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The principle of non-contradiction

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The most certain principle of all

... *the most certain principle of all is that regarding which it is impossible to be mistaken; for such a principle must be both the best known (for all men may be mistaken about things which they do not know), and non-hypothetical. For a principle which every one must have who knows anything about being, is not a hypothesis; and that which every one must know who knows anything, he must already have when he comes to a special study. Evidently then such a principle is the most certain of all; which principle this is, we proceed to say. It is, that **the same attribute cannot at the same time belong and not belong to the same subject in the same respect**; we must presuppose, in face of dialectical objections, any further qualifications which might be added. This, then, is the most certain of all principles, since it answers to the definition given above.*

The starting-point for all the other axioms

For it is impossible for any one to believe the same thing to be and not to be, as some think Heraclitus says; for what a man says he does not necessarily believe. If it is impossible that contrary attributes should belong at the same time to the same subject (the usual qualifications must be presupposed in this proposition too), and if an opinion which contradicts another is contrary to it, obviously it is impossible for the same man at the same time to believe the same thing to be and not to be; for if a man were mistaken in this point he would have contrary opinions at the same time. It is for this reason that all who are carrying out a demonstration refer it to this as an ultimate belief; for this is naturally the starting-point even for all the other axioms.

Aristotle, *Metaphysics*, libro Γ, 3, 1005 b, 10-30 (English translation by W. D. Ross).

The principle of non-contradiction in Aristotle

- ▶ In the above passage from Book Γ of *Metaphysics*, one finds one of Aristotle formulations of the principle of non-contradiction:
the same attribute cannot at the same time belong and not belong to the same subject in the same respect. . .
- ▶ Łukasiewicz (1910) calls this formulation "**the metaphysical formulation**" of the principle of non-contradiction.
- ▶ Later on, in Book Γ, one finds a further formulation, what Łukasiewicz calls **the logical formulation**:
the most indisputable of all beliefs is that contradictory statements are not at the same time true. . . (Metafisica, Γ, 6, 1011 b, 13-14).
- ▶ Finally, according to Łukasiewicz the long passage we quoted from Book Γ also contains a "**psychological formulation**" of the principle:
it is impossible for any one to believe the same thing to be and not to be. . . (Metafisica, Γ, 3, 1005 b, 23-24)

The “logical formulation”

- ▶ Let's focus on Aristotle's “logical formulation” of the principle of non-contradiction:
the most indisputable of all beliefs is that contradictory statements are not at the same time true...
Immediately after, Aristotle also says: “. . . it is impossible that contradictories should be at the same time true of the same thing. . .”.
- ▶ By “contradictory statements” Aristotle means a pairs of sentences like: <“man is trireme”, “man is not a trireme”>, <“Socrates is pale”, “Socrates is not pale”>, etc.
- ▶ Thus, as a first approximation, we may state Aristotle's “logical formulation” of the principle of non-contradiction in this way:
it is impossible that statements of the form $\lceil A \rceil$ and $\lceil \text{not } A \rceil$ be both true.
- ▶ (In fact, this way of stating Aristotle's principle is inadequate, since for Aristotle statements like “every man is mortal” and “some man is not mortal” are also contradictory. For the purposes of today's discussion we'll ignore this complication).

Formal definitions of contradiction

- ▶ From the above reconstruction, it turns out that Aristotle's logical formulation of the principle of non-contradiction is based on a *formal* (or *syntactic*) notion of contradiction, in the sense that *contradictory statements are such in virtue of their form*.
- ▶ A formal definition of contradiction was also adopted by more recent authors. For instance, Kalish, Montague, and Mar (1980) claim:
A contradiction consists of a pair of sentences, one of which is the negation of the other.
- ▶ The definition proposed by Marcus (1995) is another way of characterizing contradictions formally:
A contradiction is the conjunction of a proposition and its denial.
- ▶ But the formal definitions of contradiction are not the only ones that can be found in the philosophical literature.

Semantic definitions of contradiction

- ▶ Prior (1967) defines contradiction thus:
... contradiction ... is the relation between statements that are exact opposites, in the sense that they can be neither true together nor false together.
- ▶ Bonevac (1987) claims:
A sentence is contradictory if and only if it's impossible for it to be true.
- ▶ These are *semantic definitions* of contradiction, since *they make use of the semantic notion truth*.

Pragmatic definitions of contradiction

- ▶ Brody (1967) defines contradiction thus:
Contradiction: the joint assertion of a proposition and its denial.
- ▶ Kahane (1995) claims:
A contradiction both makes a claim and denies that very claim.
- ▶ These are *pragmatic definitions* of contradiction, since *they define contradiction by means of the pragmatic notions assertion and denial*.

