Università degli Studi di Milano

McGee's counterexample to *modus ponens*

Sandro Zucchi

2022-2023

The scenario

- A week before the 1980 U.S. presidential elections, the opinion polls showed that Republican candidate Ronald Reagan was several points ahead of Jimmy Carter, the Democratic candidate.
- The other Republican in the race, John Anderson, was far behind in third position.
- In the end, Reagan won, Carter came second, Anderson a distant third.

Truth value judgement

- Those informed of the results of the opinion polls had good reasons to believe that (1)-(2) were true and that (3) was false:

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

(2) A Republican will win.

(3) If it's not Reagan who wins, it will be Anderson.

- Retrospectively, it seems correct to say that (1) was true, since only two Republicans, Reagan and Anderson, were in the race, and (2) was also true, since Reagan won in the end.
- But (3) was false, since if Reagan had not won, *Carter* would have won, because Anderson came far behind Carter.

Credits and consequences

- Example (1)-(3) is due to McGee (1985):

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

(2) A Republican will win.

(3) If it's not Reagan who wins, it will be Anderson.

- If (1) and (2) are true but (3) is false in McGee's scenario, *modus ponens* is not unrestrictedly valid (validity requires that there be no case in which the premises of the argument are true and the conclusion false).

modus ponens

1. *if* φ, *then* ψ
2. φ
3. $\therefore \psi$

- The example raises a problem for all major truth-conditional theories of conditionals, since classical analyses of conditionals as material conditionals (Grice), strict conditionals (C. I. Lewis) or minimal change conditionals (Stalnaker) all support *modus ponens*.

A reminder

- *Modus ponens* is a valid inference for material conditionals. Indeed, suppose premises 1-2 below are true. If ⌜$\varphi \supset \psi$⌝ is true, then either φ is false or ψ is true; given that φ is true, it follows that ψ is true.
  1. $\varphi \supset \psi$
  2. φ
  3. $\therefore \psi$
- *Modus ponens* is a valid inference for strict conditionals (assuming accessibility is reflexive). Indeed, suppose premises 4-5 below are true at a world w. If ⌜$\varphi \dashv\!\!\!\!3 \; \psi$⌝ is true at a world w, then at every accessible world w' either φ is false at w' or ψ is true at w'; so, if ⌜$\varphi \dashv\!\!\!\!3 \; \psi$⌝ is true at w, either φ is false at w or ψ is true at w; given that φ is true at w, it follows that ψ is true at w.
  4. $\varphi \dashv\!\!\!\!3 \; \psi$
  5. φ
  6. $\therefore \psi$
- *Modus ponens* is a valid inference for Stalnaker's conditionals. Indeed, suppose premises 7-8 below are true at a world w. If ⌜$\varphi > \psi$⌝ is true at w, ψ is true at the world minimally different from w in which φ is true; given that φ is true at w, the world minimally different from w in which φ is true is w itself. So, ψ is true at w.
  7. $\varphi > \psi$
  8. φ
  9. $\therefore \psi$

Different predictions

- Since the material analysis, the strict analysis, and the minimal change analysis of conditionals all support the validity of *modus ponens*, it follows that the case described by McGee raises a problem for all these analyses.
- However, these analyses make different predictions about the truth-values of the sentences that make up McGee's counterexample (in this sense, they run into different problems).
- Let's see what the different predictions are.

The predictions of the material analysis

- According to the view that indicative conditionals are material conditionals, (1) is true at the time when the opinion polls were taken. Indeed, the antecedent "A Republican will win" is true (Reagan was a Republican and won), and the consequent is an indicative conditional with a false antecedent, so it's also true.

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

(2) A Republican will win.

(3) If it's not Reagan who wins, it will be Anderson.

- Moreover, (2) is true, since Reagan was a Republican and won.
- Finally, the materialist view predicts that (3) is true, since it has a false antecedent.
- So, the materialist must claim that (1)-(3) are all true. The problem is that (3) seems to be false in McGee's scenario.

The predictions of the strict analysis

- If indicative conditionals are strict conditionals, (3) is false:

(3) If it's not Reagan who wins, it will be Anderson.

- Indeed, if (3) is a strict conditional, (3) has the form (3'):

(3') $\Box(\sim$*Reagan will win* $\supset$ *Anderson will win*$)$.

