34) is stronger than (33), the speaker violates the maxim of quantity by asserting (33).

Back to indicative conditionals

- ▶ Let's now come back to indicative conditionals and let's see how the above observations help us to deal with some of the objections raised against the claim that indicative conditionals are material conditionals.
- ▶ Consider (7)-(9) again:
 - (7) If New York is in New Zealand then $2 + 2 = 4$.
 - (8) If New York is in the United States then World War II ended in 1945.
 - (9) If World War II ended in 1941 then gold is an acid.
- ▶ The objection is this: if indicative conditionals are material conditionals, then (7)-(9) should be true, but they are false.

Grice's pragmatic defense

- ▶ Grice's reply in defense of the claim that indicative conditionals are material conditionals is the following: (7)-(9) are true, but they *give the impression* of being false because they are not assertable.
 - (7) If New York is in New Zealand then $2 + 2 = 4$.
 - (8) If New York is in the United States then World War II ended in 1945.
 - (9) If World War II ended in 1941 then gold is an acid.
- ▶ They are not assertable because they violate the maxim of quantity.
- ▶ Let's see why.

Why (7) is not assertable

- ▶ If indicative conditionals are material conditionals, then (7) is equivalent to (35):
 - (7) If New York is in New Zealand then $2 + 2 = 4$.
 - (35) New York is not in New Zealand or $2+2=4$.
- ▶ Clearly, we are in a position to make a stronger statement than (35) (and thus than (7)), since we can assert that New York is not in New Zealand and $2+2=4$.
- ▶ Thus, conditional (7), like the disjunction in (35), is not assertable, because if we asserted (7) we would violate the maxim of quantity.

Why (8) is not assertable

- ▶ If indicative conditionals are material conditionals, then (8) is equivalent to (36):
 - (8) If New York is in the United States then World War II ended in 1945.
 - (36) New York is not in the United States or World War II ended in 1945.
- ▶ Clearly, we are in a position to make a stronger statement than (36) (and thus than (8)), since we can assert that World War II ended in 1945.
- ▶ Thus, conditional (8), like the disjunction in (36), is not assertable, because if we asserted (8) we would violate the maxim of quantity.

Why (9) is not assertable

- ▶ If indicative conditionals are material conditionals, then (9) is equivalent to (37):
 - (9) If World War II ended in 1941 then gold is an acid.
 - (37) World War II did not end in 1941 or gold is an acid.
- ▶ Clearly, we are in a position to make a stronger statement than (37) (and thus than (9)), since we can assert that World War II did not 1941.
- ▶ Thus, conditional (9), like the disjunction in (37), is not assertable, because if we asserted (8) we would violate the maxim of quantity.

Summing up

Grice's pragmatic defense

- ▶ The pragmatic defense is based on the idea that one can defend the claim that indicative conditionals are material conditionals by distinguishing between truth and assertability.
- ▶ The cases in which the material conditional is true, but the corresponding indicative conditional doesn't seem to be true are in fact cases in which the indicative conditional is true but not assertable.
- ▶ In particular, according to Grice, a conditional is not assertable when we are in a position to assert the consequent or the negation of the antecedent (or both the consequent and the negation of the antecedent), since in these cases, if we assert the conditional, we violate the maxim of quantity.

The pragmatic defense of Jackson and Lewis

- ▶ Beside Grice, another advocate of the view that indicative conditionals are material conditionals is Jackson (1979, 1987).
- ▶ Jackson agrees with Grice that the problematic consequences of this view may be dealt with by appealing to conditions of assertability.
- ▶ However, for Jackson the assertability conditions of indicative conditionals cannot be derived from the truth-conditions of material conditionals and from the maxims of the conversation.
- ▶ For Jackson the assertability conditions of indicative conditionals depend on conventions governing their use (a version of Jackson's theory is also endorsed by Lewis 1986).
- ▶ Before describing Jackson's theory, however, let's see some problematic examples for Grice which Jackson's theory is meant to solve.