Variation along different dimensions

- ▶ The definitions of contradiction may also vary along different dimensions.
- ▶ For example, the following definitions are both formal characterizations of contradiction. However, the first defines contradiction as a pair of sentences and the second a single sentence:
 - A contradiction consists of a pair of sentences, one of which is the negation of the other. For example, the pair $\langle \text{"Socrate is mortal"}, \text{"Socrate is not mortal"} \rangle$ is a contradiction.
 - A contradiction is the conjunction of a sentence and its negation. For example, the conjunction "Socrate is mortal and Socrate is not mortal" is a contradiction.
- ▶ Both the following definitions are semantic characterizations of contradiction. However, the first defines contradiction as a pair of sentences and the second a single sentence:
 - A contradiction is a pair of sentences that can be neither true together nor false together.
 - A sentence is contradictory if and only if it's impossible for it to be true.

Corresponding principles of non-contradiction

- ▶ Different ways of defining contradiction lead to different ways of stating the principle of non-contradiction.
- ▶ For example, if contradiction is defined as a pair of sentences one of which is the negation of the other, then it becomes natural to state the principle of non-contradiction thus:

there is no circumstance in which a sentence A and its negation $\neg A$ are both true.
- ▶ On the other hand, if contradiction is defined as the conjunction of a sentence and its negation, this leads to the following way of stating the principle of non-contradiction:

there is no circumstance in which a sentence of the form A and $\neg A$ is true.

Corresponding principles of non-contradiction II

- ▶ Now suppose we adopt the semantic definition of contradiction by which a sentence is contradictory if and only if there is no possible circumstance in which the sentence is true.
- ▶ In this case, it seems natural to state the principle of non-contradiction in this way:

there is no circumstance in which a sentence which is not true in any possible circumstance is true.
- ▶ These are only some of the possible formulations of the principle of non-contradiction that emerge from different ways of defining contradiction (see Grim 2004 for a discussion of the possible variants).

Does it make a difference?

- ▶ We saw that there are different versions of the principle of non-contradiction (based on different ways of defining contradiction).
- ▶ It's conceivable that one might accept a version and not accept another version.
- ▶ Indeed, some authors do exactly this, as we will see.

The most certain principle of all?

- ▶ Aristotle, as we have seen, regards the principle of non-contradiction as “the most certain principle of all . . . regarding which it is impossible to be mistaken”.
- ▶ Aristotle does not believe that the principle can be properly demonstrated:
... Some indeed demand that even this shall be demonstrated, but this they do through want of education, for not to know of what things one may demand demonstration, and of what one may not, argues simply want of education. For it is impossible that there should be demonstration of absolutely everything; there would be an infinite regress, so that there would still be no demonstration. But if there are things of which one should not demand demonstration, these persons cannot say what principle they regard as more indemonstrable than the present one.(*Metaphysics*, Book Γ, 4, 1006a, 6-12)
- ▶ However, Aristotle thinks that it is possible to give a *demonstration by confutation*, namely he thinks that it is possible to argue for the principle if someone denies that the principle is valid. A critical discussion of Aristotle attempt to give a demonstration by confutation can be found in Priest (1998).
- ▶ Here, we will present some attempts to deny the principle (in some some formulations).

Counterfactual conditionals

- ▶ A counterfactual conditional is a conditional whose antecedent is presupposed to be false. For example:

(1) If kangaroos had no tails, they would topple over.
- ▶ Lewis (1973) proposes the following truth-conditions for counterfactuals. A counterfactual conditional is true at a world w if one of the following conditions is satisfied:
 1. there is no (accessible) world at which the antecedent is true;
 2. some accessible world in which antecedent and consequent are true differs less from w than any world in which the antecedent is true and the consequent is false.
- ▶ According to these truth-conditions, (1) is true if and only if there is some world in which kangaroos have no tail and topple over which differs less from the real world than any world in which kangaroos have no tail and do not topple over.

A problem

- ▶ Nolan (1997) observed that Lewis's truth-conditions do not explain the contrast between (2) and (3):
 - (2) If Hobbes had (secretly) squared the circle, sick children in the mountains of South America at the time would have cared.
 - (3) If Hobbes had (secretly) squared the circle, sick children in the mountains of South America at the time would not have cared.
- ▶ Intuitively, (2) is false and (3) is true. But Lewis predicts that they are both true, since there is no possible circumstance in which Hobbes squares the circle.

Impossible circumstances

- ▶ Several authors, among them Nolan (1997) and Berto & Jago (2019), propose to solve the problem raised by (2) and (3) by supposing that there are circumstances in which the antecedent of (2) and (3) is true:
 - (2) If Hobbes had (secretly) squared the circle, sick children in the mountains of South America at the time would have cared.
 - (3) If Hobbes had (secretly) squared the circle, sick children in the mountains of South America at the time would not have cared.
- ▶ Circumstances of this kind are *impossible circumstances*, since impossible states of affairs, like that in which Hobbes squares the circle, obtain in them.
- ▶ Let's see how resorting to impossible circumstances may solve the problem.