- Formula (3') is true iff at every world accessible from the real world it is true that either Reagan will win or Anderson will win. Since Carter might win, there are accessible worlds at which neither Reagan nor Anderson will win. Thus, (3) is false, if it is a strict conditional.
- Moreover, if indicative conditionals are strict conditionals, (1) is also false (unless we restrict the accessibility relation):

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

- Let's see why.

Explaining the prediction

- If indicative conditionals are strict conditionals, (1) has the form (1'):

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

(1') $\Box$(*A Rep. will win* $\supset \Box$($\sim$ *Reagan will win* $\supset$ *Anderson will win*)).

- Formula (1') is true iff at every world w accessible from the real world either no Republican will win or at every w' accessible from w either Reagan will win or Anderson will win.
- Again, it seems that there may be a possible world w accessible from the real world in which a Republican will win such that in some world w' accessible from w neither Reagan nor Anderson will win (for example, a world w' in which both Reagan and Anderson will drop out of the race and Carter will win).
- Thus, the strict conditional analysis predicts that premise (1) is false. The problem is that (1) appears to be true in McGee's scenario.

Stalnaker's abstract semantics

- Now, let's turn to the *abstract semantics* for conditionals proposed by Stalnaker (1968):

  ⌜*if* φ, *then* ψ⌝ is true in a world w just in case ψ is true in $f(\varphi, w)$, where f is a contextually determined function which selects the world in which φ is true which differs minimally from w.

- What prediction does Stalnaker's semantics make about the truth-value of sentences (1)-(3) that make up McGee's counterexample?

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

(2) A Republican will win.

(3) If it's not Reagan who wins, it will be Anderson.

Prediction about the second premise and conclusion

- Clearly, Stalnaker must accept that (2) is true, since Reagan will win and he is a Republican:

(2) A Republican will win.

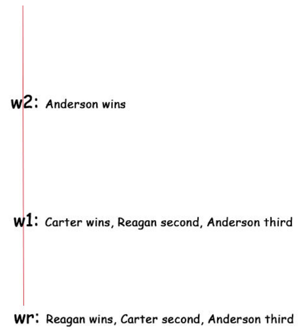
- Moreover, by Stalnaker's abstract semantics (3) is false:

(3) If it's not Reagan who wins, it will be Anderson.

- Indeed, in the world closest to the real world in which Reagan doesn't win, arguably, Anderson's position is unchanged. So, the world closest to the real world in which Reagan doesn't win is a world in which Carter wins. So, (3) is false.

In a picture

- The picture illustrates the similarity relation induced by McGee's scenario:



- Sentence (3) is false by Stalnaker's semantics, since the world closest to the real world wr in which Reagan doesn't win is $w1$.

(3) If it's not Reagan who wins, it will be Anderson.

Prediction about the first premise

- Moreover, according to Stalnaker abstract semantics, premise (1) is also false in the scenario described by McGee:

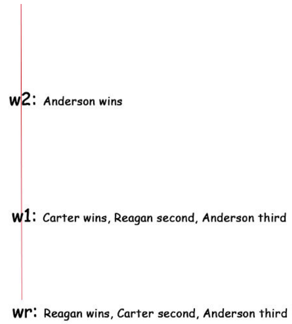
(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

- Premise (1) is false, since
  - (a) the world *w* minimally different from the real world in which a Republican wins is the real world, in which Reagan wins and Carter comes second,
  - (b) so the world minimally different from *w* in which Reagan doesn't win is a world in which Carter wins.
- The problem is that (1) seems to be true in McGee's scenario.

In a picture

- Sentence (1) is false, since the world closest to the real world *wr* in which a Republican wins is *wr* itself, and the world closest to *wr* in which Reagan doesn't win is *w*1.

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.



Rescue strategies

- We have seen that, *prima facie*, the material analysis, the strict analysis, and the minimal change analysis of conditionals all make incorrect predictions about the truth-values of (1)-(3):

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

(2) A Republican will win.

(3) If it's not Reagan who wins, it will be Anderson.

- Are there ways in which one may try to rescue these analyses? Let's consider some possibile strategies.

The pragmatic strategy

- The proponents of the view that indicative conditionals are material conditionals, as we saw, are committed to the claim that (1)-(3) are all true:

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

(2) A Republican will win.

(3) If it's not Reagan who wins, it will be Anderson.

- So, they must explain why (3), unlike (1) and (2), seems to be false.
- The strategy is to check for the assertability of (1)-(3).