Problems for Grice's defense

false antecedents

- ▶ Consider the following conditional:

(38) If the sun goes out of existence in ten minutes time, the earth will be plunged into darkness in eighteen minutes time (Jackson 1979).
- ▶ Given what we know, the antecedent of (38) is false. Thus, according to Grice, (38) should not be assertable. Yet, it is assertable.
- ▶ (Indeed, by asserting (38) one would say the truth: light travels at 300.000 km per second, the minimal distance of the sun from the earth is 146 millions of km, thus $(146 \text{ milioni} : 300.000) : 60 = 8.1$).

Problems for Grice's defense

true consequents

- ▶ Suppose that you now that Leo will be elected whether or not Lea runs.
- ▶ In this case, we can assert both (39) and (40) (Jackson 1979):

(39) If Lea does not run, Leo will be elected.
(40) If Lea does run, Leo will be elected.
- ▶ Yet, according to Grice (39) and (40) should not be assertable, since we know that the consequent is true.

What to do?

- ▶ The previous examples seem to indicate that Grice's defense of the claim that indicative conditionals are material conditionals makes incorrect predictions concerning the assertability of indicative conditionals.
- ▶ Jackson's solution consists in proposing an alternative theory of the assertability conditions of indicative conditionals, a theory that avoids the problem Grice runs into and allows one to hold on to the claim that indicative conditionals are material conditionals.

Assertability conditions for indicative conditionals

Jackson

- ▶ Jackson's view is that when we assert a conditional, we want to be able to use *modus ponens* if the occasion arises: if we learn that the antecedent is true, we conclude that the consequent is also true. Thus, learning that the antecedent is true should not lead us to revise our claim that the conditional is true.
- ▶ For this reason, while Jackson assumes that indicative conditionals have the same truth conditions as material conditionals, he also proposes that the use of indicative conditionals is governed by the following convention:
 - it is appropriate to assert a conditional only if the conditional is *robust* relative to its antecedent;
 - a conditional is *robust* relative to its antecedent if and only if the (subjective) probability of the conditional is high, and it would stay high also if it came to be known that the antecedent is true.
- ▶ (The subjective probability of a sentence is the degree of trust of a rational subject in the truth of the sentence).
- ▶ Let's see how the theory works with some examples.

Applying robustness

unassertable conditionals

- ▶ Consider again conditional (9). If indicative conditionals are material conditionals, then (9) is equivalent to (37):
(9) If World War II ended in 1941 then gold is an acid.
(37) World War II did not end in 1941 or gold is an acid.
- ▶ The probability of (37) (and thus the probability of (9)) is high, since the first disjunct is true.
- ▶ However, if we learnt that the antecedent of (9) is true, namely that World War II ended in 1941, the probability of (9) would not stay high, since we would know that both disjuncts in (37) are false and thus that (37) (and (9)) is false.
- ▶ Thus, conditional (9), although it is true, is not assertible, since (9) is not robust relative to its antecedent. This is why (9) *gives the impression* of being false.

Applying robustness

assertable conditionals

- ▶ Now consider (38) again, the conditional that was a problem for Grice's defense. For Jackson, as for Grice, (38) is equivalent to (41):
(38) If the sun goes out of existence in ten minutes time, the earth will be plunged into darkness in eighteen minutes time.
(41) Either the sun does not go out of existence in ten minutes time or the earth will be plunged into darkness in eighteen minutes time.
- ▶ The probability of (41) (thus the probability of (38)) is high, since we know that the first disjunct is true.
- ▶ Moreover, if we learnt that the antecedent of (38) is true, namely we learnt that sun will go out of existence in ten minutes time, the probability of (38) would stay high, since we would know that the second disjunct in (41) is true.
- ▶ Thus, unlike Grice's defense, Jackson's defense correctly predicts that (38) is assertable.

Summing up

- ▶ We saw an argument in support of the thesis that indicative conditionals are material conditionals.
- ▶ But we also saw that this thesis runs into several daunting difficulties.
- ▶ We outlined a pragmatic strategy to defend the thesis, based on the distinction between truth and assertability.
- ▶ We saw two versions of this strategy, one proposed by Paul Grice and one proposed by Frank Jackson.