Modifying Lewis's truth-conditions

- ▶ Let's assume that in the set W there are both possible worlds and impossible worlds.
- ▶ Given any possible world w , a possible world is always more similar to w than any impossible world. So, possible worlds are always closer to the real world than any impossible world.
- ▶ Yet, impossible worlds may also be ranked according to similarity to the real world (or to a given possible world): some impossible worlds differ less from the real world (or from a given possible world) than other impossible worlds.
- ▶ We may now restate Lewis's truth-conditions for counterfactuals as follows:
 - a counterfactual conditional is true at a world w if and only if some accessible world (possible or impossible) in which antecedent and consequent are true differs less from w than any world (possible or impossible) in which the antecedent is true and the consequent is false.
- ▶ Let's now go back to the problem raised by Nolan.

The solution

- ▶ Other things being equal, the impossible worlds in which Hobbes secretly squared the circle and sick children in the mountains of South America at the time did not care differ less from the real world than the impossible worlds in which Hobbes secretly squared the circle and sick children in the mountains of South America at the time cared.
- ▶ Thus, (2) is false and (3) is true:
 - (2) If Hobbes had (secretly) squared the circle, sick children in the mountains of South America at the time would have cared.
 - (3) If Hobbes had (secretly) squared the circle, sick children in the mountains of South America at the time would not have cared.

A counterexample to the principle of non-contradiction

- ▶ Now consider the formulation of the principle of non-contradiction based on the idea that a contradiction is a sentence that cannot be true:
 - there is no circumstance in which a sentence which is not true in any possible circumstance is true.
- ▶ We just saw that there seem to be some good reasons to think that there are circumstances in which sentence (4) is true:
 - (4) Hobbes had (secretly) squared the circle.
- ▶ But (4) is a sentence which is not true in any possible circumstance.
- ▶ Thus, the principle of non-contradiction seems to be false in the formulation given above.

No impossible circumstances

- ▶ A way of answering the objection to the principle of non-contradiction (advocated for example by D. Lewis) is to deny that there are impossible circumstances.
- ▶ If there are no impossible circumstances, the formulation of the principle of non-contradiction becomes trivially true.
- ▶ Indeed, if there are only *possible* circumstances, formulation 1 is equivalent to formulation 2, which is tautological:
 1. there is no circumstance in which a sentence which is not true in any possible circumstance is true;
 2. there is no possible circumstance in which a sentence which is not true in any possible circumstance is true.
- ▶ Of course, if one takes this line, the contrast between (2) and (3) remains unexplained:
 - (2) If Hobbes had (secretly) squared the circle, sick children in the mountains of South America at the time would have cared.
 - (3) If Hobbes had (secretly) squared the circle, sick children in the mountains of South America at the time would not have cared.

Restricting the scope

- ▶ Another way of answering the objection based on counterpossibles is to admit that there are impossible circumstances and concede that the principle of non-contradiction only concerns *possible* circumstances.
- ▶ This move amounts to rejecting formulation 1 and adopting formulation 2:
 1. there is no circumstance in which a sentence which is not true in any possible circumstance is true.
 2. there is no possible circumstance in which a sentence which is not true in any possible circumstance is true.

More “impossible” circumstances

- ▶ In addition to counterpossibles, some philosophers have described other cases of allegedly impossible circumstances.
- ▶ To deal with the embarrassment these cases pose for the principle of non-contradiction we may follow the same strategies illustrated in the case of counterpossibles (denying that they are genuine cases of impossible circumstances or restricting the scope of the principle).
- ▶ However, by examining these cases we may discover other ways in which the different formulations of the principle of non-contradiction may make a difference.

False perspective



Whoever makes a DESIGN, without the Knowledge of PERSPECTIVE will be liable to such Absurdities as are shown in this View of a House.

The fellow with the pipe and the woman at the window

- ▶ In Hogarth's engraving *False perspective*, the woman at the window on this side of the river lights the pipe of the fellow on a faraway hill on the other side of the river.
- ▶ Thus, in the picture the woman is far from the hill, (since the window she leans out of is on the other side of the river) and, at the same time, she is close to the hill (since she lights the pipe of the fellow on the hill).
- ▶ (Hogarth's picture is full of similar incongruities).

Watson and the second Afghan war

In the year 1878 I took my degree of Doctor of Medicine of the University of London, and proceeded to Netley to go through the course prescribed for surgeons in the army. Having completed my studies there, I was duly attached to the Fifth Northumberland Fusiliers as Assistant Surgeon. The regiment was stationed in India at the time, and before I could join it, the second Afghan war had broken out. . . . The campaign brought honours and promotion to many, but for me it had nothing but misfortune and disaster. I was removed from my brigade and attached to the Berkshires, with whom I served at the fatal battle of Maiwand. There I was struck on the shoulder by a Jezail bullet, which shattered the bone and grazed the subclavian artery.

A. Conan Doyle, *A Study in Scarlet*.

Watson and Holmes's vanity

More than once during the years that I had lived with him in Baker Street I had observed that a small vanity underlay my companion's quiet and didactic manner. I made no remark, however, but sat nursing my wounded leg. I had a Jezail bullet through it some time before, and, though it did not prevent me from walking, it ached wearily at every change of the weather. "My practice has extended recently to the Continent", said Holmes, after a while, filling up his old brier-root pipe.