Checking for robustness

premise (1)

- Let's assume, with Jackson and Lewis, that in order to be assertible conditionals must be robust with respect to their antecedent.
- If (1) is a material conditional, it's equivalent to (4):

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

(4) Either a Republican will not win the election or either Reagan or Anderson will.

- Given that the subjective probability that Reagan will win is high, the subjective probability of "either Reagan or Anderson will win" is high. Thus, the subjective probability of (4) is high.
- Moreover, if we learned that the antecedent of (1) is true, the subjective probability of (4) would remain high. Indeed, if we learn that a Republican will win, the subjective probability of "either Reagan or Anderson will win" would be high, since Reagan and Anderson are the only Republicans in the race.
- So, (1) is robust with respect to its antecedent.

Checking for robustness

conclusion (3)

- The conclusion (3) is equivalent to (5), if (3) is a material conditional:

(3) If it's not Reagan who wins, it will be Anderson.

(5) Either Reagan or Anderson will win.

- The subjective probability of (5) is high, given that the subjective probability that Reagan will win is high.
- However, if we learnt that the antecedent of (3) is true, the subjective probability of (5) would not remain high, since the subjective probability that Anderson will win is low.
- So, (3) is not robust with respect to its antecedent.

Rescuing the material analysis

- Given that (1) is robust with respect to its antecedent, (1) meets Jackson's condition for assertability:

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

- Moreover, (2) is also assertible, since it has a high subjective probability (Reagan is a Republican and the subjective probability that Reagan will win is high):

(2) A Republican will win.

- However, the conclusion (3) is not assertible, since it is not robust relative to its antecedent:

(3) If it's not Reagan who wins, it will be Anderson.

- So, the materialist may claim that (1), (2), and (3) are all true in McGee's scenario, but (3) seems false because it is not assertible.
- (So, the inference from (1)-(2) to (3) is valid, but it seems invalid because the premises are assertible, while the conclusion is not).

Rescuing the strict analysis

- The proponents of the view that indicative conditionals are strict conditionals, as we saw, are committed to the counterintuitive claim that (1) is false:

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

- A possible strategy to cope with this problem is to put suitable constraints on the accessibility relation.

How to constrain the accessibility relation

- More precisely, to solve the problem posed by (1), it may be suggested that *accessibility is updated as new suppositions are made*:

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

- Intuitively, the idea is this: when we evaluate (1), three types of worlds are initially accessible,
  - worlds in which Reagan will win,
  - worlds in which Carter will win,
  - worlds in which Anderson will win.
- After the supposition that a Republican will win is made, only worlds in which Reagan or Anderson will win stay accessible.
- So, when we evaluate the embedded strict conditional "$\Box$ (it's not Reagan who wins $\supset$ it will be Anderson)", the material conditional "it's not Reagan who wins $\supset$ it will be Anderson" is true in all the accessible worlds.
- A formal sketch of this proposal is given in Gillies (2010, 2012).

A consequence

- If the proposal we sketched is viable, the modified strict analysis can account for the truth of (1):

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

- Notice that now *modus ponens* is predicted to be invalid, since (1) and (2) are true, but (3) is false:

(2) A Republican will win.

(3) If it's not Reagan who wins, it will be Anderson.

- Sentence (3) remains false in the modified strict analysis, since among the worlds that are initially accessible, there are worlds in which Carter will win, thus there are some accessible worlds in which neither Reagan nor Anderson win.

How can the minimal change analysis be rescued?

- Let's now turn to Stalnaker's analysis. Sentence (1) poses a problem, since the analysis incorrectly predicts that (1) is false:

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

- However, in discussing Stalnaker's prediction about (1), we only considered his abstract semantics for conditionals, namely the core semantics which is common to both indicative and counterfactual conditionals, and we ignored the further constraint that Stalnaker imposes on the selection function for indicative conditionals.
- One might think that, in order to cope with (1), one should somehow amend Stalnaker's semantics for indicative conditionals.
- In fact, there are reasons to believe that this is not the right strategy.

A similar problem with subjunctive conditionals

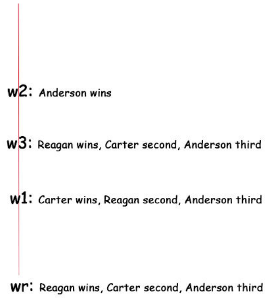
- Imagine that one utters (6) *after* the 1980 elections:

(6) If Reagan hadn't won the election, then if a Republican had won, it would have been Anderson.

- Again, it seems that (6) is true, since Reagan and Anderson were the only Republican candidates.
- Yet, as McGee points out, (6) is false by Stalnaker's theory. The reason is that the world *w* minimally different from the real world in which Reagan did not win is a world in which Carter won and Reagan came second. So, the world minimally different from *w* in which a Republican won is one in which Reagan won.

In a picture

- The picture illustrates the similarity relation induced by McGee's scenario:



- Sentence (6) is false, since the world closest to the real world w_r in which a Reagan didn't win is $w1$, but the world closest to w_1 in which a Republican won is $w3$.

(6) If Reagan hadn't won the election, then if a Republican had won, it would have been Anderson.

Stalnaker theory and nested conditionals

- It seems that Stalnaker theory has a problem with nested conditionals of the form ⌜if A, then if B then C⌝, *whether they are indicative or subjunctive*:

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

(6) If Reagan hadn't won the election, then if a Republican had won, it would have been Anderson.

- Why is that?

The root of the problem

- The root of the problem is this: for Stalnaker to determine whether ⌜if A, then if B then C⌝ is true, first we must move to the closest world w in which A is true, then we must move to the world w' closest to w in which B is true. But w' may not be a world in which A is true. This is why (1) and (6) are predicted to be false.

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

(6) If Reagan hadn't won the election, then if a Republican had won, it would have been Anderson.

- Indeed, to determine whether (1) is true, first we must move to the world w closest to the real world in which a Republican will win, then we must move to the world w' closest to w in which Reagan will not win, but w' is not a world in which a Republican will win. Thus, (1) is false.
- Similarly, to determine whether (6) is true, first we must move to the world w closest to the real world in which Reagan didn't win, then we must move to the world w' closest to w in which a Republican won, but w' is not a world in which Reagan didn't win. Thus, (6) is false.

An intuitive equivalence

- Notice that (7)-(8) are correctly predicted to be true by Stalnaker's semantics:

(7) If a Republican wins the election and it's not Reagan who wins, it will be Anderson.

(8) If Reagan hadn't won the election and a Republican had won, it would have been Anderson.

- Indeed, (7) is true since the world closest to the real world in which Republican will win and it's not Reagan is a world in which Anderson will win. And (8) is true since the world closest to the real world in which Reagan didn't win and a Republican won is a world in which Anderson won.
- Intuitively, (7) and (8) are equivalent to (1) and (6). The problem is that Stalnaker semantics fails to support this equivalence.

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

(6) If Reagan hadn't won the election, then if a Republican had won, it would have been Anderson.

The Import-Export Law

- Let's call the thesis that (a) and (b) are necessarily equivalent *the Import-Export law* (the name is from Gillies 2009 and Kaufmann & Kaufmann 2015):
  (a) if A, then if B, then C
  (b) if A and B, then C
- *Prima facie*, the Import-Export law seems to hold for natural language conditionals, whether they are indicatives or counterfactuals.
- However, as we have just seen, Stalnaker's abstract semantics fails to validate this law.
- McGee (1985) suggests a way of modifying Stalnaker's semantics in order to validate Import-Export. Before looking a McGee's proposal, however, let's point out a natural consequence of this move.

The Import-Export Law and *modus ponens*

- Suppose we modify Stalnaker's core semantics for conditionals in order to validate the Import-Export law and capture the equivalence of (7) and (8) with (1) and (6):

(7) If a Republican wins the election and it's not Reagan who wins, it will be Anderson.

(8) If Reagan hadn't won the election and a Republican had won, it would have been Anderson.

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

(6) If Reagan hadn't won the election, then if a Republican had won, it would have been Anderson.

- A natural consequence of this move is that we should reject *modus ponens*. Let's see why.

StrImp

- Consider the following principle, called StrImp:

  StrImp. If φ logically implies ψ, then the indicative conditional $\ulcorner$if φ, then $\psi\urcorner$ is true.
- StrImp says that an indicative conditional is true if its antecedent logically implies its consequent.
- For example, "it is not the case that the butler or the gardener did it" logically implies that the butler didn't do it. By StrImp we can conclude that the conditional "If it is not the case that the butler or the gardener did it, then the butler didn't do it" is true.
- It is hard to imagine how StrImp could fail.