A. Conan Doyle, *The Sign of the Four*.

The strange case of the traveling war wound

- ▶ Where is Watson's war wound?
- ▶ In *The Sign of the Four* Watson says it's in the leg. However, in *A Study in Scarlet* he says it's not in the leg, but in the shoulder.
- ▶ Clearly, Conan Doyle was careless: in the stories of Sherlock Holmes, Watson contradicts himself when writes about his war wound.
- ▶ (In fact, as we can see from the above passages, Watson does not contradict himself, strictly speaking: however improbable, it is possible that he is talking about two different war wounds. For sake of discussion, ignore this possibility and assume he is contradicting himself.)

Fictions as representations of circumstances

- ▶ It seems plausible to claim that Hogarth's *False perspective* and Conan Doyle's stories represent circumstances.
- ▶ These circumstances support the truth of pairs of sentences where one sentence in the pair is the negation of the other:
 - in the circumstances described by *False perspective* it is true that the woman at the window is far from the hill and it is also true that the woman at the window is close to (thus not far from) the hill;
 - in the circumstances described by Conan Doyle's stories it is true that Watson's war wound is in the leg and it is also true that Watson's war wound is in the shoulder (thus not in the leg).

The distributive formulation of the principle

- ▶ Suppose now that we define contradiction as a pair of sentences where one sentence is the negation of the other.
- ▶ In this case, the natural formulation of the principle of non-contradiction is the following (Varzi 2004 calls this the *distributive formulation* of the principle):

There is no circumstance in which a sentence A and its negation $\neg A$ are both true.
- ▶ If Hogarth's *False perspective* and Conan Doyle's stories describe circumstances, the principle of non-contradiction is false in this formulation.

Well-known strategies and more

- ▶ We have already seen two possible strategies to avoid counterexamples to the principle of non-contradiction.
- ▶ One strategy consists in denying that the circumstance described by Hogarth's *False perspective* and the circumstance described by Conan Doyle's stories are genuine circumstances. According to this view, the only genuine circumstances are those in which no impossible events occur.
- ▶ The other strategy consists in conceding that *False perspective* and the stories of Sherlock Holmes describe genuine circumstances and assume that the principle of contradiction only applies to possible circumstances.
- ▶ A third option, advocated by Varzi for cases like the stories of Sherlock Holmes, is to assume that they describe genuine circumstances and adopt the *collective* formulation of the principle of non-contradiction in place of the distributive formulation.

The collective formulation of the principle

- ▶ Varzi (2004) suggests that, while 1 is a plausible claim, 2 is not:
 1. in the circumstance described by Conan Doyle's stories it is true that Watson's war wound is in the leg and it is also true that Watson's war wound is not in the leg;
 2. in the circumstance described by Conan Doyle's stories it is true that Watson's war wound is in the leg and is not in the leg;
- ▶ Varzi regards 1, but not 2, as a plausible claim since since the statements concerning the location of Watson's war wound are "pairs of inadvertently contradictory statements rather than blatantly self-contradictory conjunctions".
- ▶ For this reason, Varzi thinks that cases like Watson's war wound are good reasons to reject the distributive formulation of the principle of non-contradiction, and adopt the following *collective formulation* instead:

There is no circumstance in which a sentence of the form $\neg(A \text{ and } \neg A)$ is true.

A consequence

- ▶ Notice that, according to Varzi, the collective formulation of the principle of non-contradiction admits no exception.
- ▶ Since the incongruences of *False perspective* are not involuntary, but intentional, it would seem that *False perspective* describes a circumstance in which a contradictory conjunction is true:

(5) The woman at the window is far from the hill and is not far from the hill.
- ▶ Thus, it seems that Varzi must either concede that the collective formulation of the principle is not true or deny that *False perspective* describes a genuine circumstance.

Dialetheism

- ▶ The cases considered so far as possible objections to (some formulations of) the principle of non-contradiction are cases in which some contradictions are true in possible, but not actual, circumstances.
- ▶ Some philosophers claim however that some contradictions are true *in the real world*. This thesis is called *dialetheism*.
- ▶ Let's see some instances of contradictions that, according to these philosophers, are true in the real world.

The instant of change

... when I exit the room, I am inside the room at one time, and outside of it at another. Given the continuity of motion, there must be a precise instant in time, call it t , at which I leave the room. Am I inside the room or outside at time t ? Four answers are available: (a) I am inside; (b) I am outside; (c) I am both; and (d) I am neither. There is a strong intuition that (a) and (b) are ruled out by symmetry considerations: choosing either would be completely arbitrary. ... As for (d): if I am neither inside nor outside the room, then I am not inside and not-not inside; therefore, I am either inside and not inside (option (c)), or not inside and not-not inside (which follows from option (d)); in both cases, a dialethic situation [namely, a situation in which a sentence and its negation are both true]. (Priest, Berto, and Weber 2018).