A dilemma for Stalnaker

- Now, Stalnaker claims that indicative conditionals are not material conditionals.
- We also saw that indicative nested conditionals seem to obey the Import-Export law (ImpExp, for short):

  Imp-Exp. (a) and (b) are logically equivalent:
  (a) if φ, then if χ, then ψ
  (b) if φ and χ, then ψ
- And we saw that it is reasonable to assume StrImp:

  StrImp. If φ logically implies ψ, then the indicative conditional $\ulcorner$if φ, then $\psi\urcorner$ is true.
- The problem for Stalnaker, as McGee points out, is that in the present setting if we assume ImpExp, StrImp, and *modus ponens*, indicative conditionals are logically equivalent to material conditionals. Let's see why.

From indicatives to ⊃

- Suppose indicative conditionals are represented by the connective $\Rightarrow$.Then, 3 follows from 1-2:
  1. ImpExp, StrImp, *modus ponens* hold for $\Rightarrow$.
  2. $\ulcorner\varphi \Rightarrow \psi\urcorner$ is true.
  3. $\ulcorner\varphi \supset \psi\urcorner$ is true.
- Indeed, suppose $\ulcorner\varphi \supset \psi\urcorner$ is false. Then, φ is true and ψ is false. But, if φ is true, then by 2 and *modus ponens* for $\Rightarrow$, we conclude ψ is true, which contradicts the assumption that ψ is false.

From ⊃ to indicatives

1. ImpExp, StrImp, *modus ponens* hold for $\Rightarrow$.
2. $\ulcorner\varphi \supset \psi\urcorner$ is true.
3. Show: $\ulcorner\varphi \Rightarrow \psi\urcorner$ is true.
   4. $\ulcorner(\varphi \supset \psi) \wedge \varphi\urcorner$ logically implies ψ (indeed, if $\ulcorner(\varphi \supset \psi) \wedge \varphi\urcorner$ is true, then $\ulcorner(\varphi \supset \psi)\urcorner$ and φ are both true and, since *modus ponens* holds for $\supset$, it follows that ψ is true).
   5. Thus, $\ulcorner((\varphi \supset \psi) \wedge \varphi) \Rightarrow \psi\urcorner$ is true (by StrImp and 4).
   6. Thus, $\ulcorner(\varphi \supset \psi) \Rightarrow (\varphi \Rightarrow \psi)\urcorner$ is true (by ImpExp and 5).
   7. Thus, $\ulcorner\varphi \Rightarrow \psi\urcorner$ is true (by *modus ponens* for $\Rightarrow$, 2 and 6).

A consequence for Stalnaker's theory

- So, if we want to retain Stalnaker's claim that indicative conditionals are not material conditionals, then for indicative conditionals we have to give up on StrImp, or on ImpExp, or on *modus ponens*.
- But ImpExp seems to be valid and it is reasonable to accept StrImp.
- On the other hand, McGee's counterexample *prima facie* shows that *modus ponens* is not unrestrictedly valid.
- So, if we want retain to Stalnaker's claim that indicative conditionals are not material conditionals, the natural move is modify Stalnaker's semantics to validate ImpExp and let *modus ponens* fail.
- McGee's presents a modification of Stalnaker's semantic which achieves exactly this result.

McGee's semantics

informal description

- The central idea of McGee's semantics is that truth is relative to a world and *a set of hypotheses* (formulae).
- Antecedents of conditionals introduce hypotheses in this set.
- To check whether a nested conditional $\ulcorner\varphi > (\psi > \xi)\urcorner$ is true at world *w* relative to a set of hypotheses Γ, we must add the antecedent φ to Γ and then check whether $\ulcorner\psi > \xi\urcorner$ is true in the world closest to *w* in which all hypotheses in Γ are true.

A natural consequence

- By the informal description of McGee's semantics, we can see how (1) and (6) are now predicted to be true:

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

(6) If Reagan hadn't won the election, then if a Republican had won, it would have been Anderson.

- To check whether (1) is true we must move to the world w closest to the real world in which a Republican will win. Since in the real world Reagan will win and is a Republican, w is the real world. Now, we must insert the antecedent of (1) ("a Republican wins the election") in the set of hypotheses and move to the world w' closest to w (i.e. closest to the real world) in which Reagan will not win and the hypothesis that a Republican wins the election is also true. Since in the real world there are only two candidates in the race, the world closest to it in which Reagan will not win and a Republican wins the election is a world in which Anderson wins. Thus, (1) is true.
- With a similar reasoning, we may also conclude that (6) is true.