Contradictory laws

Suppose, for instance, that some norm states that a marriage performed by the captain of a ship counts as a legal marriage only if the ship was in open water throughout the ceremony. It turns out, then, that some other law has established that such a marriage is valid also if the ceremony has only begun with the ship in open water, but has ended with the ship in the port. Then someone may turn out to be both a married man and not a married man. ... If one accepts the plausible view that statements concerning legal rights, obligations, and statuses, can be truth-value apt, we seem to have a dialetheia [namely, a sentence A such that both A and its negation are true]. (Priest, Berto, and Weber 2018).

The Liar

In its standard version, the Liar paradox arises by reasoning on the following sentence:

(1): (1) is false

where the number to the left is the name of the sentence to the right. As we can see, (1) refers to itself and tells us something about (1) itself. Its truth value? Let us reason by cases. Suppose (1) is true: then what it says is the case, so it is false. Then, suppose (1) is false: this is what it claims to be, so it is true. If we accept the ... Law of Bivalence, that is, the principle according to which all sentences are either true or false, both alternatives lead to a contradiction: (1) is both true and false, that is, a dialetheia, contrary to the Law on Non-Contradiction. (Priest, Berto, and Weber 2018).

The structure of the Liar

- ▶ Priest states the paradox of the Liar by means of the method of *proof by cases*:
given $\lceil A \text{ or } B \rceil$, if C follows from A and C follows from B, we may conclude C.
- ▶ The structure of the argument is this:
 1. Every sentence is true or false (Principle of Bivalence).
 2. If sentence M is true, then sentence M is false: (Premise)
M. Sentence M is false.
 3. If sentence M is false, then sentence M is true (Premise)
 4. **Show:** Sentence M is true and false (Direct Derivation)
 5. Sentence M is true or false (from 1 by Univ. Inst.)
 6. **Show:** If sentence M is true, then sentence M is true and false (Cond. proof)
 7. Sentence M is true (Assumption)
 8. Sentence M is false (from 2, 7 by *modus ponens*)
 9. Sentence M is true and false (from 7, 8 by conj. intro.)
 10. **Show:** If sentence M is false, then sentence M is true and false (Cond. proof)
 11. Sentence M is false (Assumption)
 12. Sentence M is true (from 3, 11 by *modus ponens*)
 13. Sentence M is true and false (from 11, 12 by conj. intro.)
- 14. Sentence M is true and false (from 5, 6, 10 by proof by cases).

Gluts and contradictions

- ▶ If we accept the previous proof, sentence M is a case of **truth value glut**), namely M is a sentence which is both true and false:

M. Sentence M is false.
- ▶ For Priest, **the thesis that there are true contradiction follows from the existence of sentences that are both true and false.**
- ▶ Here's Priest's reasoning:
If M is true and false, then M is true.
If M is true and false, then M is also false.
If M is false, then the negation of M is true.
Thus, if M is true and false, M is true and its negation is true.
Thus, "M and not M" (a contradiction) is true.

Dialetheism and the principle of non-contradiction

- ▶ **Nota bene:** according to Priest's reasoning, the contradiction "M and not M" (where M is the Liar sentence) is true. But it's also false! Indeed, if M is true, then the negation of M is false, thus "M and not M".
- ▶ Thus, for Priest **all contradictions are false**. But **some contradictions are also true**.
- ▶ This means that the following formulation of the principle of non-contradiction is acceptable also for the dialetheists:
In all possible circumstances, a sentence of the form $\lceil A \text{ and not } A \rceil$ is false.
- ▶ On the other hand, the following formulation of the principle of non-contradiction is false for the dialetheists:
there is no circumstance in which a sentence of the form $\lceil A \text{ and not } A \rceil$ is true.

Gluts and gaps

- ▶ According to Priest, in the proof that sentence M is true and false, there is no fallacy:

M. Sentence M is false.
- ▶ In other words, Priest *embraces* this conclusion: for him the Liar shows that there are true contradictions.
- ▶ Those philosophers who are not dialetheists, on the other hand, find the conclusion that M is true and false unacceptable. These philosophers reject some premise of the proof that sentence M is true and false.
- ▶ Several philosophers who are not dialetheists hold the view that M is neither true nor false, namely they regard M as a case of **truth value gap** (a similar solution has been proposed also for the other cases of true contradictions that Priest's proposed).
- ▶ Thus, the gap theorists reject the proof that sentence M is true and false, because, in their view, a premise of the proof is false: the principle of bivalence (by which every sentence is either true or false).
- ▶ For a comparison between the gap view and the glut view see Parsons (1990), Priest (1990), and McGee (2004).

Objections to dialetheism

- ▶ We are now going to examine some objections to dialetheism, the thesis that there are some true contradictions.
- ▶ We will see how Graham Priest, the main proponent of dialetheism, answers these objections.