McGee's semantics

formal description

Let F be a function that assigns a truth value at a world to the atomic sentences of the language. The function $[\![\ ]\!]$, which assigns truth values to sentences of arbitrary complexity relative to a world w and a set of hypotheses Γ, is now defined thus:

- if there is no world (accessible from w) at which every member of Γ is true, $[\![\varphi]\!]^{w,\Gamma} = 1$, for any sentence φ;
- if there is some world (accessible from w) at which every member of Γ is true, then
  1. if φ is atomic, $[\![\varphi]\!]^{w,\Gamma} = 1$ iff $F(\varphi)(w') = 1$, where w' is the world most similar to w at which all members of Γ are true,
  2. $[\![\sim\varphi]\!]^{w,\Gamma} = 1$ iff $[\![\varphi]\!]^{w,\Gamma} = 0$,
  3. $[\![\varphi \vee \psi]\!]^{w,\Gamma} = 1$ iff it not the case that: $[\![\varphi]\!]^{w,\Gamma} = 0$ and $[\![\varphi]\!]^{w,\Gamma} = 0$,
  4. $[\![\varphi \wedge \psi]\!]^{w,\Gamma} = 1$ iff $[\![\varphi]\!]^{w,\Gamma} = 1$ and $[\![\varphi]\!]^{w,\Gamma} = 1$,
  5. $[\![\varphi > \psi]\!]^{w,\Gamma} = 1$ iff $[\![\psi]\!]^{w,\Gamma\cup\{\varphi\}} = 1$.

Truth at a world

Truth at a world *simpliciter* is defined in terms of truth relative to a world and a set of hypotheses:

- a sentence φ is true at w iff $[\![\varphi]\!]^{w,\Gamma} = 1$ where $\Gamma = \oslash$.

Empty set of hypotheses

- Now, consider again clause 1:
  1. if φ is atomic, $[\![\varphi]\!]^{w,\Gamma} = 1$ iff $F(\varphi)(w') = 1$, where w' is the world most similar to w at which all members of Γ are true,
- What happens when $\Gamma = \oslash$?
- When $\Gamma = \oslash$, clause 1 reduces to the condition that $[\![\varphi]\!]^{w,\Gamma} = 1$ iff $F(\varphi)(w') = 1$, where w' is the world closest to w. Since w is the world closest to w, it follows that an atomic sentence which does not occur as part of a more complex sentence is true at w iff $F(\varphi)(w) = 1$.
- Things change when we evaluate a conditional $\ulcorner\varphi > \psi\urcorner$ at a world w. In this case, $\ulcorner\varphi > \psi\urcorner$ is true at w iff the conditional is true at w relative to the empty set iff (by clause 5) ψ is true at w relative to $\{\varphi\}$.

An example

- Let's assume that conditional (1) is of the form $\ulcorner\varphi > (\chi > \xi)\urcorner$:

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

- According to the definition of truth at a world, (1) is true at w iff $[\![(1)]\!]^{w,\varnothing} = 1$
  iff $[\![\sim\textit{Reagan wins} > \textit{Anderson wins}]\!]^{w,\{\textit{a Rep. wins}\}} = 1$
  iff $[\![\textit{Anderson wins}]\!]^{w,\{\textit{a Rep. wins}, \sim\textit{R. wins}\}} = 1$
  iff Anderson wins the election in the possible world closest to w in which a Republican wins and it's not Reagan.
- Given that the real world is one in which Reagan and Anderson are the only Republican candidates, the world closest to it in which Reagan doesn't win but a Republican does is a world in which Anderson wins.
- Thus, McGee's semantics correctly predicts that (1) is true in the scenario he described.

Back to the troubling subjunctive

- For the same reason, McGee's semantics correctly predicts that (6) is true:

(6) If Reagan hadn't won the election, then if a Republican had won, it would have been Anderson.

- Given that the real world is one in which Reagan and Anderson are the only Republican candidates, the world closest to it in which Reagan doesn't win but a Republican does is a world in which Anderson wins.
- Again, in this semantics *modus ponens* fails. The reason is that, while the embedded conditional in (6) is evaluated relative to a set which includes the assumption that Reagan didn't win, the same conditional in the conclusion (9) is not.

(9) If a Republican had won, it would have been Anderson.

Summary

- We presented McGee's counterexample to *modus ponens*.
- The counterexample raises a problem for all the analyses of conditionals we have considered: the material analysis, the strict analysis, and the minimal change analysis.
- We examined some ways in which each analysis can cope with McGee's counterexample.

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