The problem *ex contradictione quodlibet*

- ▶ An objection to the thesis that there are true contradictions (dialetheism) is this:
 - from a contradiction anything follows; thus whoever believes that there are true contradictions should believe anything.
- ▶ One may show that from a contradiction anything follows in this way:
 1. A and not A (Premise)
 2. Show: B (Direct Demonstration)
 3. A (from 1 by conjunction elimination)
 4. A or B (from 3 by disjunction introduction)
 5. not-A (from 1 by conjunction elimination)
 6. B (from 4-5, by disjunction elimination, also called disjunctive syllogism)

The dialetheist's reply

- ▶ The dialetheist's reply is articulated as follows:
 - the proof that from a contradiction anything follows assumes that the connectives "not", "and", "or" are interpreted as in classical logic;
 - if there are sentences that are both true and false, classical logic does not say how one should interpret complex sentences of which they are part;
 - given suitable assumptions about how to interpret these complex sentences, the proof that from a contradiction anything follows is fallacious.
- ▶ Let's examine each point of the dialetheist's reply in detail.

The classical interpretation of the connectives

- ▶ The classical interpretation of the connectives "not", "and", "or" is given by the following truth-tables:

A	B	(A and B)	A	B	(A or B)
T	T	T	T	T	T
T	F	F	T	F	T
F	T	F	F	T	T
F	F	F	F	F	F

A	not A
T	F
F	T

The dialetheist's interpretation of the connectives

Priest 1979

- ▶ According to the dialetheist, no sentence lacks a truth-value, but some sentences may have two truth-values (true and false).
- ▶ The semantics of the connectives “not”, “and”, “or” is the following:
 1. $\lceil \text{not-}A \rceil$ is true iff one of the truth values of A is the value false; $\lceil \text{not-}A \rceil$ is false iff one of the truth values of A is the value true.
 2. $\lceil A \text{ and } B \rceil$ is true iff one of the truth values of A is the value true and one of the truth values of B is the value true; $\lceil A \text{ and } B \rceil$ is false iff one of the truth values of A is the value false or one of the truth values of B is the value false.
 3. $\lceil A \text{ or } B \rceil$ is true iff one of the truth values of A is the value true or one of the truth values of B is the value true; $\lceil A \text{ or } B \rceil$ is false iff one of the truth values of A is the value false and one of the truth values of B is the value false.
- ▶ According to clauses 1-3, the classical interpretation of the connectives “not”, “and”, “or” is a *particular case* in which the sentences that make up complex sentences formed by these connectives have exactly one truth-value. For complex sentences of this sort, the dialetheist agrees with the classical logician.

Truth-tables for the dialetheist

A	B	$(A \text{ and } B)$	A	B	$(A \text{ or } B)$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	F	F	T	T
F	F	F	F	F	F
T, F	T, F	T, F	T, F	T, F	T, F
T, F	T	T, F	T, F	T	T
T, F	F	F	T, F	F	T, F
T	T, F	T, F	T	T, F	T
F	T, F	F	F	T, F	T, F

A	$\text{not-}A$
T	F
F	T
T, F	T, F

Definition of validity

- ▶ The dialetheist's definition of *valid argument* is the same as the classical logician's:

an argument from a set of premises Γ and conclusion A is valid iff any valuation that makes every premise in Γ true also makes A true.

A counter-model to disjunction elimination

- ▶ Now suppose that
 1. S is both true and false,
 2. Q is false (and not true).
- ▶ By the dialetheist's truth-tables it follows that
 - a. $\lceil S \text{ or } Q \rceil$ is true (and also false),
 - b. $\lceil \text{not } S \rceil$ is true (and also false),
 - c. Q is false (and not true).
- ▶ This shows that **disjunction elimination** (B follows from $\lceil A \text{ or } B \rceil$ and $\lceil \text{not } A \rceil$) is *not valid for the dialetheist*: the valuation in 1-2 makes $\lceil S \text{ or } Q \rceil$ and $\lceil \text{not } S \rceil$ true and Q false.

Rejecting *ex contradictione quodlibet*

- ▶ Thus, from the dialetheist's point of view, the proof that from a contradiction anything follows contains a fallacy.
- ▶ The dialetheist rejects step 6 of the proof, because it makes use of an invalid inference rule:
 1. A and not A (Premise)
 2. Show: B (Direct Demonstration)
 3. A (from 1 by conjunction elimination)
 4. A or B (from 3 by disjunction introduction)
 5. not-A (from 1 by conjunction elimination)
 6. B (from 4-5, by disjunction elimination)

The problem of content

- ▶ Another objection to the thesis that there are true contradictions is this:

... *contradictions have no content, no meaning. If so, then, a fortiori, they have no true content: contradictions cannot be true.* (Priest 1998)
- ▶ The problem with this objection is that different contradictions seem to say different things. Sentences (6) and (7) have different contents:
 - (6) Socrates is sitting and is not sitting.
 - (7) Plato is sitting and is not sitting.
- ▶ If contradictions have no content, how come that (6) and (7) seem to say different things?

Priest's reply

- ▶ Moreover, Priest observes:

If contradictions had no content, there would be nothing to disagree with when someone uttered one, which there (usually) is. Contradictions do, after all, have meaning. If they did not, we could not even understand someone who asserted a contradiction, and so evaluate what they say as false (or maybe true). We might not understand what could have brought a person to assert such a thing, but that is a different matter – and the same is equally true of someone who, in broad daylight, asserts the clearly meaningful “It is night”. (Priest 1998)

McTaggart's problem

- ▶ Another objection (attributed to McTaggart 1922) may be stated thus:

If contradictions were true, any assertion would lack meaning. Indeed, *an assertion has a meaning only if it excludes something (an assertion that excludes nothing says nothing).* Now, if contradictions are true, any assertion A fails to exclude $\lceil \text{not } A \rceil$. But, if A fails to exclude $\lceil \text{not } A \rceil$, it excludes nothing. Thus, if contradictions are true, any assertion would lack meaning.

First reply

- ▶ A first reply is this. The objection is based on the premise:
if contradictions are true, then any assertion A fails to exclude $\lceil \text{not } A \rceil$.
- ▶ One may understand this premise in two ways:
 1. If *all* contradictions are true, then any assertion A fails to exclude $\lceil \text{not } A \rceil$.
 2. If *some* contradictions are true, then any assertion A fails to exclude $\lceil \text{not } A \rceil$.
- ▶ If we understand it as in 2, the premise is clearly false, since the existence of some true contradictions leaves open the possibility that for some A , the contradiction $\lceil A \text{ and not } A \rceil$ is necessarily false, and in this case the assertion of A would exclude $\lceil \text{not } A \rceil$.
- ▶ If we understand it as in 1, the premise is true, but the dialetheist, as Priest points out, does not claim that all contradictions are true, but only that *some* are. Thus, the consequences of 1 cannot be blamed on the dialetheist.

Second reply

- ▶ Moreover, Priest argues that McTaggart's argument has an even more fundamental flaw:
The premiss that a proposition is not meaningful unless it rules something out is just plain false. Merely consider the claim "Everything is true". This rules nothing out: it entails everything. Yet it is quite meaningful (it is, after all, false). If you are in any doubt over this, merely consider its negation "Something is not true". This is clearly true—and so meaningful. And how could a meaningful sentence have a meaningless negation? (Priest 1998)

The problem of induction

- ▶ Here is a further objection considered by Priest (1998):
As we review the kinds of situations that we witness, very few of them would seem to be contradictory. Socrates is never both seated and not seated; Brisbane is firmly in Australia, and not not in it. Hence, by induction, no contradictions are true. (Priest 1998)
- ▶ Priest's reply is this:
The flaws of this argument are apparent enough, though. It is all too clear that the argument may be based on what Wittgenstein called "an inadequate diet of examples". Maybe Socrates is both sitting and not sitting sometimes: at the instant he rises. This, being instantaneous, is not something we observe. We can tell it to be so only by a-priori analysis. Worse, counter-examples to the principle are staring us in the face. Think, for example, of the Liar. Most would set an example such as this aside, and suppose there to be something wrong with it. But this may be short-sighted. (Priest 1998)

The problem of irrational belief

- ▶ Still another objection considered by Priest is based on the thesis that believing a contradiction would be irrational:
..... even if contradictions could be true, they can't be believed rationally, consistency being a constraint on rationality; hence one ought not to believe a contradiction since this would be irrational. (Priest 1998)
- ▶ Priest argues that *coherence is not an absolute requisite for rationality*.
- ▶ That coherence is not an absolute requisite for rationality is shown, according to Priest and others, by the so-called *Paradox of the preface*.

The paradox of the preface

Makinson 1965

It is customary for authors of academic books to include in their prefaces statements such as this: "I am indebted to ... for their invaluable help; however, any errors which remain are my sole responsibility".

Occasionally an author will go further. Rather than say that if there are any mistakes then he is responsible for them, he will say that there will inevitably be some mistakes and he is responsible for them. For example, in the preface to his Introduction to the Foundations of Mathematics (1952) R. L. Wilder writes

"To those of my colleagues and students who have given me encouragement and stimulation, I wish to express sincere thanks. I am especially grateful to ... for suggestions and criticisms; but the errors and shortcomings to be found herein are not their fault, and are present only in spite of their wise counsel".

Although the shouldering of all responsibility is usually a social ritual, the admission that errors exist is not—it is often a sincere avowal of belief. But this appears to present a living and everyday example of a situation which philosophers have commonly dismissed as absurd; that it is sometimes rational to hold logically incompatible beliefs..

The paradox of the preface (cont.)

Makinson 1965

Suppose that in the course of his book a writer makes a great many assertions, which we shall call s_1, \dots, s_n . Given each one of these, he believes that it is true. If he has already written other books, and received corrections from readers and reviewers, he may also believe that not everything he has written in his latest book is true. His approach is eminently rational; he has learnt from experience. The discovery of errors among statements which previously he believed to be true gives him good ground for believing that there are undetected errors in his latest book.

However, to say that not everything I assert in this book is true, is to say that at least one statement in this book is false. That is to say that at least one of s_1, \dots, s_n is false, where s_1, \dots, s_n are the statements in the book; that $(s_1 \& \dots \& s_n)$ is false; that $\sim(s_1 \& \dots \& s_n)$ is true. The author who writes and believes each of s_1, \dots, s_n and yet in a preface asserts and believes $\sim(s_1 \& \dots \& s_n)$ is, it appears, behaving very rationally. Yet clearly he is holding logically incompatible beliefs: he believes each of s_1, \dots, s_n , $\sim(s_1 \& \dots \& s_n)$, which form an inconsistent set. The man is being rational though inconsistent. More than this: he is being rational even though he believes each of a certain collection of statements, which he knows are logically incompatible.

The problem of rational criticism

- ▶ Another objection Priest discusses is this:

... if contradictions were acceptable, no one could be rationally criticized for the views that they hold. The thought here is that if you hold some view, and I object to it, there is nothing, rationally, to stop you maintaining both your original view and my objection. (Priest 1998).

- ▶ Priest replies that this objection presupposes, once again, that all contradictions are acceptable, while the dialetheist endorses only the weaker view that some contradictions are. When a contradiction $\lceil A \text{ and not } A \rceil$ is not acceptable, it is not acceptable to hold that A is true and at the same time hold that $\text{not } A$ is true:

The mere fact that some contradictions are rationally acceptable does not entail that all are. The charge "you accept some contradictions to be true, so why shouldn't you believe any contradiction to be so?" is as silly as the charge "you believe something to be true, so why shouldn't you believe anything to be so?". (Priest 1998)

How can one know?

- ▶ If only some, but not all, contradictions are true, how can one distinguish between true and false contradictions?
- ▶ According to Priest, there is no general criterion to determine which contradictions are acceptable and which are not.
- ▶ Generally speaking, Priest admits that it is very unlikely that a contradiction is true: if we have contradictory opinions, it is very likely that we are wrong.
- ▶ But in some cases, like the Liar, Priest claims that there is no better alternative to admitting that a contradiction is true.

The problem of disagreement

- ▶ The last objection considered by Priest is this:
... if contradictions were acceptable, then no one would have a way of denying anything: whenever they asserted $\sim\alpha$, this would not show that they rejected α , for they might accept both α and $\sim\alpha$. (Priest 1998)
- ▶ The problem of disagreement was raised by Parsons (1990) and discussed further by Shapiro (2004).

Parsons's formulation of the problem of disagreement

- ▶ Parsons (1990) presents the problem in this way:
Suppose that you say ' β ', and Priest replies ' $\sim\beta$ '. Under ordinary circumstances you would think that he had disagreed with you. But you remember that Priest is a dialetheist, and it occurs to you that he might very well agree with you after all—since he might think that ' β ' and ' $\sim\beta$ ' are both true. How can he indicate that he genuinely disagrees with you? The natural choice is for him to say ' β is not true'. However, the truth of this assertion is also consistent with β 's being true—for a dialetheist anyway ... (Parsons 1990)

Denial vs. asserting the negation

- ▶ Priest's (1998) reply is based on the distinction between *denying* a proposition and *asserting its negation*.
- ▶ *Denial* is a *speech act*, like asserting, ordering, asking.
- ▶ Often we deny something by asserting its negation. But denying a proposition is not the same thing as *asserting the negation* of that proposition.
- ▶ We may assert the negation of a proposition without denying that the proposition is true. The dialetheist that asserts the negation of the Liar's sentence ("sentence M is false") accepts at the same time that the sentence is true ("... but sentence M is also true").
- ▶ Moreover, we can deny something without asserting its negation. For example, we can deny that a proposition is true simply by means of a facial expression (much as we show by means of a facial expression our opinion that someone said something stupid).

Priest's solution to the problem of disagreement

- ▶ Let's now go back to the objection:
... if contradictions were acceptable, then no one would have a way of denying anything: whenever they asserted $\sim\alpha$, this would not show that they rejected α , for they might accept both α and $\sim\alpha$.
- ▶ Priest's reply to the objection is this:
 1. The dialetheist accepts that asserting $\sim\alpha$ is not the same as denying that α is true (indeed, the dialetheist admits the possibility that α and $\sim\alpha$ are both true);
 2. However, this does not mean that it is impossible to deny α : as we have seen, it is possible to deny something without asserting its negation.

Summing up

- ▶ We considered different formulations of the notion of contradiction and of the principle of non-contradiction.
- ▶ We saw some objections to the principle based on the idea that there are some non actual circumstances in which some contradictions are true.
- ▶ We examined the dialetheist's view, by which some contradictions are true (in the actual world).
- ▶ We saw how the dialetheists reply to some objections to their view.
- ▶ *Nota bene*: the aim of our discussion was not to resolve the issue whether the dialetheists are right or wrong, the aim was to show how the dialetheists can coherently argue for their view.
- ▶ As we saw, an alternative analysis of the Liar (and of other alleged true contradictions) has been proposed in the literature by rejecting the Principle of Bivalence.
- ▶ For a discussion of dialetheism and references to the relevant literature see Priest, Berto and Weber (2018).